

# Acceleration-to-torque Ratio based Anti-skid Control for Electric Vehicles

Zhiyang Cai  
Department of Automation  
Shanghai Jiaotong University  
Email: cxqyxp@sjtu.edu.cn

Chengbin Ma  
Univ. of Michigan-Shanghai Jiaotong Univ.  
Joint Institute  
Shanghai Jiaotong University

Qunfei Zhao  
Department of Automation  
Shanghai Jiaotong University

**Abstract**—Anti-skid control for electric vehicles has attracted the attention of many researchers over the past years. Most of the approaches proposed need to measure or estimate the vehicle velocity to calculate the slip ratio. In this paper, an approach based on the ratio of wheel acceleration to motor torque for electric vehicle anti-skid control is proposed. Instead of the vehicle velocity, only two easily measured parameters for electric vehicles, wheel acceleration and motor torque, are used to estimate the slip level. Two types of controllers, a rules-based controller and a fuzzy logic controller are designed for the vehicle anti-skid control. Simulation results show that both of them can successfully prevent vehicle skidding.

## I. INTRODUCTION

Carbon emission mitigation has been a central issue for relieving the global warming effects. One major source of carbon emission is from billions of conventional internal combustion engine vehicles (ICEVs). It has been widely recognized that electrifying vehicles can provide a solution for both the emission and oil shortage problem. Consequently, most of the current researches on electric vehicles (EVs) including the hybrid EVs are focusing on the environment and energy aspects.

At the same time, due to the much higher control performance of electric motors than internal combustion engines, EVs are not only “cleaner” and “higher energy efficient”, but also have potential to become “safer” with “higher driving performance”. Compared with the internal combustion engines, the electric motors have the following advantages:

- 1) Fast torque response(10 to 100 times faster)
- 2) Accurate feedback of the generated motor current/torque (motor torque is proportional with motor current)
- 3) Small size but powerful output (easy to implement distributed motor location using in-wheel motors)

With the above advantages, high performance ABS (Anti-lock Braking System) and TCS (Traction Control System) for EVs can be realized with flexible and simplified configuration [1]. Additional hardware may not be needed for implementations of ABS and TCS systems. And with fast and accurate feedback control, it is possible to have a higher vehicle motion control performance than the conventional ICEVs.

Lots of researches on EV anti-skid control have been done. The most common method is based on slip ratio, a variable to describe the slip level of vehicles. Vehicle velocity is needed

to be measured or estimated for the calculation of the slip ratio. Then a slip ratio controller such as a PID controller can be applied to control the ratio to stay within a certain given range. Fuzzy logic control can also be introduced for vehicles anti-skid control. In M.Bauer’s paper, a fuzzy logic controller is designed for ICEVs [2]. The change rate of slip ratio and adhesion coefficient are selected as the inputs of the controller. A fuzzy logic controller specifically designed for EVs is proposed by V.D.Colli [3]. An index “A” is defined based on the change of adhesion coefficient and slip ratio. And the index is controlled to stay at zero by fuzzy logic. The methods mentioned above both have good performance for the anti-skid control. But the vehicle velocity is required to calculate the slip ratio, which is not easy to accurately measured or estimated. K.Fujii proposes an approach on EV anti-skid control based on slip ratio estimation without using vehicle velocity [4]. In this approach, a method based on compensation is proposed to estimate the slip ratio without using vehicle velocity. But it is not easy to determine the compensation gain. In K.Hartani’s paper, a behavior model is introduced to estimate wheel velocity when the vehicle is not skidding [5]. The difference between the estimated wheel velocity and the true wheel velocity is used to calculate the necessary compensation torque. The behavior model must be accurate enough in this approach, which is also not easy to guarantee. Another approach for anti-skid control is proposed by D.Yin [6]. Maximum transmissible torque is calculated based on a fixed ratio of estimated vehicle acceleration to wheel acceleration. The motor torque is limited within the maximum transmissible torque. However, the fixed ratio and model-based estimation of vehicle acceleration may cause robust problem in real applications. Xu proposed an approach based on the PI control of wheel velocity-to-torque ratio to prevent vehicle skidding[7]. However, it is difficult to directly estimate the slip level from the relationship between wheel velocity and torque. And the PI control may also have robust problems.

In this paper, a new approach to skidding prevention for electric vehicle in the longitudinal direction is proposed. This approach does not need to know the vehicle velocity to prevent skidding. By calculating the ratio of wheel acceleration to motor torque which are both easy to measure, the slip level can be estimated. Two anti-skid controllers, a rule-based controller

and a fuzzy logic controller are designed to control the motor torque. The simulation results show the two controllers can both successfully prevent the vehicle skidding; while the fuzzy logic controller has better accuracy than the rule-based controller.

