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#### Editor's Choice

# Electronic structure, anisotropic elastic and thermal properties of the $\eta$ phase Fe<sub>6</sub>W<sub>6</sub>C



## XiaoYu Chong<sup>a</sup>, YeHua Jiang<sup>a,\*</sup>, Rong Zhou<sup>a</sup>, Hong Zhu<sup>b</sup>, Jing Feng<sup>a,c,\*</sup>

<sup>a</sup> Faculty of Material Science and Engineering, Kunming University of Science and Technology, Kunming 650093, People's Republic of China <sup>b</sup> Department of Materials Science and Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139, USA <sup>c</sup> School of Engineering and Applied Sciences, Harvard University, Cambridge, MA 02138, USA

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#### 1. Introduction

The tungsten-containing  $\eta$ -carbides Fe<sub>3</sub>W<sub>3</sub>C, Co<sub>3</sub>W<sub>3</sub>C, Co<sub>6</sub>W<sub>6</sub>C and Fe<sub>6</sub>W<sub>6</sub>C, etc. possess interesting properties as refractory and hard materials, and recently as potential catalysts. These phases could occur at the interfaces in heterogeneous composite materials, i.e., the interfaces between WC and transition metals (or their alloys) or tungsten-containing steels as secondary phases [1,2]. These compounds can also be prepared using special synthetic routes such as mechanical alloying [3]. Among the several phases, Fe<sub>6</sub>W<sub>6</sub>C is an important one and has attracted much attention to be investigated. Waki et al. systematically studied electronic properties of the  $\eta$ -carbide-type compounds, synthesized the Fe<sub>6</sub>W<sub>6</sub>C compounds and measured its magnetic susceptibility [4]. Suetin et al. investigated the structural, electronic, magnetic properties and stability of  $\eta$  carbides by first-principles FLAPW-GGA calculations [5]. Liu et al. explored the stability, electronic and mechanical properties of Fe–W–C system [6]. But as a common phase in the interface region of WC/Fe composites, a good understanding on thermodynamic and elastic properties of Fe<sub>6</sub>W<sub>6</sub>C is important to

#### ABSTRACT

The optimized structure, chemical bonding characteristics, elastic and thermal properties of  $Fe_6W_6C$  are investigated by the first principle calculations with and without dispersion-corrected methods combined with the quasi-harmonic approximation. The bonding behaviors of  $Fe_6W_6C$  are discussed by the density of states and Mulliken population analysis. Anisotropy of shear and Young's moduli are characterized by three-dimensional surface contours and the planar projections on different planes. Anisotropy of the minimum thermal conductivity of  $Fe_6W_6C$  is discussed based on Cahill's model and Clarke's model and the values are 1.38 and 1.26 W m<sup>-1</sup> K<sup>-1</sup> predicted by these two models. Moreover, the 3D representation of the anisotropic thermal conductivity of  $Fe_6W_6C$  is obtained based on the Clarke's model and anisotropic Young's modulus.

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better control and improve the properties of the composites. In this paper, both the density functional theory and quasi-harmonic approximation are employed to obtain the anisotropic elastic and thermal properties of  $Fe_6W_6C$ .

#### 2. Methods and details

The first-principles calculations based on the density functional theory (DFT) are implemented in Cambridge Serial Total Energy Package (CASTEP) code, with a plane-wave cutoff energy of 400 eV and a  $6 \times 6 \times 6$  Monkhorst–Pack *k*-point grid. Ultra-soft pseudo potentials (USPPs) are used to represent the interactions between ionic cores and valence electrons. The Broydene-Fletche re-Goldarbe-Shanno (BFGS) method is applied to optimize the crystal structure until the total energy changes are converged to  $1 \times 10^{-6}$  eV and the forces per atom are less than 0.02 eV/Å [7]. In this work, generalized gradient approximation (GGA) within two different functional is used for exchange-correlation energy calculations. One is the Perdew, Burke and Ernzerhof approach [8] and the other one is the Perdew and Wang parameterization (PW91) combined with the OBS method for the calculations of dispersion interactions such as the Van Der Waals (VDW) interactions [9]. The elastic properties are determined using the stress-strain relations by deforming the unit cell. The density functional theory (DFT) method combined with the quasi-harmonic approximation (QHA) and Debye model is used to calculate the thermodynamic



<sup>\*</sup> Corresponding authors at: Faculty of Material Science and Engineering, Kunming University of Science and Technology, Kunming 650093, People's Republic of China (Y. Jiang), School of Engineering and Applied Sciences, Harvard University, Cambridge, MA 02138, USA. Tel.: +1 6174964295; fax: +1 857 259 2445 (J. Feng).

