OMNI Interpolator for High-performance Motion Control System
Part I: Algorithm of Interpolator

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Abstract

This paper presents the OMNI-Interpolator design based on the NURBS interpolation algorithm. The five main issues regarding CNC interpolator have been discussed. First, a unified NURBS interpolator is proposed to simplify the CNC interpolator design thoroughly. Then, the geometry chord error that usually occurs during interpolation calculation is well controlled to obtain high accuracy. Third, variable feedrate is introduced to adjust the speed profile to adapt with the cutting trajectory geometry, especially around sharp corners. Fourth, limitation settings for the limitations on servo motor power and overcut/undercut are discussed. Finally, the machine tool dynamics are considered and integrated in the interpolation stage by filtering out natural frequency components contained in the compositional feedrate profile of the interpolated motion command. A simulation has been conducted using a X-Y table to evaluate the feasibility and effectiveness of the proposed interpolation scheme.

Keywords: NURBS, Interpolation, Machine dynamics

1 INTRODUCTION

The interpolator is one of the key components for CNC systems to conduct complex and accurate part surface machining. As of today, linear and circular interpolators are the most commonly used interpolation algorithms for CNC [1]. These interpolation algorithms usually approximate a tool path by a series of linear and circular segments. The more complex the trajectory is, the larger the amount of these tiny segments is needed. Usually, large storage space and high speed communication are needed to store and transfer a large NC program. In the occasion that a complex free-form surface is to be machined or high accuracy is required, the situation becomes even worse.

So far, NURBS has become the industrial standard for representing the trajectory [2]. Meanwhile, the study on NURBS curve interpolation has been conducted for many years. However, there is still missing an overall interpolator solution in which both the advantage of NURBS and other factors actually impacting interpolation have been considered. In addition, no solution is available at all for the traditional linear/circular represented NC program. Therefore, this study is proposed to seek a thorough solution based on NURBS interpolation, which is named as the OMNI-Interpolator. Basically five aspects should be considered in this new interpolator design. They are:

(1) How to deal with the existing linear, circular or other shape segments so that the advantage of NURBS can be fully utilized. This corresponding work has been performed by the associates of the author [3].

(2) How to consider controlling the geometry chord error under a certain tolerance to guarantee machining accuracy. The curvature for NURBS curve can be accurately calculated. Adaptive adjustment of feedrate according to the chord error can guarantee the chord error restriction [4].

(3)The current fixed interpolation feedrate doesn't match the actual trajectory geometry shape very well. The desired feedrate profile should be varied according to chord error restriction on the machining accuracy, also feedrate and acceleration constraints on the machine tool and machining quality demands. For NURBS interpolation, it is easy to carry out variable feedrate interpolation by using pre-designed feedrate profiles. This should be considered in the new interpolator. Also the feedrate profiles can be represented by cubic B-spline with the same parameter u of NURBS. Therefore, the interpolated position and its feedrate can be obtained simultaneously.

(4) Overcut/undercut during machining processes where the tool and workpiece are in contact are only allowed within the specified tolerances due to the demands from machining surface quality. There are also limitations of motor torque and power constrains on motor. Correct limitation settings like jerk limitations are therefore very important. Jerk limitation with proper selected jerk limitation setpoint and its time are needed to match the feed axis's damping and natural frequency. Thus, machine dynamics should be considered in the interpolation stage.

(5) Therefore, besides the geometric features considered in the above interpolation algorithms, machining dynamics should also be integrated into the interpolation algorithm, especially under the demands of high-speed CNC machining. For machine tools, resonance modes usually exist in their natural frequencies. If the motion of the tool coincides with one of machine's natural frequencies, it will result in the vibration of the machine. This vibration adversely affects machining quality and tool life, and therefore must be avoided. The natural frequency components contained in interpolation points should be filtered out to avoid the excitement of unwanted machine dynamics, i.e. its resonance modes.

In the current state of technology for CNC, the machining dynamics are mainly considered by tuning parameters of servo control loop, i.e. feedback control approach. However, an optimized tuning of servo parameters for complex CNC machine tools is usually quite difficult. An improperly tuned servo control loop may introduce flexibility of its own to CNC machine tools, even if the mechanical components of the machine tool are very stiff. Techniques based on using a specially designed motion command in which machine dynamics are considered may be one of the most appealing options for high-speed CNC machining. However, the dynamics of each machine's axis might be different. How to integrate each axis's dynamics in the CNC interpolation algorithm and avoid the distortion of the original cutting trajectory simultaneously is still largely unresolved. A systematic approach on this aspect is needed.

