Experimental Verification of Fractional Order Controller's Robustness Using Torsion Test Bench

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Abstract: Fractional Order Control (FOC) is still an unfamiliar concept. But it is actually a natural generalization of classical Integer Order Control (IOC) theory, in which the orders of controlled systems and/or controllers can be any real numbers including the conventional integer numbers. This generalization provides insight and more flexibility for robust control design. This paper reviews the applications of various FOC approaches including fractional order filter and disturbance observer into the gear backlash vibration control of a torsion test bench, in which a trade-off exists between stability and the strength of vibration suppression. The theoretical analysis and implementation results show FOC's superior robustness against gear backlash nonlinearity and more importantly its possibility of enabling a clear-cut robust control design.

Key Words: Fractional Order Control, Robustness, Nonlinearity, Torsion

1 INTRODUCTION

The concept of Fractional Order Control (FOC) means controlled systems and/or controllers are described by fractional order differential equations. Expanding derivatives and integrals to fractional orders is by no means new and actually has a firm and long standing theoretical foundation. Interest in this subject was evident almost as soon as the ideas of the classical calculus were known. The earliest more or less systematic studies seem to have been made in the beginning and middle of the 19th century by $Liouville$, $Holmgren$ and $Riemann$, although $Eular$, $Lagrange$, and others made contribution even earlier. Parallel to these theoretical beginnings was the development of applying fractional calculus to various problems [1].

As to fractional calculus' application in control engineering, FOC was introduced by $Tustin$ for the position control of massive objects half a century ago in 1958, where actuator saturation requires sufficient phase margin around and below the critical point [2]. Some other pioneering works were also carried out around 60's by $Manabe$ [3][4][5]. However, the FOC concept was not widely incorporated into control engineering mainly due to the unfamiliar idea of taking fractional order, so few physical applications and limited computational power available at that time [6].

In last few decades, researchers pointed out that fractional order differential equations could model various materials more adequately than integer order ones and provide an excellent tool for describing complex dynamic features [1][7]. Especially for the modeling and identification of flexible structures with increasing application of lighter materials, fractional order differential equations could provide a natural solution since these structures are essentially distributed-parameter systems [8] [9]. Obviously, the fractional order models need fractional order controllers for more effective control of the "real" systems. This necessity motivated renewed interest in various applications of FOC [10] [11] [12] [13]. At the same time with the rapid development of computer performances, realization of FOC systems also became much easier than before.

For control design, due to the intrinsic tradeoff among performance and robustness, FOC could be a general and effective approach, which has a unique "in-between" characteristics. Author believes there are three major advantages for introducing fractional order calculus:

- Adequate modeling of control plant's dynamics
- Effective and clear-cut robust control design
- Reasonable realization by approximation

Like other engineering researches, parallel to the development in theoretical aspects of FOC, it's applications in real control problems are of the same importance. In this paper, author demonstrates the applications of FOC in motion control using a torsion test bench. Since there are already lots of references for fractional order PID controller [14]; in this paper the applications of fractional order low-pass filter and disturbance observer are mainly reviewed [15] [16]. The experimental results verified FOC's superior robustness against gear backlash nonlinearity and more importantly its possibility of enabling a clear-cut robust design.

2 TORSION TEST BENCH

The experimental setup of the torsion test bench is depicted in Fig. 1. A torsional shaft connects two flywheels while driving force is transmitted from driving servomotor

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to the shaft by gears with gear ratio 1:2. Some system parameters are changeable, such as gear inertia, load inertia, shaft's elastic coefficient and gears' backlash angle. The encoders and tacho-generators are used as position and rotation speed sensors.

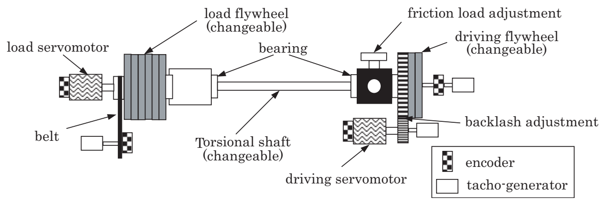


Figure 1: Experimental setup of test bench

The simplest model of the torsion test bench with gear backlash is the three-inertia model depicted in Fig. 2 and Fig. 3, where J_m, J_g and J_l are driving motor, gear with flywheels and load's inertias, K_s shaft elastic coefficient, ω_m and ω_l motor and load rotation speeds, T_m input torque and T_l disturbance torque. In the modeling, the gear backlash is simplified as a deadzone factor with backlash angle band [-δ,+δ] and elastic coefficient K_g. Frictions between components are neglected due to their small values.

