

DETC2009-86569

DYNAMIC EMULATION OF TIRE/ROAD FRICTION FOR DEVELOPING ELECTRIC VEHICLE CONTROL SYSTEMS

Chengbin Ma

Univ. of Michigan-Shanghai Jiaotong Univ. Joint Institute
No. 800 Dongchuan Road, Shanghai 200240, P. R. China
Email: chengbin.ma@ieee.org

ABSTRACT

Current researches on electric vehicles are focusing on the environment and energy aspects. However, electric motors also have much better control performance than the internal combustion engines. Electric vehicles could not only be “cleaner” and “more energy efficient”, but also become “safer” with “better driving performance”. In this paper, a discrete elasto-plastic friction model is proposed for a dynamic emulation of tire/road friction for developing control systems of electric vehicles. The friction model can capture the transient behavior of the friction force during braking and acceleration, therefore the model-based emulation could enable more reliable verifications for various electric vehicle control methods.

1 INTRODUCTION

It has been widely recognized that electrifying vehicles can provide a solution to the emission and oil shortage problems brought by billions of conventional vehicles today, which are propelled by internal combustion engines. Consequently the most of current researches on EVs (Electric Vehicles including the hybrid electric vehicles in this paper) are focusing on the environment and energy aspects. And one of the key issues to commercializing EVs is considered to largely rely on the development of long-term energy storage devices with competitive cost.

However the most fundamental difference between EVs and the conventional vehicle is that EVs are the vehicles with one or more electric motors for propulsion instead of using the internal combustion engines. Namely the motion of EVs is pro-

vided either by wheels driven or partly driven by electric motors, i.e. EVs are actually typical mechatronic systems just like hard disks, robots, machine tools, etc. By introducing the highly-developed mechatronic technologies especially the mechatronic control, EVs could not only be “cleaner” and “more energy efficient”, but also become “safer” with “better driving performance” compared to the conventional vehicles. Few researches have been done on this aspect of EVs [1]; however it is still not well recognized by public.

From the viewpoint of control, the most distinct advantages of well-controlled electric motors over the internal combustion engines and hydraulic braking systems are:

1. Millisecond-level torque response (10 to 100 times faster)
2. Accurate feedback of the generated motor current/torque (motor torque \propto motor current)
3. Continuously variable speed in nature (see the torque-speed characteristics of electric motors depicted in Fig. 1)
4. Small size but powerful output (easy to implement distributed motor location using in-wheel motors)

The above unique characteristics make it possible to achieve high-performance motion control of EVs with flexible and simplified configurations.

Like the development of other mechatronic systems, model-based simulation/emulation is essential for efficient designs of EV control systems such as the Anti-lock Braking (ABS) and yaw dynamics control systems. Test benches have been developed for serving the purposes ranging from the design and test of propulsion motor drives to the implementation of Hardware-In-

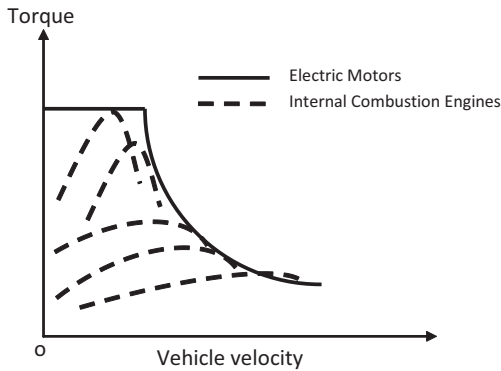


Figure 1. The torque-speed characteristics of electric motors and internal combustion engines

the-Loop (HIL) powertrain control strategies [2, 3]. As depicted in Fig. 2, the test benches usually have the following three main components: a dynamometer, a real-time data acquisition and digital control system, and a propulsion drive motor under test.