This paper mentions and discusses the above five important items for the OMNI-interpolator. An overall interpolator design is proposed based on NURBS interpolation algorithm. This paper is organized as follows: in section 2, review of NURBS curve interpolation is given. Unified representation of different types of curves by NURBS curve is mentioned; in section 3, the adaptive algorithm of NURBS curve is mentioned for the chord error control; in section 4, the variable feedrate interpolation based on NURBS curve interpolation is discussed; in section 5, overcut/undercut control and motor power constraints are discussed; in section 6, a systematic discussion and approach for integrating machine dynamics in NURBS curve interpolation is carried out. Verification of the proposed approach by simulation is also introduced; in section 7, conclusions and the future work about realizing the proposed interpolation scheme in FPGA are mentioned.

2 NURBS CURVE INTERPOLATION

NUBRS has become the industry standard. NURBS is widely used as a fundamental geometry representation in computer-aided design (CAD), manufacturing (CAM), and engineering (CAE) due to its good characteristics:

1) Offers one common mathematical form for both standard analytical shapes (e.g., conics) and free-form shapes.
2) Provides the flexibility to design a large variety of shapes.
3) Can be evaluated reasonably quickly by numerically stable and accurate algorithms.
4) Invariant under affine as well as perspective transformations.
5) A generalization of non-rational B-splines, and nonrational and rational Bezier curves and surfaces.

A NURBS curve is defined by its order, a set of weighted control points and a knot vector. The knot vector determines where and how the control points affect the NURBS curve. NURBS curves evolve into only one parametric direction, usually called u. By evaluating a NURBS curve at various values of the parameter u, the curve can be represented in Cartesian 2D or 3D space [2].

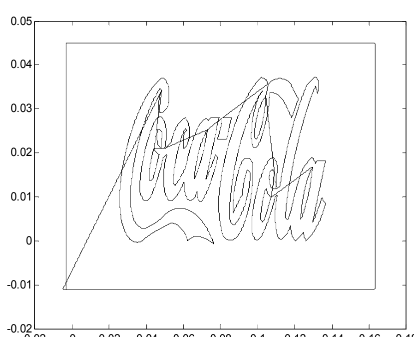


Figure 1: Example of a complicated curve represented by a single NURBS curve.

The significance of the NURBS interpolator is that it can unify linear and circular segments into a single NURBS trajectory [3]. For example, the complicated curve shown in Fig. 1 can be represented by a single NURBS curve. Therefore only one NURBS interpolator is necessary. For traditional linear and circular interpolations, hundreds of G-code lines (G01, G02 and G03) are needed. With a single NURBS curve representation of the cutting trajectory, high feedrate and low storage requirements can be realized simultaneously.

In this paper, the NURBS curve shown in Fig. 2 is introduced as a sample curve. This curve has a straight-line segment and a curve segment joining together. The joint point is tangent-discontinuity and leads to a sharp corner. The NURBS curve is defined by following parameters:

Control Points:	(0.0, 0.0, 0.0), (0.2, 0.2, 0.0), (0.4, 0.4, 0.0), (0.5, 0.5, 0.0), (0.7, 0.5, 0.0), (0.9, 0.5, 0.0), (1.0, 1.0, 0.0)
Knot Vector:	{0, 0, 0, 0, 0.5, 0.5, 0.5, 1, 1, 1, 1}
Weights:	{1, 1, 1, 1, 1, 1, 1}

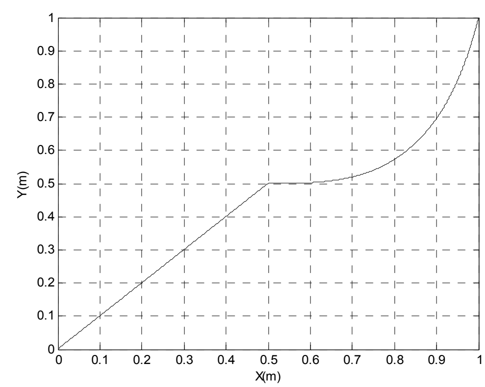


Figure 2: The sample NURBS curve.