*E-mail addresses: jiangyehua@kmust.edu.cn* (Y. Jiang), jfeng@seas.harvard.edu (J. Feng).



**Fig. 1.** Crystal structure of Fe<sub>6</sub>W<sub>6</sub>C. The large orange ball, medium green ball and small blue ball represent the tungsten atom, carbon atom and iron atom, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

properties of  $Fe_6W_6C$  at finite temperatures. The studied crystal structure of  $Fe_6W_6C$  is shown in Fig. 1. The type of lattice of  $Fe_6W_6C$  belongs to the complex cubic structure and the space group is FD–3M. The unit cell contains 48 iron atoms, 48 tungsten atoms and 8 carbon atoms, in which the C atoms occupy the octahedral interstice of W atoms and the strong W–C bonds surround the Fe atoms.

#### 3. Results and discussions

#### 3.1. Equilibrium crystal parameters and mechanical modulus

The calculated equilibrium lattice parameters are shown in Table 1, together with the experimental and other theoretical results [5,10]. From Table 1, it is clearly seen that the calculated results are close to experimental ones. The lattice constants calculated using PW91 functional combined with the dispersion-corrected method are a little smaller than the results obtained from other methods. The tiny difference between experimental and theoretical results can be attributed to the thermodynamic effects on the crystal and the lattice defect. The obtained elastic constants and mechanical modulus evaluated within Viogt-Reuss-Hill approximation are summarized in Table 2. It is obvious that the mechanical parameters estimated using PW91 functional and OBS method are larger than that obtained with PBE functional. Furthermore, the bulk (B) and shear modulus (G) of  $Fe_6W_6C$  are all smaller than that of h-WC (393.0 and 286.2 GPa), but equivalent to h-W<sub>2</sub>C (330.6 and 190.6 GPa) [11]. The value of B/G and Poisson's ratio ( $\sigma$ ) are calculated within the common expression [12], which are all larger than the critical value (1.75 and 0.26), indicating Fe<sub>6</sub>W<sub>6</sub>C is more ductile than other transition metal carbides ceramic such as h-WC(1.37 and 0.21) [11], VC (1.53 and 0.23) [12] and TiC (1.22 and 0.18) [13]. The transverse and longitudinal sound velocities ( $v_t$  and  $v_l$ ), mean sound velocity ( $v_m$ ), Debye temperature ( $\Theta_D$ ) are also evaluated in this work using the following relations in order to calculate the thermal conductivity in the next part.

$$\Theta_D = \frac{h}{k_B} \left[ \frac{3n}{4\pi} \left( \frac{N_A \rho}{M} \right) \right]^{1/3} v_m \tag{1}$$

$$\nu_m = \left[\frac{1}{3}\left(\frac{2}{\nu_t^3} + \frac{1}{\nu_l^3}\right)\right]^{-1/3} \tag{2}$$

$$\nu_l = \sqrt{\frac{(B + (4/3)G)}{\rho}} \tag{3}$$

$$v_t = \sqrt{\frac{G}{\rho}} \tag{4}$$

here *h* is the Planck's constant,  $k_B$  is the Boltzmann constant,  $N_A$  is the Avagadro's constant, *n* is the number of atoms per formula, *M* is the molecular weight,  $\rho$  is the theoretical density of the compound. The calculated results are shown in Table 2. Moreover, the lattice parameters and bulk modulus as the function of temperature are obtained from the fitting of the Birch–Murnaghan equation of state [14] and QHA approximation, which is presented in Fig. 2. The effects of thermal energies on the lattice parameter and bulk modulus are caused mainly by lattice vibrations or phonons. With the temperature increasing from 0 K to 1200 K, the lattice parameter rises from 10.871 to 10.886 Å and the bulk modulus decreases from 327.8 to 326.7 GPa, suggesting the strong thermal stability for Fe<sub>6</sub>W<sub>6</sub>C.