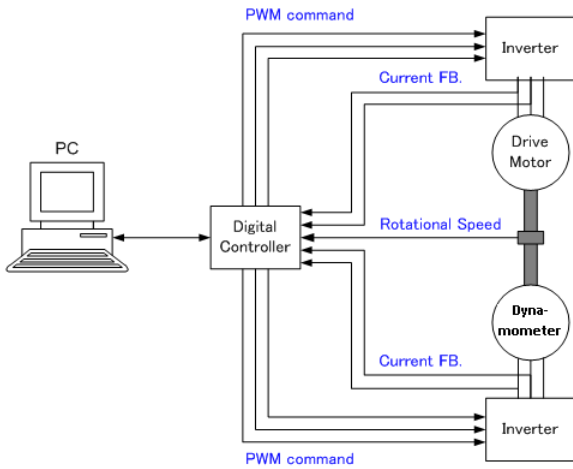


Figure 2. Electric vehicle test bench

For those test benches, no tire sliding (complete tire/road adhesion) is assumed. Therefore the equivalent equation to emulate EV's longitudinal dynamics is:

$$J_e \dot{\omega} = T_m - T_g - T_a - T_r \quad (1)$$

where J_e is the equivalent inertia of the vehicle, ω is the angular velocity of motor, T_m is the motor torque, T_g , T_a and T_r are

the equivalent torques for the slope resistance, aerodynamic drag and rolling resistance, respectively. The equivalent inertia J_e is calculated as:

$$J_e = J_w + Mr^2 \quad (2)$$

where M is the vehicle mass and J_w , r are the inertia and radius of the wheel, respectively.

As emphasized above, the high-performance vehicle control of EVs could be achieved by taking full advantage of electric motor's unique characteristics. Since the friction force in the tire/road interface is the main mechanism for converting wheel angular acceleration to forward acceleration, i.e. generating longitudinal force, the emulation of friction force characteristics at the road/tire interface is important for developing electric vehicle control systems. The friction model for emulating the road/tire interaction need be able to accurately capture the transient behavior of the friction force during braking and acceleration. The dynamic emulation of tire/road friction would be very helpful for the designs of vehicle control systems such as ABS and traction control, which rely on the knowledge of the friction characteristics.

2 REVIEW OF ROAD/TIRE FRICTION MODELS

In this paper a simplified motion dynamics of a quarter-vehicle model is considered. Unlike in Eqn. (1), the longitudinal dynamics is of the form:

$$\frac{M}{4} \dot{v} = F \quad (3)$$

$$J_w \dot{\omega} = -rF + T_m \quad (4)$$

where v is the linear velocity of the vehicle and F is the tire/road friction force. For simplicity's sake, the slope resistance, aerodynamic drag and rolling resistance are neglected.

A common assumption in the most tire friction models is that the normalized tire friction μ :

$$\mu = \frac{F}{F_n} = \frac{\text{Friction Force}}{\text{Normal Force}} \quad (5)$$

is a nonlinear function of the relative velocity between the tire and the road with a distinct maximum.

2.1 Static Slip/Friction Models

The static slip/friction models are the most common road/tire friction models used in the simulation of vehicle longitudinal dynamics. As depicted in Fig. 3, they are defined as

one-to-one maps between the friction F , and the longitudinal slip rate s which is defined as

$$\begin{cases} s = \frac{r\omega}{v} - 1, & \text{if } v > r\omega \text{ and } v \neq 0 \text{ for braking} \\ s = 1 - \frac{v}{r\omega}, & \text{if } v < r\omega \text{ and } \omega \neq 0 \text{ for driving} \end{cases} \quad (6)$$