Most of existing methods to calculate an appropriate u for the desired feedrate along the NURBS curve are based on Taylor's expansion. The second-order Taylor's expansion interpolator is obtained as [4]:

$$u_{i+1} = u_i + \frac{T_s \cdot V_i}{\left\| \frac{dP(u)}{du} \right\|_{u=u_i}} + \frac{T_s^2}{2}\left\{ \frac{A_i}{\left\|\frac{dP(u)}{du}\right\|} - \frac{V_i^2\left[\frac{dP(u)}{du}\cdot\frac{d^2P(u)}{du^2}\right]}{\left\|\frac{dP(u)}{du}\right\|^4}\right\}_{u=u_i} \quad (1)$$

where $P(u)$ is the curve and $u_i = u(t_i)$ is the value of u at the time instant of $t_i = iT_s$. V_i and A_i represent the feedrate and acceleration at t_i. T_s denotes the sampling time of the digital interpolator.

3 CHORD ERROR CONTROL

In order to maintain the permissible machining accuracy, chord error due to the interpolation of the original continuous NURBS curve must be controlled under a certain tolerance. The feedrate should be adaptively adjusted where chord error exceeds the prescribed tolerance. As shown in Fig. 3, chord error can be approximately calculated as [4]:

$$\delta_i = \rho_i - \sqrt{\rho_i^{\,2} - \left(\frac{L_i}{2}\right)^2} \tag{2}$$

where $\rho_i = 1/K_i$, ρ_i and K_i are the radius of curvature and curvature at the parameter value u_i, respectively. L_i equals $\|P(u_{i+1}) - P(u_i)\|$. The curvature K_i of NURBS curve can be calculated as following equation:

$$K_i = \left.\frac{\dfrac{dx}{du}\dfrac{d^2y}{du^2} - \dfrac{dy}{du}\dfrac{d^2x}{du^2}}{\left[\left(\dfrac{dx}{du}\right)^2 + \left(\dfrac{dy}{du}\right)^2\right]^{3/2}}\right|_{u=u_i} \tag{3}$$

If the chord error is below its permissible maximum chord error, the constant feedrate is maintained; while if the chord error exceeds its permissible value, the feedrate must be adaptively adjusted according to following equation:

$$V_i = \frac{2}{T_s}\sqrt{\rho_i^2 - (\rho_i^2 - \delta_{\max})^2} \tag{4}$$

where δ_{max} is the permissible chord error.

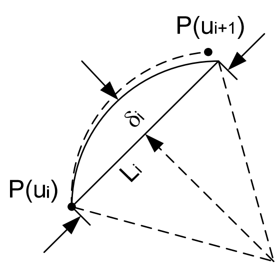


Figure 3: Approximation of chord error.

Fig. 4 shows the feedrate profile after applying the above adaptive interpolation algorithm. The constant feedrate cannot be maintained in the period of 5.2-7.6 sec due to the chord error restriction. An abrupt jump at 4.2 sec can also be observed which is caused by the sharp corner. Obvious sharp changes in feedrate also occur in the feed starting and stopping positions.

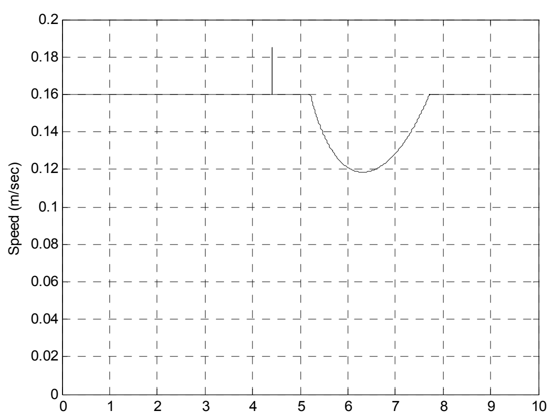


Figure 4: Feedrate profile of the sample NURBS curve for chord error control.

4 VARIABLE FEEDRATE INTERPOLATION

The feedrate for curve interpolation should be varied according to feedrate and feedrate-acceleration constraints on the machine tool, chord error restriction on the machining accuracy or the machining quality demands. For the example in Fig. 4, besides the feedrate variation due to the chord error restriction, the feedrate for traversing the sharp corner should also be properly restricted based on the chord error tolerance. Since this feedrate is usually very small, for simplicity here the feedrate in sharp corner is set to zero and the machine tool should smoothly decelerate before the sharp corner in advance and accelerate to the constant feedrate again after it. The redesigned feedrate profile is shown in Fig. 5, in which machine dynamics are also considered that will be discussed in section 6.