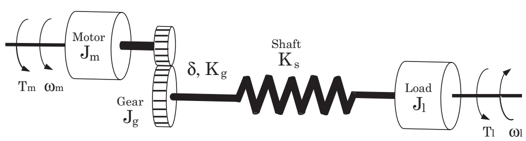


Figure 2: Torsion test bench's three-inertia model

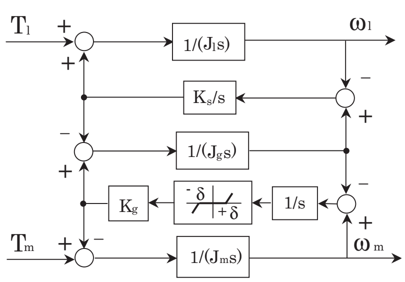


Figure 3: Block diagram of the three-inertia model

The open-loop transfer function from T_m to ω_m is

$$P_{3m}(s) = \frac{(s^2+\omega_{h1}^2)(s^2+\omega_{h2}^2)}{J_m s(s^2+\omega_{o1}^2)(s^2+\omega_{o2}^2)} \tag{1}$$

where ω_{o1} and ω_{o2} are resonance frequencies while ω_{h1} and ω_{h2} are anti-resonance frequencies. ω_{o1} and ω_{h1} correspond to torsion vibration mode, while ω_{o2} and ω_{h2} correspond to gear backlash vibration mode.

3 TRADEOFF: STABILITY VS VIBRATION SUPPRESSION

As mentioned by Ref. [14], a well designed set-point-I PI controller can give a satisfactory performance for speed control in nominal case (see Fig. 4 and Fig. 5). The PI controller is designed by Coefficient Diagram Method (CDM) [14].

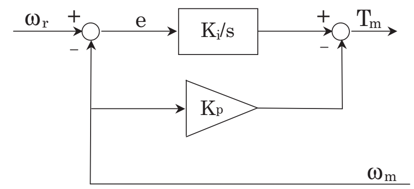


Figure 4: Set-point-I PI controller

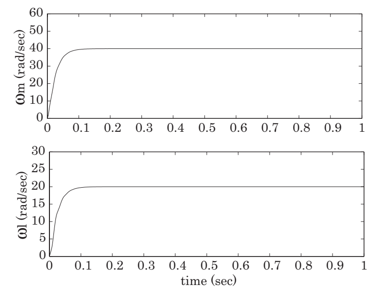


Figure 5: Simulation results with nominal three-inertia model

For the nominal three-inertia model $P_{3m}(s)$, the close-loop transfer function of integer order PI control system from ω_r to ω_m is

$$G_{close}(s) = \frac{C_I(s)P_{3m}(s)}{1 + C_I(s)P_{3m}(s) + C_P(s)P_{3m}(s)} \tag{2}$$

where $C_I(s)$ is I controller and $C_P(s)$ is P controller in minor loop; therefore $G_{close}(s)$ is stable if and only if $G_l = C_I(s)P_{3m}(s)+C_P(s)P_{3m}(s)$ has positive gain margin and phase margin. At the same time, for torsion system's speed control, suppressing vibration caused by the gear backlash must be concerned.

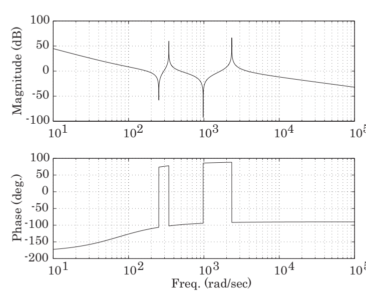


Figure 6: Bode plots of $G_l(j\omega)$ with PI controller

As depicted in Fig. 6, the PI speed control system has enough stability margin; while in order to recover some vibration performance, additional factors with negative slope and phase-lag are needed. However introducing these factors will simultaneously lead to phase margin loss. Namely, there exists a trade-off between stability margin loss and the strength of vibration suppression in the testing torsion system's speed control.

In order to be robust against the backlash non-linearity while keep being stable, several methods have been proposed. But their design processes are very complicated. As an example, for PID control introducing a low-pass filter $K_d s/(T_d s + 1)$ and redesigning the whole control system with three-inertia model can be a solution [17]. Due to the necessity of solving high order equations, the design is not easy to carry out. Clear and straightforward design concept is required in practical applications.