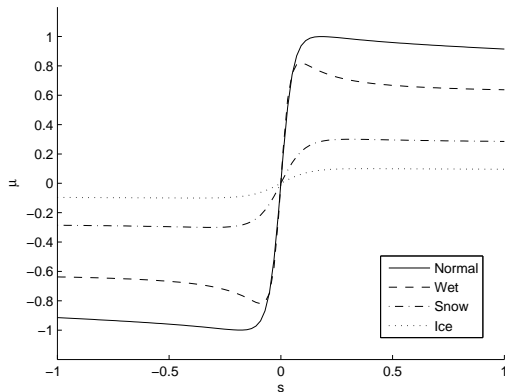


Figure 3. Plot of static slip/friction characteristics

One of the most well-known models of this type is Pacejka's model, also known as the "Magic Formula" [4]. This model has been shown to suitably match experimental data, obtained under particular conditions of constant linear and angular velocity. The Pacejka model has the form:

$$F(s) = c_1 \sin(c_2 \arctan(c_3 s - c_4 (c_3 s - \arctan(c_3 s)))) \quad (7)$$

where the parameters c_1, \dots, c_4 can be identified by fitting a given set of tire test data.

The static friction models assume the idealized steady-state conditions for the linear and angular velocities, i.e. v and ω in Eqn. (3) and Eqn. (4). In reality the linear and angular velocities can never be controlled independently especially during acceleration and braking.

2.2 Dynamic Friction Models

The dynamic friction models attempt to describe the transient behavior of the tire/road friction. There are two types of dynamic friction models, lumped friction models and distributed friction models. The lumped friction models assume a point tire/road contact; while the distributed ones assume a contact patch existing between the tire and the road [5]. Naturally the distributed friction models are represented by partial differential equations. Only the lumped friction model is discussed in this paper.

A number of dynamic friction models have been proposed such as the Bristle model, Dahl model, etc [6]. Among those models, LuGre model is the one of the most popular models for control system design with friction [7]. The LuGre model is an extension of the Dahl model with the Stribeck effect.

In the LuGre model, the mechanism of friction is described by two rigid bodies making contact through elastic bristles. If the deflection of the bristles is large enough, the bristles start to slip. An internal state z denotes the average deflection which is modeled by:

$$\dot{z} = v_r - \frac{\sigma_0 |v_r|}{g(v_r)} z \quad (8)$$

$$g(v_r) = \mu_c + (\mu_s - \mu_c) e^{-|v_r/v_s|^\eta} \quad (9)$$

where $v_r = r\omega - v$ is the relative velocity, σ_0 the stiffness of the bristles, μ_c the normalized Coulomb friction, μ_s normalized static friction ($\mu_s \geq \mu_c$), v_s the Stribeck relative velocity, η to capture the steady-state slip/friction characteristics. Typical value for η is between 0.5 and 2 (η is taken as 0.5 in this paper). Therefore the steady state of z , i.e. when v_r is constant, is:

$$z_{ss}(v_r) = \frac{g(v_r)}{\sigma_0 |v_r|} v_r = \frac{g(v_r)}{\sigma_0} \text{sgn}(v_r) \quad (10)$$

The friction force is:

$$F = F_n (\sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v_r) \quad (11)$$

The first two terms describe the elastic and dissipative forces generated by the deflection of the bristles (see Fig. 4), where σ_0 is the stiffness and σ_1 is the damping. The last term is for the vicious force proportional to v_r with coefficient σ_2 .

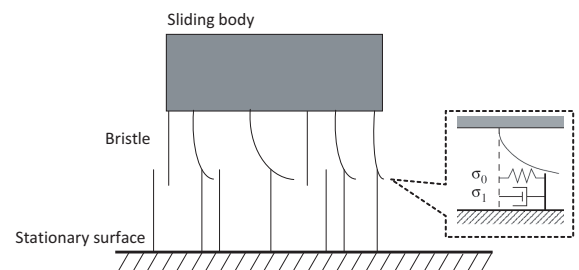


Figure 4. The friction mechanism of the bristle deflection

For the level of tire/road adhesion, it can be modeled by in-

roducing a parameter θ in the function $g(v_r)$:

$$\tilde{g}(v_r) = \theta g(v_r) \quad (12)$$

The LuGre model is popular because its parameters have a physical significance and the velocity-dependency is also physically consistent. However, the LuGre model exhibits drift for arbitrarily small external forces, which is not a physically consistent behavior [8]. This non-physical phenomenon results from inaccurate modeling of presliding as a combination of elastic and plastic placement.