As shown in Eq. 1, for the NURBS interpolator, the variable feedrate interpolation is easy and natural by using the re-designed feedrate profile of V_i. However, for traditional linear and circle interpolations, a long series of G-code like "G01 F" is needed, where F should be varied according to the position of the interpolation points.

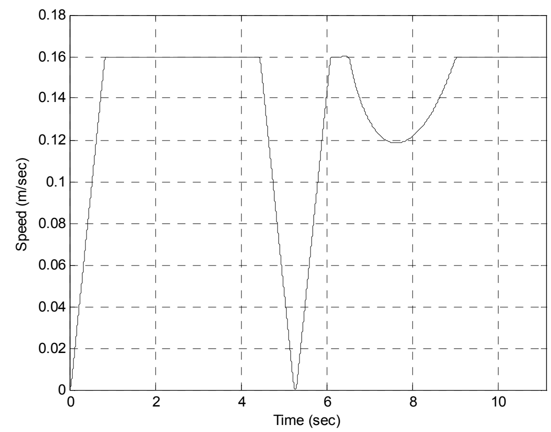


Figure 5: An example of the variable feedrate interpolation's feedrate profile.

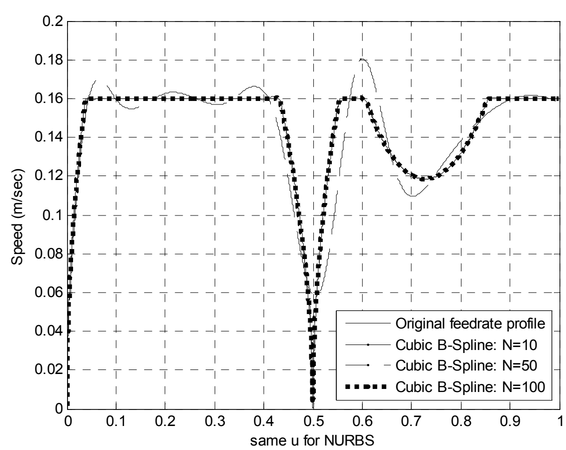


Figure 6: Cubic B-spline approximations of the variable feedrate profile with a different number N of polynomial pieces.

The redesigned feedrate profile can be represented by cubic B-splines for data reduction and the second order continuity of the profile. As shown in Fig. 6, the variable feedrate profile represented by the cubic B-spline can use the same parameter u of the NURBS curve. Therefore, for an arbitrary u, the corresponding spatial position and its feedrate can be calculated simultaneously.

5 LMITATIONS ON MOTOR AND OVERCUT

There are two limitations on motor, motor torque and power constraints. Torque (actually current) limits impose bounds on the magnitude of the axis acceleration. Suppose $a(t)$ is the compositional acceleration for a X-Y table, therefore:

$$a(\mathrm{t})\cos(\theta(\mathrm{t})) \le a_{\mathrm{x,max}}, \quad a(\mathrm{t})\sin(\theta(\mathrm{t})) \le a_{\mathrm{y,max}} \tag{8}$$

where $a_{x,max}$ and $a_{y,max}$ are the maximum accelerations of X-Y axes due to the torque limitation on each axis.

Similarly, power limits impose bounds on the multiplication of the axis's speed and acceleration. Again suppose $v(t)$ and $a(t)$ are the compositional speed and acceleration for the X-Y table, therefore:

$$v(\mathrm{t})a(\mathrm{t})\cos^2(\theta(\mathrm{t})) \le p_{\mathrm{x,max}}, \quad v(\mathrm{t})a(\mathrm{t})\sin^2(\theta(\mathrm{t})) \le p_{\mathrm{y,max}} \tag{9}$$

where $p_{x,max}$ and $p_{y,max}$ are the maximum power of X-Y axes.

As mentioned in the above sections, checking whether the commanded movement violates the limits on the motor can be easily carried out. However, it is known that the motor torque limit changes with the motor's angular speed. Different values of a_{max} and p_{max} for each axis are appropriate for different motor angular speeds.