#### 3.2. Heat capacity and thermal expansion coefficient

In order to study the thermal properties of Fe<sub>6</sub>W<sub>6</sub>C, the volume dependence of total energy is obtained by the DFT calculations. The heat capacity at constant pressure ( $C_p$ ) can be achieved by analyzing the phonon frequencies of the crystal structure and the Debye model is applied to evaluate the heat capacity at constant volume ( $C_V$ ) [15]. Then the calculated difference between the two heat capacities can be correlated to thermal expansion and temperature through  $C_P(T) - C_V(T) = \beta^2 V(T)TB_0(T)$  [16], where  $\beta$  is volumetric thermal expansion coefficient; V(T) is the equilibrium cell volume at temperature T obtained from lattice parameter and  $B_0(T)$  is the isothermal bulk modulus which have been calculated in the previous part. For cubic crystal class, the linear expansion coefficient ( $\alpha$ )

Table 1

Lattice parameters of the studied  $Fe_6W_6C$  structure determined both by experiment and theoretical calculation.

Method	Lattice con	istants (Å)	Lattice a	angles (°)		$\rho$ (g/cm <sup>3</sup> )	$V(Å^3)$	Reference	
	a	b	с	α	β	γ			
GGA-PW91 + OBS	10.81	10.81	10.81	90	90	90	15.24	1264.5	This work
GGA-PBE	10.87	10.87	10.87	90	90	90	15.00	1284.0	This work
EOS	10.87	10.87	10.87	90	90	90	15.00	1284.7	This work
FLAPW-GGA-PBE	10.90	10.90	10.90	90	90	90	14.87	1295.2	[5]
Experiment	10.93	10.93	10.93	90	90	90	14.73	1307.2	[11]

 $\label{eq:alpha} \begin{array}{l} \textbf{Table 2} \\ \textbf{Elastic constants (GPa), mechanical modulus (GPa), Poisson's ratio, acoustic velocities and Debye temperature for Fe_6W_6C. \end{array}$ 

Method	C <sub>ij</sub>			В	G	Е	B/G	σ	$v_l$	$v_t$	$v_m$	$\Theta_D$
	C <sub>11</sub>	C44	C <sub>12</sub>									
GGA-PW91+OBS GGA-PBE EOS	609.6 593.5	173.7 168.0	204.8 203.9	339.8 333.8 327.8	184.7 178.3	469.0 454.0	1.84 1.87	0.27 0.27	6201 6172	3481 3447	3874 3838	501.5 494.2



Fig. 2. Temperature dependence of lattice parameter and bulk modulus of Fe<sub>6</sub>W<sub>6</sub>C.

and volumetric coefficient are related by  $\beta = 3\alpha$  [16]. All the results can be found in Fig. 3. From Fig. 3(b) and (c), we can clearly find that the  $C_V$  and  $C_p$  of Fe<sub>6</sub>W<sub>6</sub>C increase sharply below the Debye temperature (501.5 K), but above the Debye temperature,  $C_V$  and  $C_p$  increase linearly as a function of temperature and the  $C_V$  approaches to a constant which is well described by the classic Dulong–Petti rule, i.e., 3NR, at very high temperature. For Fe<sub>6</sub>W<sub>6</sub>C, the value is 324.25 J mol<sup>-1</sup> K<sup>-1</sup>.

The thermal expansion characterizes the anharmonic lattice vibrations of a crystal. The calculated linear expansion coefficient ( $\alpha$ ) is shown in Fig. 3(d), which exhibits similar variation tendency as  $C_V$  and  $C_p$ . The propensity of increment of  $\alpha$  becomes very moderate at high temperature, which means that  $\alpha$  changes slowly and increases linearly with respect to temperature. For Fe<sub>6</sub>W<sub>6</sub>C, the  $\alpha$  value approach 7.53  $\times$  10<sup>-6</sup> K<sup>-1</sup> at 1200 K, indicating that it is thermal expansion coefficient is low compared with the steel and iron materials.



**Fig. 3.** The curve of the volume dependence of total energy (a); the temperature dependence of  $C_V$ ,  $C_p$  and  $\alpha$  (b)–(d).