The drawback of the LuGre model can be overcome by having an elasto-plastic presliding. A piecewise continuous function $\alpha(z, v_r)$ is introduced which controls \dot{z} to avoid the drift:

$$\dot{z} = v_r - \alpha(z, v_r) \frac{\sigma_0 |v_r|}{g(v_r)} z \quad (13)$$

where $\alpha(z, v)$ is defined as:

$$\alpha(v_r, z) = \begin{cases} 0, & |z| \leq z_{ba}, \text{sgn}(v_r) = \text{sgn}(z) \\ \alpha_m(v_r, z), & z_{ba} < |z| < z_{ss}(v_r), \text{sgn}(v_r) = \text{sgn}(z) \\ 1, & |z| \geq z_{ss}(v_r), \text{sgn}(v_r) = \text{sgn}(z) \\ 0, & \text{sgn}(v_r) \neq \text{sgn}(z) \end{cases} \quad (14)$$

where z_{ba} is the breakaway displacement below which the presliding is purely elastic, and a specific example of $\alpha_m(v_r, z)$ for a smooth transition between the elastic and plastic behavior (see Fig. 5) is:

$$\alpha_m(v_r, z) = \frac{1}{2} \sin \left(\pi \frac{z - \frac{z_{ss}(v_r) + z_{ba}}{2}}{z_{ss}(v_r) - z_{ba}} \right) + \frac{1}{2} \quad (15)$$

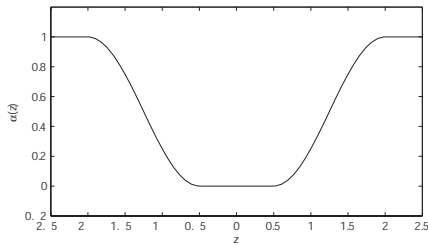


Figure 5. An example of the smooth elastic-to-plastic transition provided by $\alpha_m(v, z)$ where $z_{ba} = 0.5$ and $z_{ss} = 2.0$

For small displacements $\alpha = 0$ and thus $\dot{z} = v_r$ (purely elastic presliding), while for larger displacements the mixed elastic-plastic sliding is entered; and finally transitions to purely plastic is achieved with $\alpha = 1$, $\dot{z} = 0$ and $z = z_{ss}(v_r)$ at steady state.

3 EMULATION USING ELASTO-PLASTIC MODEL

As mentioned in the introduction section of this paper, the emulation of friction force's dynamic characteristics at the road/tire interface is important for developing electric vehicle control systems. The dynamic friction models need to be discretized for the digital controller to calculate the generated friction force. The bilinear transform can be applied to the LuGre model:

$$z_k = \frac{1 - \frac{T}{2} \frac{\sigma_0 |v_{r,k-1}|}{g(v_{r,k-1})}}{1 + \frac{T}{2} \frac{\sigma_0 |v_{r,k}|}{g(v_{r,k})}} z_{k-1} + \frac{\frac{T}{2}}{1 + \frac{T}{2} \frac{\sigma_0 |v_{r,k}|}{g(v_{r,k})}} (v_{r,k} + v_{r,k-1}) \quad (16)$$

$$F_k = F_n \left[\left(\sigma_0 - \sigma_1 \frac{\sigma_0 |v_{r,k}|}{g(v_{r,k})} \right) z_k + (\sigma_1 + \sigma_2) v_{r,k} \right] \quad (17)$$

where T is the sampling time.