Overcut/undercut during machining processes where the tool and workpiece are in contact, is only allowed within the specified tolerances due to the demands from machining surface quality. Therefore, correct limitation settings including the above motor power limitations are very important. These limitations can be used to achieve smooth movement of feed axes with low stress on the mechanical system. At the same time, the productivity requires a specific acceleration. Jerk limitation in motion command interpolation can be introduced to match the specified acceleration and the overcut limitation.

A jerk r is defined as:

$$r = \frac{da}{dt} \tag{5}$$

Therefore the relationship between the constant jerk limitation r_0 and final acceleration value a_0 is:

$$a_0 = r_0 \cdot t_R \tag{6}$$

where t_R is the jerk limitation t_R. The setting of the jerk limitation should be $t_R \ge 1/f_{NF\ \mathrm{min}}$, where $f_{NF\ min}$ is the lowest natural frequency of the feed axes [5][6].

For smaller jerk limitation times, the overcut will increase due to the excitation of the machine's natural frequency vibrations. Jerk limitation and its time can be empirically determined. For example, for a feed axis that equals to a two order element with the lowest natural frequency $f_{NF\ min}$ and the damping ration D_{mech}, the above correlation when $t_R \ge 1/f_{NF\ \mathrm{min}}$ can be obtained as [8]:

$$r_0 = \frac{4\pi^2}{0.15} \cdot e^{\frac{\pi D_{mesh}}{\sqrt{1-D_{mesh}^2}} - 0.316} \cdot f_{NF\ \mathrm{min}}^3 \cdot \Delta s_m \tag{7}$$

where Δs_m is the overcut expressed as a distance.

The jerk limitation to be set is therefore proportional to the third power of the lowest natural frequency $f_{NF\ min}$, i.e. for a specific tolerable overcut besides the motor power limitations the acceptable jerk should be determined by machine dynamics.

6 MACHINE TOOL DYNAMICS

As mentioned in previous sections, in order to have high machining quality, the machine tool's dynamics must be considered and integrated in the interpolation stage. Axes' natural frequency components contained in the interpolated motion command should be filtered out in advance to guarantee that the natural frequency vibrations will not be excited.

The preconditioning of the motion command by filtering out natural frequency components has been verified to be an effective method for a machine's natural frequency vibration suppression. A lot of research has been done on this aspect [4][7].

Obviously for CNC machining, the most important thing is to keep the permissible accuracy of the cutting trajectory. Simply filtering out the natural frequency components of each axis will lead to distortion of the original cutting trajectory and should be avoided as much as possible. Zero trajectory distortion after the filtering must be guaranteed. However, this problem has not well been solved yet.

In this paper, a method that can realize the filtering of each axis's natural frequency components contained in the interpolated motion command and zero trajectory distortion simultaneously is proposed. It is actually straightforward to consider that in order to avoid the trajectory distortion, only the feedrate along the curve should be re-designed. The direction of the feedrate must be decided only by the geometric features of the trajectory.

As shown in Fig. 7, the relationships between compositional feedrate Vc and its resolutional feedrates on the X and Y axes Vx, Vy are:

$$V_x(t) = V_c(t)\cdot\cos(\theta(t)), \quad V_y(t) = V_c(t)\cdot\sin(\theta(t)) \tag{9}$$

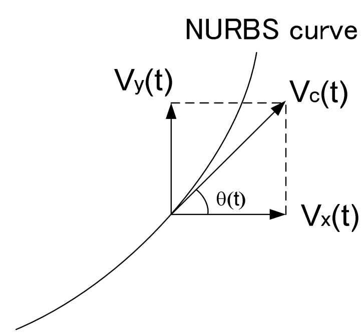


Figure 7: Compositional feedrate V_c and resolutional feedrates of the X and Y axis, V_x and V_y.

Since in most cases except for sharp corners, i.e. tangent discontinuities, the changes of $cos(\theta t)$ and $sin(\theta t)$ are much slower than the machine's natural frequencies which are basically several 10Hz, if both X and Y axis's natural frequency components are filtered out in the compositional feedrate profile $V_c(t)$, obviously:

- $V_x(t)$ and $V_y(t)$, the resolutional feedrates of $V_c(t)$, will not have the natural frequency components of the X and Y axes anymore.

- Since V_c(t) is the compositional feedrate profile for NURBS curve calculation, trajectory distortion will also be zero.