4 FRACTIONAL ORDER CONTROL APPROACHES

4.1 Fractional Order Low-pass Filter

Fractional order low-pass filter $\frac{1}{(Ts+1)^{\alpha}}$ can be introduced to achieve a proper controller, which is neither conservative nor aggressive (see Fig. 7) [15]. The trade-off between stability margin loss and the strength of vibration suppression can be easily adjusted by choosing proper fractional order α only, as depicted in Fig. 8. T need to be selected to give control system enough large band width for a fast time response. Here considering the frequency range of torsion vibration mode, T is taken as 0.005(=1/200).

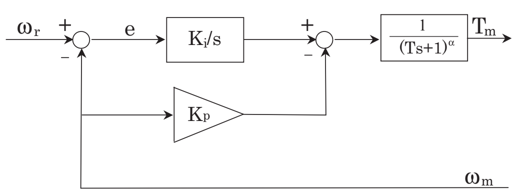


Figure 7: PI controller with fractional order low-pass filter

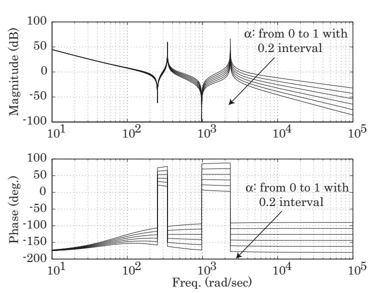


Figure 8: Bode plots of $G_l(j\omega)$ with fractional order low-pass filters

4.2 Fractional Order Disturbance Observer

Disturbance observer can also be applied, as depicted in Fig. 9 [16]. In the simple inverse model Js, the three masses of driving motor, gear and load are considered to be connect with a rigid shaft and can be described as a single mass. The disturbance observer is applied to estimate disturbance torque $\hat{T}_d$, which is generated due to unmodeled dynamics in the single inertia model Js. The Q-filter is actually a low-pass filter to restrict the effective bandwidth of the disturbance observer, where τ is the cutoff frequency and n is the relative degree of Q-filter.

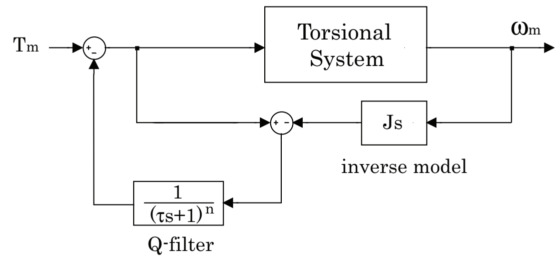
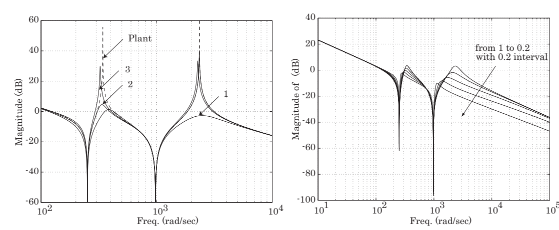


Figure 9: Conventional integer order disturbance observer

Due to the negative feedback of the estimated signal in disturbance observer, larger n will give more phase margin, but also larger gain in control system's open-loop frequency responses, and vice versa (see Fig. 10.a). Namely, the tradeoff between stability margin loss and vibration suppression also exists for applying disturbance observer in torsion test bench control.

For conventional integer order disturbance observer, the only tradeoff tuning knob is Q-filters relative degree n. The possibility for achieving a better tradeoff is quite restricted since only integral order n can chosen. Taking n as 1, the smallest value for n, gives the best vibration suppression performance for the integer order disturbance observer. To further improve vibration suppression performance while keep enough phase margin, introducing a fractional order Q-filter, whose order α is between 0 and 1, is actually a natural choice (see Fig. 10.b):

$$Q(s) = \frac{1}{(\tau s + 1)^{\alpha}} \tag{3}$$



(a) Integer order n (b) Fractional order α

Figure 10: Open-loop gain plots for integer order and fractional order disturbance observer

5 REALIZATION METHODS

Although it is not difficult to understand FOC's theoretical superiority, realization issue keeps being somewhat problematic. It is perhaps one of the most doubtful points for applying FOC. Since the fractional order systems have an infinite dimension, proper approximation by finite difference equation is needed to realize the designed fractional order controllers.