However, the function $\alpha(z_k, v_{r,k})$ and z_k are depend on each other. In order to avoid this interaction (i. e. algebraic loop), a simple solution is to adopt $\dot{z} \approx \frac{z_k - z_{k-1}}{T}$. But the one sample delay introduced by taking back-forward difference is known to lead to inaccuracy and even instability in the simulations/emulations. Therefore in this paper an iterative Newton-Raphson technique is used to find a local zero (i.e. z_k) of the below function using z_{k-1} as the initial guess:

$$f(x) = x - \frac{A}{1 + B \cdot \alpha(x, v_{r,k})} \quad (18)$$

where

$$A = \left(1 - \frac{T}{2} \alpha(z_{k-1}, v_{r,k-1}) \frac{\sigma_0 |v_{r,k-1}|}{g(v_{r,k-1})} \right) z_{k-1} + \frac{T}{2} (v_{r,k} + v_{r,k-1}) \quad (19)$$

$$B = \frac{T}{2} \frac{\sigma_0 |v_{r,k}|}{g(v_{r,k})} \quad (20)$$

with derivative:

$$f'(x) = 1 + \frac{A}{[1 + B \cdot \alpha(x, v_{r,k})]^2} B \cdot \alpha'(x, v_{r,k}) \quad (21)$$

The derivative of $\alpha(x, v_{r,k})$ is also a piecewise function:

$$\alpha'(x, v_{r,k}) = \begin{cases} 0, & \alpha(x, v_{r,k}) = 1 \text{ or } 0 \\ \frac{1}{2} \cos \left(\pi \frac{x - \frac{z_{ss}(v_r) + z_{ba}}{2}}{z_{ss}(v_r) - z_{ba}} \right) \frac{\pi}{z_{ss}(v_r) - z_{ba}}, & \text{else} \end{cases} \quad (22)$$

Simulations show that five iterations of the Newton-Raphson method typically have a good convergence to the solution. After finding z_k , the friction force F_k is calculated as:

$$F_k = F_n \left[\left(\sigma_0 - \alpha(v_{r,k}, z_k) \sigma_1 \frac{\sigma_0 |v_{r,k}|}{g(v_{r,k})} \right) z_k + (\sigma_1 + \sigma_2) v_{r,k} \right] \quad (23)$$

The block diagram of emulating the dynamic friction force using the dynamometer is depicted in Fig. 6. The current command of the dynamometer is:

$$i_{d,k}^* = F_k \cdot r / K_t \quad (24)$$

where K_t is dynamometer's torque constant. Again vehicle speed v_k is dependent on F_k . Here v_k is simply approximated by the 1-order Taylor series expansion using F_{k-1} :

$$a_{k-1} = \frac{F_{k-1}}{M/4} \quad (25)$$

$$v_k \approx v_{k-1} + a_{k-1} T = \frac{T}{2} \frac{1+z^{-1}}{1-z^{-1}} a_{k-1} + a_{k-1} T \quad (26)$$

where v_{k-1} is calculated by the trapezoidal integration of the acceleration $\{a_1, \dots, a_{k-1}\}$.

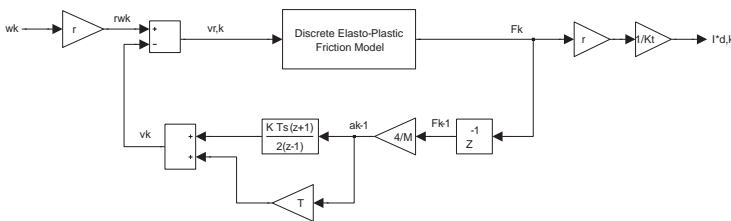


Figure 6. Emulation of the dynamic friction using the dynamometer

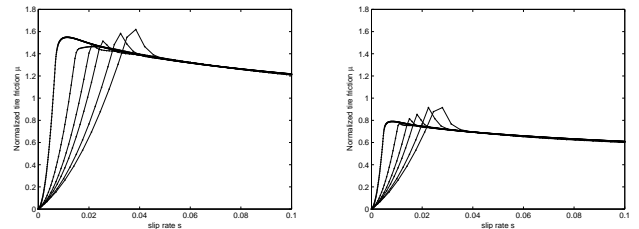
4 Simulations

Simulations are carried out using the proposed discrete elasto-plastic dynamic friction model. Data used for simulating the dynamic friction model is shown in Table. 1, which is from the experimental data and Ref. [8].