In section 7, the feasibility and effectiveness of the proposed interpolation scheme will be verified by simulation using a X-Y table model.

7 SIMULATION RESULTS

7.1 The X-Y Table Model

The X-Y table is modeled using two two-mass models as shown in Fig 8. The load (inertia J_l) is connected to servomotor (inertia J_m) by a spring (elastic coefficient K). In order to make the X-Y table model easier to vibrate, the damping between two masses is set to zero.

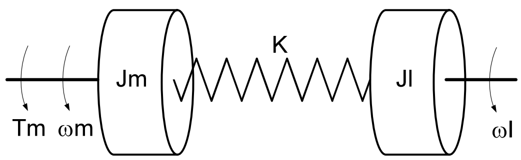


Figure 8: the two-mass model for X-Y table's axes.

The Bode magnitude plot for the transfer function between T_m and ω_l is shown in Fig. 9, where the natural frequency of the two-mass model is 20Hz. T_m is the torque generated by servomotor and ω_l is the rotation speed of the load.

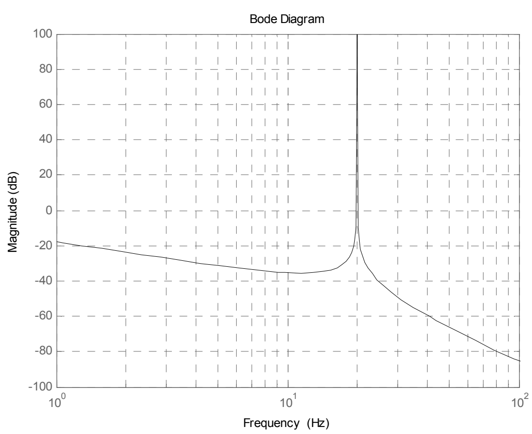


Figure 9: Bode magnitude plot of the two-mass model.

The natural frequency ω can be calculated as:

$$\omega = \sqrt{K\left(\frac{1}{J_m} + \frac{1}{J_l}\right)} \quad (11)$$

In this simulation, the natural frequencies of the X and Y axes are set to 40Hz and 20Hz, respectively, by a different combination of J_m, J_l and K for each axis.

7.2 Integration of Machine Dynamics

As shown in Fig. 4, sudden changes of feedrate occur in three positions:

1) Feed starting position
2) Sharp corner
3) Part of curve with large curvature

For feed starting position, the feedrate should be smoothly accelerated from 0 to the predefined constant feedrate. As mentioned in Ref. [6][7], the longest possible time T_{rlim} (duration time for rectangular jerk limitation) should be taken as 1/ω sec, where ω's unit is Hz. Since the X and Y axes have different natural frequencies, in order to filter out both 40Hz and 20Hz frequency components in the compositional feedrate profile, T_{rlim} should be taken as the longer one, 1/20=50msec.

As to the sharp corner, the feedrate should be decelerated to zero at the sharp corner and then accelerated again to the constant feedrate. Therefore T_{rlim} for the sharp corner is also taken as 1/20=50msec.

For the sudden change of feedrate caused by chord error restriction, both the 40Hz and 20Hz frequency components contained in the original feedrate profile are filtered out.

The redesigned feedrate profile based on the proposed design is shown in Fig. 10, in which the machine dynamics are integrated.

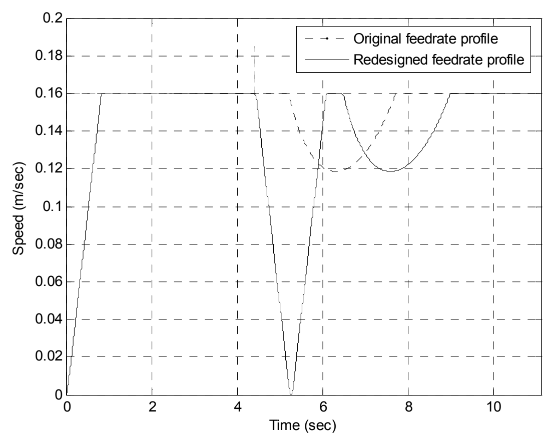
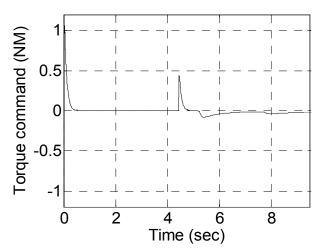
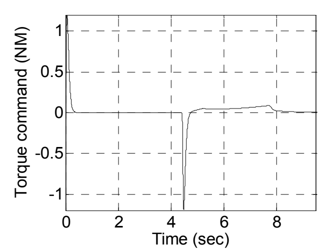