Fig. 4. The partial density of states (PDOS) for  $Fe_6W_6C$ . The dash line represents the Fermi energy.

#### 3.3. Electronic structure

In order to reveal the electronic origin of the calculated properties of Fe<sub>6</sub>W<sub>6</sub>C, the density of states (DOS) and electron density distribution maps are calculated and plotted in Figs. 4 and 5, respectively. From Fig. 4, we can clearly see that  $Fe_6W_6C$  is an insulator. The d bands of metal atoms dominate the Fermi level. Furthermore, the 3d band of Fe and 5d band of W are overlapped with the 2p band of C near Fermi level, implying the covalent interactions between the C and metal atoms by p-d hybridization and the metallic bonding between metal atoms. So the bonding behaviors of Fe<sub>6</sub>W<sub>6</sub>C are the combinations of covalent and metallic bonds. Another evidence can be found in Fig. 5(a), in which the core regions for all the W, Fe and C have large values and smaller in interstitial area. Covalent bond features can be seen clearly in the picture between W and C atoms, and interactions of metallic bonds between Fe and W are also evidence. More details can be found from electron density difference distribution in Fig. 5(b). Some electrons are delocalized and distributed through the matrix map in the area between Fe and W atoms, suggesting the characteristics of metallic bonding. The electrons in the blue color area are effectively localized around the C atoms and strong polarized covalent bonds can be concluded between W and C atoms. The large mechanical modulus and low thermal expansion coefficient can be attributed to the strong bonding between W and C atoms.

The Mulliken population analysis on the chemical bonds of  $Fe_6W_6C$  is conducted by DFT calculations with the GGA-PBE functional, as shown in Fig. 6. The Mulliken populations of W–C bonds are higher than those of other bonds, indicating high level of covalency of bonds. There is only one type C–W bond in CW<sub>6</sub> octahedron, while two different bond lengths and Mulliken populations exist in Fe–Fe and Fe–W bonds. Moreover, the bond lengths as well as the Mulliken population of C–W bonds are quite different from those of Fe–W and Fe–Fe bonds, which demonstrate anisotropic chemical bonding nature of Fe<sub>6</sub>W<sub>6</sub>C. The lengths of metallic bonds are longer and the Mulliken populations are smaller than those of C–W bonds. In general, Fe<sub>6</sub>W<sub>6</sub>C show heterogeneous bonding nature and complex structures. These structural features play an essential role in the properties and performances of these two materials.

#### 3.4. Anisotropy of mechanical modulus and thermal conductivity

The calculated elastic compliance constants and anisotropic index are summarized in Table 3. For the universal anisotropic index ( $A^{U}$ ) and percent anisotropic index ( $A_{B}$  and  $A_{G}$ ), the large

discrepancies from zero refer to the highly mechanical anisotropic properties and the values of  $A_1$ ,  $A_2$  and  $A_3$  should be one for an isotropic crystal [17]. From the  $A_B$  value we can conclude that the bulk modulus is isotropic for Fe<sub>6</sub>W<sub>6</sub>C. But the Young's modulus is more dependent on the direction which is confirmed by  $A_B$  and  $A_G$  values.  $A_1$ ,  $A_2$  and  $A_3$  values are 0.86 for Fe<sub>6</sub>W<sub>6</sub>C, indicating that Fe<sub>6</sub>W<sub>6</sub>C has weak anisotropy of shear modulus.

Another more intuitive and simple way to describe the anisotropic behavior of elastic properties for  $Fe_6W_6C$  is to plot the three-dimensional (3D) surface contour for mechanical modulus in spherical coordinates as a function of the crystallographic orientation. For the shear modulus, the shear stress direction and the shear plane must be considered in the 3D coordinate, which make it difficult to draw the 3D anisotropic picture of shear modulus. In this paper, the anisotropy of torsion shear modulus for  $Fe_6W_6C$  ( $G_T$ ) is discussed to solve the problem. The directional dependence of torsion shear modulus and Young's modulus is given by [18,19]:

$$\frac{1}{G_T} = S_{44} + 4 \left[ (S_{11} - S_{12}) - \frac{1}{2} S_{44} \right] (l_1^2 l_2^2 + l_2^2 l_3^2 + l_1^2 l_3^2)$$
(5)