As depicted in Fig. 7, the transient behavior of the friction force with a constant vehicle velocity 60km/hr and various changing rate of s is different. This dynamic behavior can not be captured using the static slip/friction models such as the well-known ‘‘Magic Formular’’. And the level of tire/road adhesion can also properly modeled by introducing the parameter θ in the function of $g(v_r)$.

Table 1. Data used for simulations

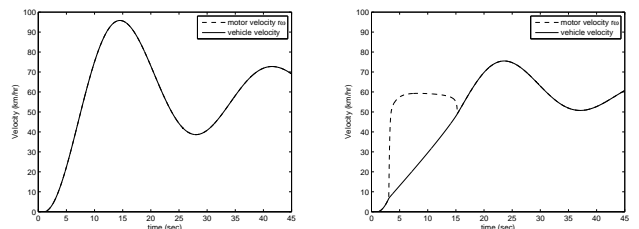
Parameter	Value
σ_0	316 m^{-1}
σ_1	1.0 s/m
σ_2	0.0005 s/m
μ_c	0.69
μ_s	1.779
v_s	3.5 m/s
z_{ba}	0.7



(a) $\theta = 1.0$ (b) $\theta = 0.5$

Figure 7. Normalized friction μ vs slip ratio s

For test purpose, the times responses of a step angular velocity command (equivalent longitude velocity is 60km/hr) are simulated with velocity and current PI controllers for the drive motor. Besides $\theta = 1.0$, a very slippery road surface with $\theta = 0.1$ is also simulated. As shown in Fig. 8, the step velocity command leads to oscillatory angular and vehicle velocity responses due to the large nonlinear friction force generated at the beginning (see Fig. 11.a and Fig. 11.c), especially for the slippery road surface.



(a) $\theta = 1.0$ (b) $\theta = 0.1$

Figure 8. Velocities $r\omega$ and v without DOB

The nonlinear friction force can be compensated by a ‘‘friction observer’’ [7]. However, as mentioned in the introduction

section, the accurate feedback of motor current can be used to estimate the torque by friction T_f . A simpler and more robust observer, the disturbance observer (DOB), can be introduced in which the friction force T_f is considered as a unmodeled dynamics in the wheel inertia model $J_w s$ and estimated as \hat{T}_f .

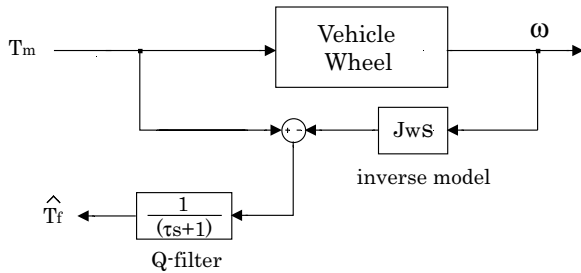


Figure 9. Estimation of T_f by the disturbance observer

As depicted in Fig. 10, the velocity responses are greatly improved by introducing the DOB for the both two tire/road adhesion levels. With the accurate feedback of motor current, the estimated \hat{T}_f by DOB well matches the torque by friction T_f (see Fig. 11). With the dynamic emulation of the tire/road friction, more reliable verifications for various EV control methods could be carried out.

5 Summary

In this paper, a discrete dynamic friction model is proposed to have a dynamic emulation of the tire/road friction for developing EV control systems. The model is an elasto-plastic model based on the LuGre model. For the bilinear discretization of the original continuous model, the Newton-Raphson method is introduced to solve the algebraic loop problem. Simulation results show the model can capture the transient behavior of the friction force, therefore the model-based emulation could enable more reliable verifications for various EV control methods. Future works include implementing the discrete dynamic friction model to EV test bench and also the verification of its performance by comparing with real experimental data.