For frequency-band fractional order controllers, they can be realized by broken-line approximation in frequency-domain [18]. As to direct discretization, several methods have been proposed such as Short Memory Principle, Sampling Time Scaling, Tustin Taylor Expansion and Lagrange Function Interpolation method; while all the approximation methods need truncation of the expansion series. It was found that the Short Memory Principle had good approximation with simple algorithm [20].

5.1 Broken-line approximation

FOC system's frequency response can be exactly known. It is natural to approximate fractional order controllers by frequency domain approaches. The broken-line approximation method can be introduced to approximate $\frac{1}{(Ts+1)^{\alpha}}$ in frequency range $[\omega_b,\ \omega_h]$, where $T = \frac{1}{\omega_b}$. ω_h can be taken as high as $10^4 Hz$ to give an enough frequency range for a good approximation. Let

$$\left(\frac{\frac{s}{\omega_h}+1}{\frac{s}{\omega_b}+1}\right)^{\alpha} \approx \prod_{i=0}^{N-1} \frac{\frac{s}{\omega_i'}+1}{\frac{s}{\omega_i}+1} \tag{4}$$

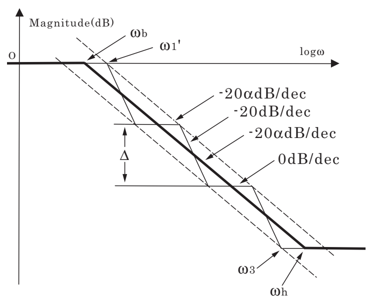


Figure 11: An example of broken-line approximation ($N = 3$)

Based on Fig. 11, two recursive factors ζ and η are introduced to calculate ω_i and $\omega_i^{'}$. Finally

$$\omega_i = \left(\frac{\omega_h}{\omega_b}\right)^{\frac{i+\frac{1}{2}-\frac{\alpha}{2}}{N}} \omega_b,\ \omega_i^{'} = \left(\frac{\omega_h}{\omega_b}\right)^{\frac{i+\frac{1}{2}+\frac{\alpha}{2}}{N}} \omega_b \tag{5}$$

Figure. 12 shows the Bode plots of ideal frequency-band case ($\alpha = 0.4,\ \omega_b = 200Hz,\ \omega_h = 10000Hz$) and its 1st-order, 2nd-order and 3rd-order approximations by broken-line approximation method. Even taking $N = 2$ can give a satisfactory accuracy in frequency domain. For digital implementation, the bilinear transformation method can be used.

5.2 Short memory principle

The mathematical definition of fractional derivatives and integrals has been a subject of several different approaches[1] [7]. One of the most frequently encountered definitions is called Grünwald-Letnikov definition, in which the fractional α order derivatives are defined as

$${}_{t_0}D_t^{\alpha} = \lim_{\substack{h\to 0\\ nh=t-t_0}} h^{-\alpha} \sum_{r=0}^{n} (-1)^r \begin{pmatrix} \alpha \\ r \end{pmatrix} f(t-rh) \tag{6}$$

where h is time increment and binomial coefficients are

$$\begin{pmatrix} \alpha \\ 0 \end{pmatrix} = 1,\ \begin{pmatrix} \alpha \\ r \end{pmatrix} = \frac{\alpha(\alpha-1)\dots(\alpha-r+1)}{r!} \tag{7}$$

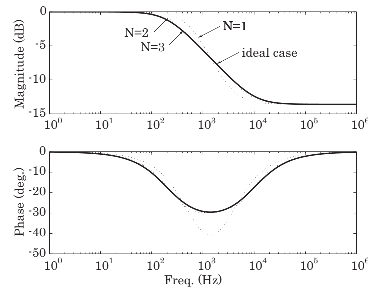


Figure 12: Bode plots of ideal case, 1st, 2nd and 3rd-order approximations

Short Memory Principle is inspired by Grünwald-Letnikov definition. Based on the approximation of the time increment h through the sampling time T, the discrete equivalent of D^k controller is given by

$$Z\{D^k[f(t)]\} \approx \left[T^{-k} \sum_{j=1}^{\infty} c_j^k z^{-(j-1)}\right] X(z) \tag{8}$$

where $X(z) = Z\{f(t)\}$ and the binomial coefficients are

$$c_1^k = 1,\ c_j^k = (-1)^{(j-1)} \begin{pmatrix} k \\ j-1 \end{pmatrix} \tag{9}$$

The semi-log chart of Fig. 13 shows binomial coefficients value versus term order j in Equ. (8) when approximating $k = 0.5$ derivative. The observation of the chart gives that the values of binomial coefficients near "starting point" t_0 in Grünwald-Letnikov definition is small enough to be neglected or "forgotten" for large t. Therefore the finite dimension approximation of D^k controller can be arrived by

$${}_{t_0}D_t^k[f(t)] \approx_{t-L} D_t^k[f(t)],\ (t > t_0 + L) \tag{10}$$

where L is the length of "memory". From Fig. 13, empirically memorizing 10 past values should have good approximation. Clearly, in order to have a better approximation, smaller sampling time and longer memory length are preferable.