$$\frac{1}{E} = l_1^4 S_{11} + l_2^4 S_{22} + l_3^4 S_{33} + 2l_1^2 l_2^2 S_{12} + 2l_1^2 l_3^2 S_{13} + 2l_2^2 l_3^2 S_{23} + l_1^2 l_2^2 S_{66} + l_1^2 l_3^2 S_{55} + l_2^2 l_3^2 S_{44}$$
(6)

where  $S_{ij}$  is the elastic compliance constants and  $l_1$ ,  $l_2$  and  $l_3$  are the directional cosines ( $l_1 = \sin\theta\cos\varphi$ ,  $l_2 = \sin\theta\sin\varphi$ ,  $l_3 = \cos\varphi$ ). The obtained contour images are shown in Fig. 6(a) and (b). For an isotropic crystal, a sphere is anticipated. As we can see from Fig. 7, Young's modulus of Fe<sub>6</sub>W<sub>6</sub>C has stronger directional dependence than shear modulus. Projections of Young's modulus on the (001) and (110) planes for Fe<sub>6</sub>W<sub>6</sub>C are also shown in Fig. 6(d), which are all deviated from the regular ellipses. The shapes of planar contours on (001) and (110) planes are alike and show the maximum Young's modulus as 506.6 GPa along [010] and [001] directions, respectively. Moreover, the minimum Young's modulus for Fe<sub>6</sub>W<sub>6</sub>C on (001) and (110) planes are 459.1 and 445.2 GPa.

The anisotropy of the minimum thermal conductivity for  $Fe_6W_6C$  is now addressed based on Cahill's model and Clarke's model. Both models can give the lower limit of the thermal conductivity of a crystal. Then the thermal conductivities ( $\kappa$ ) can be evaluated as [20,21]:

Clark's model: 
$$\kappa_{\min} = 0.87 k_B \overline{M_a}^{-2/3} E^{1/2} \rho^{1/6}, \ \overline{M_a} = [M/(m \cdot N_A)]$$
(7)

Cahill's model : 
$$\kappa_{\min} = \frac{k_B}{2.48} n^{2/3} (v_l + v_{t1} + v_{t2})$$
 (8)

where  $\overline{M_a}$  is the average mass per atom,  $\rho$  is the density, M is the molar mass, m is the total number of atoms per formula,  $N_A$  is Avogadro's number,  $k_B$  is Boltzmann's constant, *n* is the density of number of atoms per volume,  $v_l$  is the longitudinal sound velocity,  $v_{t1}$  and  $v_{t2}$  are the transverse sound velocity. For the Clark's model, we found a simple parameter of Young's modulus (E) in it. Thus we replace the simple E with the 3D expression of anisotropy of Young's modulus (Eq. (2)) to obtain the directional dependence of the minimum thermal conductivity. The 3D surface contour of the minimum thermal conductivity of Fe<sub>6</sub>W<sub>6</sub>C is presented in Fig. 7(c), which can be seen that the shape of the contour is not a sphere. Furthermore, we can also plot the anisotropic minimum thermal conductivity of a crystal at different crystal planes. For Fe<sub>6</sub>W<sub>6</sub>C with cubic crystal, the trajectories of the minimum thermal conductivity at the (001) and (110) crystal planes are given in spherical coordinates [22]:



Fig. 5. Total electron density distribution contour (a) and electron density difference distribution contour (b) at (1-10) plane for Fe<sub>6</sub>W<sub>6</sub>C.



Fig. 6. Mulliken population and bond length of chemical bonds in Fe<sub>6</sub>W<sub>6</sub>C.

$$\kappa_{\min}^{(001)} = 0.87 k_B \overline{M_a}^{-2/3} \rho^{1/6} (S_{11} (\sin^4 \varphi + \cos^4 \varphi) + (S_{44} + 2S_{12}) \cos^2 \varphi \sin^2 \varphi)^{1/2}$$
(9)

$$\kappa_{\min}^{(1\,1\,0)} = 0.87 k_B \overline{M_a}^{-2/3} \rho^{1/6} \left( S_{11} \left( \frac{1}{2} \sin^4 \theta + \cos^4 \theta \right) + (S_{44} + 2S_{12}) \left( \cos^2 \theta \sin^2 \theta + \frac{1}{4} \sin^4 \theta \right) \right)^{1/2}$$
(10)