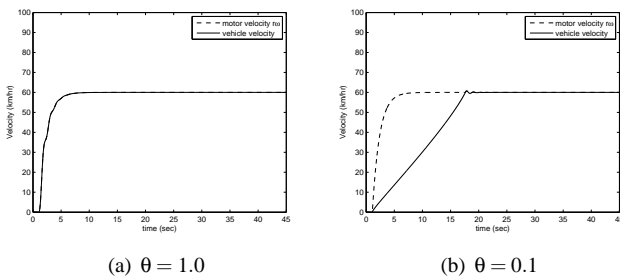


Figure 10. Velocities $r\omega$ and v with DOB

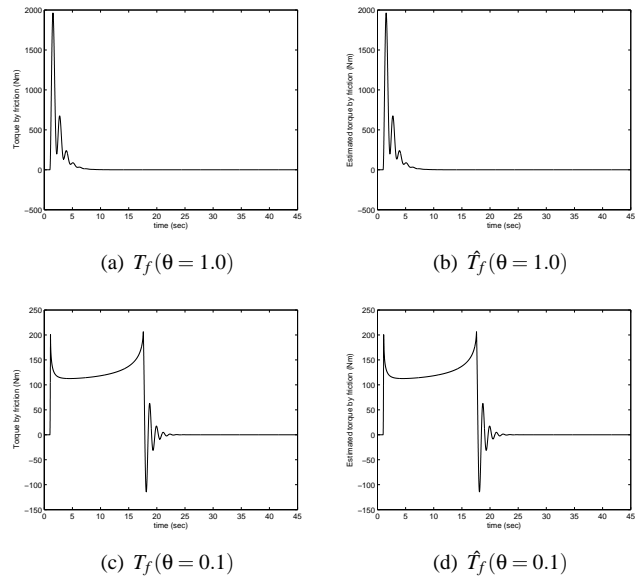


Figure 11. The torque by friction T_f and the estimated \hat{T}_f

REFERENCES

- [1] Hori, Y., 2004. "Future Vehicle Driven by Electricity and Control - Research on Four-Wheel-Motored "UOT Electric March II"". *IEEE Transactions on Industrial Electronics*, **51**(5), October, pp. 954-962.
- [2] Schupbach, R. M., Balda, J. C., 2002. "A Versatile Laboratory Test Bench for Developing Powertrains of Electric Vehicles". *Proceedings of Vehicular Technology Conference 2002-Fall*, Vol. 3, pp. 1666-1670.
- [3] Alcalá, I., Claudio, A., Guerrero, G., 2008. "Test Bench to Emulate An Electric Vehicle through Equivalent Inertia and Machine DC". *Proceedings of Power Electronics Congress 2008*, pp. 198-203
- [4] Short, M., Port, M. J., Huang, Q., 2004. *Simulation of Vehicle Longitudinal Dynamics*. Technical Report ESL 04/01, Embedded Systems Laboratory, University of Leicester
- [5] Canudas de Wit, C., Tsiotras, P., Velenis, E., Basset, M., Gissinger, G., 2003. "Dynamic Friction Models for Road/Tire Longitudinal Interaction". *Vehicle System Dynamics*, **39**(3), pp. 189-226.
- [6] Olsson, H., Åström, K. J., Canudas de Wit, C., Gäfvert, M., Lischinsky, P., 1998. "Friction Models and Friction Compensation". *European Journal of Control*, **4**(3), pp. 176-195.
- [7] Canudas de Wit, C., Olsson, H., Åström, K. J., Lischinsky, P., 1995. "A New Model for Control of Systems with Friction". *IEEE Transactions on Automatic Control*, **40**(3), March, pp. 419-425.
- [8] Dupont, P., Hayward, V., Armstrong, B., Altpeter, F., 2002. "Single State Elastoplastic Friction Models". *IEEE Transactions on Automatic Control*, **47**(5), May, pp. 787-792.