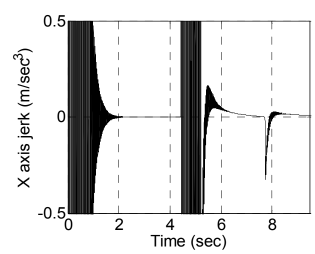
Figure 10: The original feedrate profile (dotted line) and redesigned feedrate profile (solid line) in which the machine dynamics are integrated.



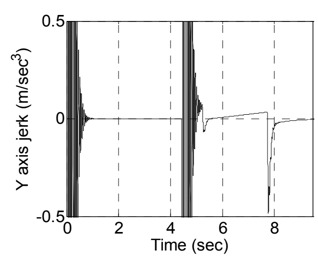
(a) X-axis torque command



(b) Y-axis torque command



(c) Jerk response of X axis

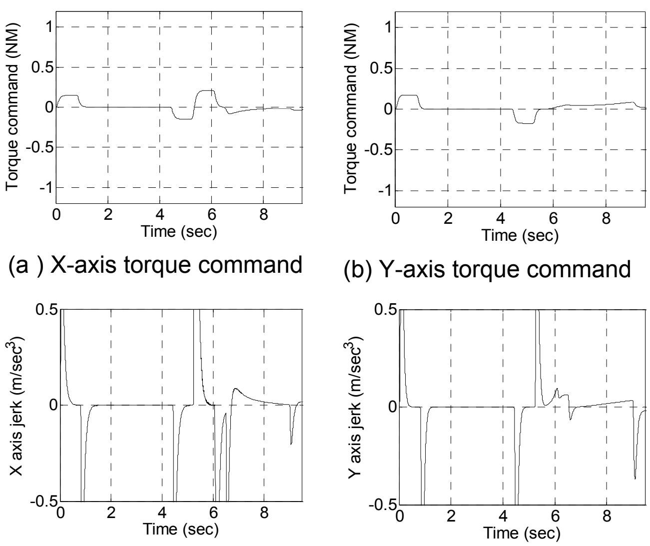


(d) Jerk response of Y axis

Figure 11: Simulation results for the original feedrate profile.

As shown in Fig. 11, using the original feedrate profile will require sharp changes of the axes' torque command in which strong natural frequency components are contained. As a result, severe natural frequency vibrations occur in the three positions mentioned above. In order to observe the vibrations more easily, jerk responses of the X and Y axes are plotted in Fig. 10(c)(d).

Compared to the simulation results in Fig. 11, using the redesigned feedrate profile gives much smoother changes in the torque command, as shown in Fig. 12. Since the 40Hz and 20Hz frequency components contained in the original feedrate profile have already be filtered out, the natural frequency vibrations of both the X and Y axes can be observed to be effectively suppressed. Most significantly, because only the compositional feedrate is redesigned, there is no distortion of the original cutting trajectory caused by filtering.



(a) X-axis torque command (b) Y-axis torque command

(c) Jerk response of X axis (d) Jerk response of Y axis

Figure 12: Simulation results for redesigned feedrate profile, in which the machine dynamics are integrated.

8 CONCLUSIONS

In this paper, the five main issues of the OMNI-interpolator are systematically mentioned and discussed. A sample NURBS curve and simulation with a X-Y table model are introduced to verify the feasibility and effectiveness of the proposed interpolation scheme. Especially, filtering out all the axes' natural frequency components in the compositional feedrate profile rather than the resolutional feedrate profile for each axis can realize excellent natural vibration suppression for all the axes and zero distortion of the original cutting trajectory simultaneously.

With the computation complexity of the NURBS interpolation, it is difficult for a motion controller to complete NURBS curve interpolation in a tight sampling period, for example 1 ms under normal CPU and DSP architectures. The FPGA can be an alternative solution for the implementation of NURBS interpolation because of its parallel computing and flexible programming. Realizing the whole proposed interpolation scheme of the OMNI-interpolator based on FPGA technologies will be carried out in the future.

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