The results are illustrated in Fig. 7(e) and (f). From the trajectory images one can clearly see that the minimum thermal conductivity along the [010] direction is smaller than that along other directions and the peak value  $\sim 1.48 \ W \ m^{-1} \ K^{-1}$  occurs along the [110] direction at the (001) plane. Furthermore, the minimum thermal conductivity close to the [111] direction is larger than that along other directions and the minimum thermal conductivity as 1.24 W m^{-1} \ K^{-1} along [110] direction is the smallest at the (110) plane.

For the Cahill's model, the sound velocities of both longitudinal and transverse waves along [100], [110] and [111] directions

must be calculated using the following relations [23] to obtain the minimum thermal conductivities along different directions:

for 
$$[100]v_l = \sqrt{C_{11}/\rho}; \quad [010]v_{t1} = [001]v_{t2} = \sqrt{C_{44}/\rho}$$
 (11)

for 
$$[110]v_l = \sqrt{(C_{11} + C_{12} + 2C_{44})/2\rho}; \quad [1\bar{1}0]v_{t1}$$
  
=  $\sqrt{(C_{11} - C_{12})/\rho}; \quad [001]v_{t2} = \sqrt{C_{44}/\rho}$  (12)

for 
$$[1\,1\,1]v_l = \sqrt{(C_{11} + 2C_{12} + 4C_{44})/3\rho};$$
  $[1\,1\,\bar{2}]v_{t1} = v_{t2}$   
=  $\sqrt{(C_{11} - C_{12} + C_{44})/3\rho}$  (13)

The anisotropic sound velocities of h-WC and h-W<sub>2</sub>C [24] are also quoted to estimate the thermal conductivities. All the obtained anisotropic sound velocities of Fe<sub>6</sub>W<sub>6</sub>C, h-WC and h-W<sub>2</sub>C are shown in Table 4. The minimum thermal conductivities estimated by Clark's and Cahill's models are tabulated in Table 5. The value obtained by Cahill's model is  $1.38 \text{ W m}^{-1} \text{ K}^{-1}$ , which are larger than the value  $1.26 \text{ W m}^{-1} \text{ K}^{-1}$  predicted by Clark's model. The thermal conductivity of the investigated Fe<sub>6</sub>W<sub>6</sub>C is almost as low as that of the thermal insulators Ln<sub>2</sub>Zr<sub>2</sub>O<sub>7</sub> (1.2- $1.5 \text{ W m}^{-1} \text{ K}^{-1}$ ), which are also evaluated by Cahill's and Clark's model and compared with the experimental results [23]. The reason is that W atom is much heavy and atomic masses and radius for W, Fe and C atoms vary widely, which may weaken the vibration of the atom and lead to different vibrational frequency for different atom. So the phonon transmission is hampered resulting in low conductivity of the crystal. As an important phase in WC/Fe composite, the thermal conductivity of Fe<sub>6</sub>W<sub>6</sub>C is larger than h-W<sub>2</sub>C, but smaller than h-WC. Furthermore, the anisotropy of minimum thermal conductivity calculated within Cahill's model is in consistent with that estimated by Clark's model. For Cahill's model, the minimum thermal conductivity along the [110] direction is the largest among all the calculated values along different directions, which is in good agreement with the result from Clark's model.

**Table 3** The calculated elastic compliance matrix ( $S_{ij}$ ), universal anisotropic index ( $A^U$ ), percent anisotropic index ( $A_B$  and  $A_G$ ) and shear anisotropic factors ( $A_1$ ,  $A_2$  and  $A_3$ ) for Fe<sub>6</sub>W<sub>6</sub>C.

Method	S <sub>ij</sub>			Shear an	isotropic facto	ors	Anisotropic index		
	S <sub>11</sub>	S44	S <sub>12</sub>	A <sub>1</sub>	<i>A</i> <sub>2</sub>	<i>A</i> <sub>3</sub>	$A^U$	$A_B$	$A_G(\%)$
GGA-PW91+DFT-D	0.0019738	0.0057582	-0.0004964	0.86	0.86	0.86	0.02821	0	0.28
GGA-PBE	0.0020440	0.0059525	-0.0005227	0.86	0.86	0.86	0.02636	0	0.26



Fig. 7. Surface contour of anisotropic shear modulus, Young's modulus and thermal conductivity (a)-(c); planar projections of Young's modulus on (001) and (110) plane (d); the anisotropic thermal conductivities for (001) plane and (110) plane (e) and (f).