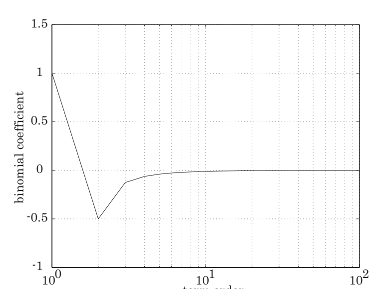


Figure 13: Binomial coefficients value versus term order j when approximating $D^{0.5}$

6 EXPERIMENTAL RESULTS

As depicted in Fig. 14, the test bench is controlled by a PC with 1.6GHz Pentium IV CPU and 512M memory. Realtime operating system $\mathrm{RTLinux}^{\mathrm{TM}}$ distributed by Finite State Machine Labs, Inc. is used to guarantee the timing correctness of the realtime tasks. The control programs are written in RTLinux C threads which can be executed with strict timing requirement of control sampling time. A 12-bit AD/DA multi-functional board is used whose conversion time per channel is $10\mu sec$. All the experiments were carried out with sampling time T=0.001sec. Two encoders (8000$pulse/rev$) are used as rotation speed sensors with coarse quantization $\pm 0.785 rad/sec$.

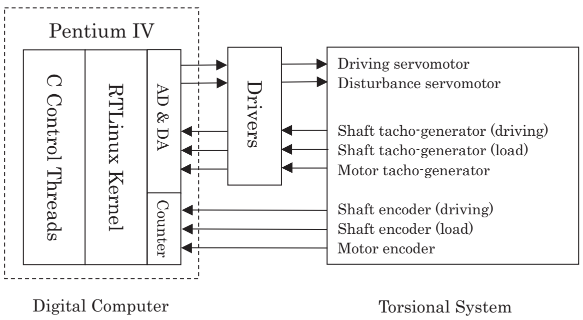


Figure 14: Digital control system of the experimental setup

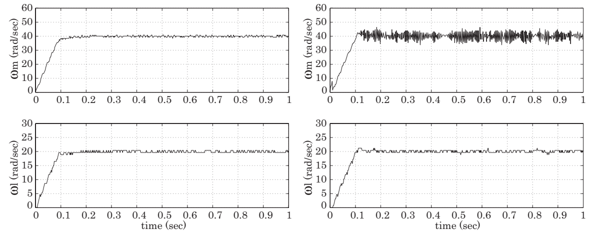
As depicted in Fig. 15 the PI control system can achieve satisfactory time responses when backlash angle is adjusted to zero degree ($\delta = 0$); while persistent vibration occurs when gear backlash non-linearity exists (see $\delta = 0.6$ case). PI control only can not provide enough strength for suppressing backlash vibration.

Figure 16 shows experimental results with different α order low-pass filters. Vibration occurred in PI-only control is effectively suppressed. Taking α as 0.4 gives the best time response. For other higher α order cases, their time responses are not such satisfied due to larger phase margin loss.

While Fig. 17 shows that compared with PI-only control, introducing integer order disturbance observer (n=1) can give better vibration suppression performance. However, this performance improvement is not enough to effectively suppress the vibration caused by gear backlash. For higher order n, like n=2 and n=3, the vibration suppression performance is actually deteriorated. Having fractional order α=0.6 gives a better tradeoff between stability and vibration suppression. The fractional order low-pass filters and Q-filters are realized by broken-line method with $N = 2$ and approximation band $[200Hz\ 10000Hz]$.

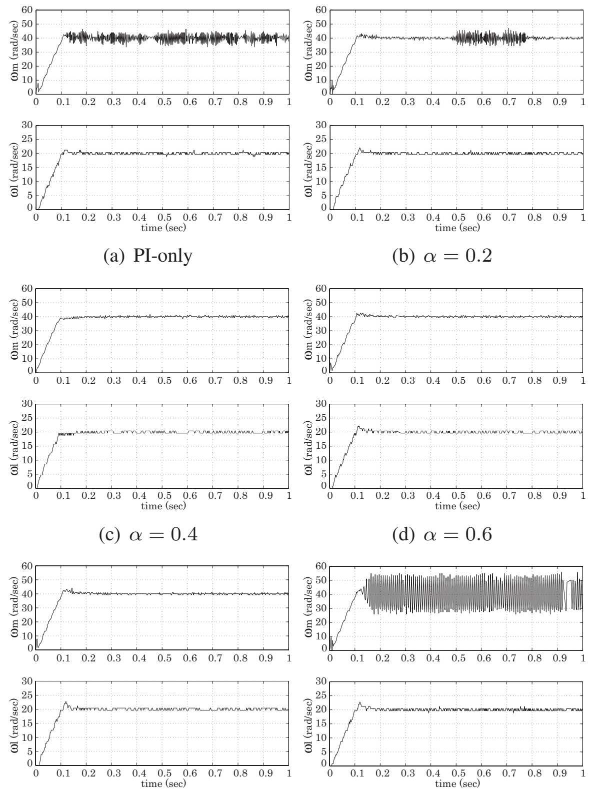
7 SUMMARY

This paper reviews the fractional order low-pass filter and disturbance observer for giving a straightforward trade-off adjustment between the control system's stability margin loss and the strength of vibration suppression. In oscillatory system control, this kind of trade-off is a common



(a) $\delta = 0deg.$ (b) $\delta = 0.6deg.$

Figure 15: Time responses of integer order PI control system



(a) PI-only (b) $\alpha = 0.2$

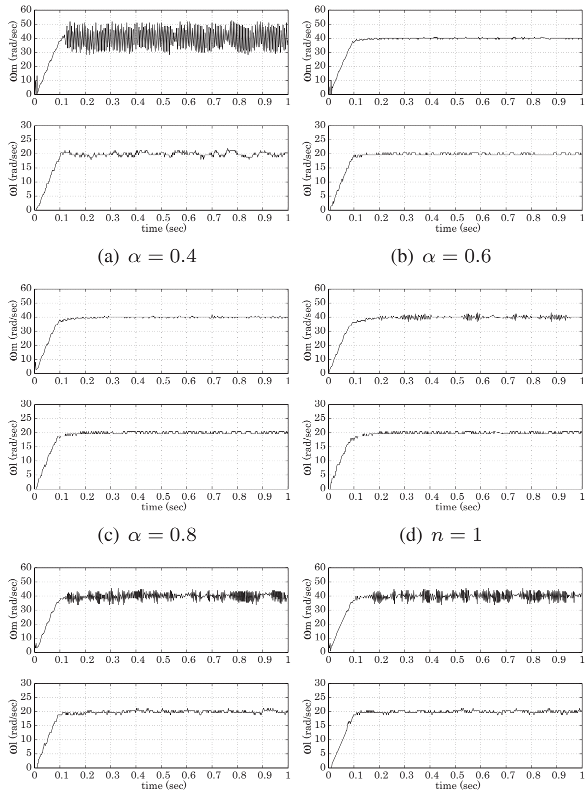
(c) $\alpha = 0.4$ (d) $\alpha = 0.6$

(e) $\alpha = 0.8$ (f) $\alpha = 1.0$

Figure 16: Time responses with fractional order $\frac{1}{(\tau s+1)^{\alpha}}$ filters

problem. As shown in the above theoretical analysis and experimental results, by introducing FOC concept, we can design control system in a clear-cut way since control system's frequency response can be continuously adjusted. Using fractional order controller could be a general method to trade off inconsistent control demands, which is not limited to the current specific problem. The implementation of fractional order controllers need proper approximation. However, as verified in experimental results, the implementation issue actually is not problematic.

FOC is actually not an abstract concept, but a natural expansion of the well-developed Integer Order Control (IOC) theory. Knowledge and design tools developed in IOC can still be made good use of in FOC research, as explained in this paper. For example, upgrading traditional



(a) $\alpha = 0.4$ (b) $\alpha = 0.6$

(c) $\alpha = 0.8$ (d) $n = 1$

(e) $n = 2$ (f) $n = 3$

Figure 17: Time responses with various integer and fractional order Q-filter

PID controller by introducing fractional order factors, such as fractional order I^{α}, D^{β} controllers or fractional order filters, could give a more effective control of complex dynamics. It is interesting to notice that actually the designed fractional order controllers can only be implemented by approximation using their integer order counterparts. The author does believe some well-designed IOC system might in fact be a unconscious approximation of FOC system.

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