Table 4					
The calculated anisotropic sound	velocities of Fe <sub>6</sub> W <sub>6</sub> C,	together with the	values of h-WC and h	-W2C. The unit of	velocity is m/s.

Species	[100]			[010]			[001]			[110]			[111]		
	$v_l$	$v_{t1}$	$v_{t2}$	$v_l$	$v_{t1}$	$v_{t2}$	$v_l$	$v_{t1}$	$v_{t2}$	$v_l$	$v_{t1}$	$v_{t2}$	$v_l$	$v_{t1}$	$v_{t2}$
Fe <sub>6</sub> W <sub>6</sub> C h-WC h-W <sub>2</sub> C	6325 4034 3466	3376 6799 5928	3376 4412 3586	6325	3376	3376	6325 4412 3586	3376 7875 5657	3376 4412 3586	6174	5154	3376	6123	3557	3557

#### Table 5

The thermal conductivity ( $\kappa$ ) of Fe<sub>6</sub>W<sub>6</sub>C, together with the values of h-WC and h-W<sub>2</sub>C calculated by Clark model and Cahill's model. The unit of the minimum thermal conductivity is W m<sup>-1</sup> K<sup>-1</sup>.

Species	Model	Μ	$M_a(10^{-26})$	$n(10^{28})$	$[100]\kappa_{\min}$	$[010]\kappa_{\min}$	$[001]\kappa_{\min}$	$[110]\kappa_{\min}$	$[111]\kappa_{\min}$	$[1\bar{1}0]\kappa_{min}$	$[1\bar{1}1]\kappa_{min}$	$\kappa_{\min}$
Fe <sub>6</sub> W <sub>6</sub> C	Cahill			8.22	1.38	1.38	1.38	1.55	1.39			1.38
	Clark	1449.6	18.52		1.31	1.31	1.31	1.48		1.24	1.47	1.26
h-WC	Cahill			9.68	1.79		1.96					1.85
	Clark	195.8	16.26									1.68
h-W <sub>2</sub> C	Cahill			7.97	1.34		1.32					1.32
	Clark	379.6	21.02									1.19

#### 4. Conclusions

In this paper, we find that the lattice constant and bulk modulus of  $\eta$  phase Fe<sub>6</sub>W<sub>6</sub>C change slightly with the temperature increasing from 0 K and 1200 K. The heat capacity at constant pressure ( $C_p$ ) and constant volume ( $C_V$ ) are calculated and the  $C_V$  approaches 324.25 J mol<sup>-1</sup> K<sup>-1</sup> for Fe<sub>6</sub>W<sub>6</sub>C. The temperature dependence of linear expansion coefficient ( $\alpha$ ) for Fe<sub>6</sub>W<sub>6</sub>C is calculated and the value is  $7.53 \times 10^{-6}$  K<sup>-1</sup> at 1200 K. The bonding behaviors of Fe<sub>6</sub>W<sub>6</sub>C are the combinations of covalent and metallic bonds. The 3D surface contour of torsion shear modulus and Young's modulus

are obtained and the anisotropy of Young's modulus of Fe<sub>6</sub>W<sub>6</sub>C is stronger than shear modulus. Furthermore, the 3D representation of minimum thermal conductivity of Fe<sub>6</sub>W<sub>6</sub>C is evaluated based on the Clark model. The minimum thermal conductivities at the (001) and (110) planes are plotted and we found that the directional dependence of thermal conductivity is determined by the anisotropy of elasticity of Fe<sub>6</sub>W<sub>6</sub>C. On the other hand, the anisotropy of minimum thermal conductivities is also evaluated by Cahill's model, which is in good agreement with the result from Clark's model. The  $\kappa_{min}$  values estimated by Cahill's and Clark's model are 1.38 and 1.26 W m<sup>-1</sup> K<sup>-1</sup>, respectively.

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