## II. EV ANTI-SKID CONTROL APPROACH

### A. Model of Longitudinal Dynamics

In this paper, only the dynamics in the longitudinal direction is considered. Because the air drag is the main factor to effect the longitudinal motion of vehicles only when the velocity of vehicle is higher than 60 mph [8], for the sake of simplicity it is neglected here.

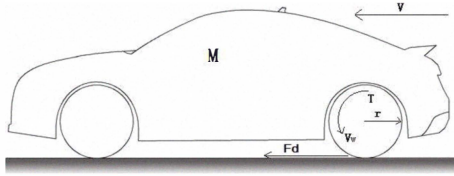


Fig. 1. The dynamic model of the vehicle in the longitudinal direction

As shown in Figure 1, based on Newton's second law, the longitudinal dynamic differential equations can be described as follows:

$$J_w \dot{\omega} = T - f \cdot r \quad (1)$$

$$M \dot{V} = F_d \quad (2)$$

$$\lambda = \frac{V_w - V}{\max(V, V_w)} \quad (3)$$

$$f = \phi(\lambda) \quad (4)$$

$$F_d = f \quad (5)$$

where  $J_w$  is wheel inertia,  $\omega$  wheel angular velocity,  $T$  the motor torque,  $r$  wheel radius,  $f$  friction force,  $M$  vehicle mass,  $V$  vehicle velocity,  $F_d$  driving force,  $\lambda$  slip ratio and  $V_w$  wheel velocity, respectively.

Here only the acceleration case is discussed, namely  $V_w$  is always higher than  $V$ . Therefore the slip ratio  $\lambda$  can be represented as:

$$\lambda = \frac{V_w - V}{V_w} \quad (6)$$

The famous "Magic Formula" is introduced to describe the relationship between the slip ratio and the friction coefficient. Using the Pacejka magic model proposed in 1989 [9], friction force between tire and road can be modeled as Equ. 7:

$$f = \phi(\lambda) = Mg \cdot D(\sin(\text{Carctan}(B\lambda - E(B\lambda - \arctan(B\lambda)))))) \quad (7)$$

where the coefficients B, C, D and E are determined by a set of experimental data [9] as shown in Table I:

The Magic Formula can describe the friction force in various road surfaces by choosing a given set of model coefficients. For example, the date set of B=12, C=2.3, D=0.82 and E=1 is

TABLE I  
THE WHEEL MODEL COEFFICIENTS

Wheel Model Coefficients				
Surface	B	C	D	E
Normal	10	1.9	1	0.97
Wet	12	2.3	0.82	1
Snow	5	2	0.3	1
Ice	4	2	0.1	1

describing the road/tire friction on the wet tarmac road. Based on the magic model, Equ. 7, and the date sets in Table I, the one-to-one maps between the slip ratio and friction coefficient under the four various road surfaces are shown in Figure 2.

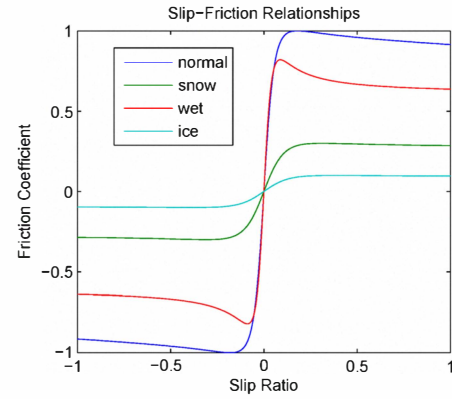


Fig. 2. The relationship between slip ratio and friction coefficient

### B. Acceleration-to-torque Ratio based Anti-skid Control

As mentioned above, for conventional anti-skid control, the vehicle velocity need to be directly measured or estimated. However, there are problems on increased hardware cost, complicated algorithm, insufficient accuracy, etc. Therefore in this paper, a new approach for EV anti-skid control is proposed, in which the information of vehicle velocity is not needed.

The driving force for the vehicle is the friction between road and tire. According to Figure 2, the friction can not be infinity. It is limited within a certain range. Namely the vehicle acceleration is also limited. If the wheel acceleration is beyond the certain range, the vehicle can be considered skidding.

The anti-skid control strategy based on the wheel acceleration is used in traditional ABS devices [10]. However, the wheel acceleration is the only available parameter, which causes an inaccurate estimation of the slip level. As mentioned in the introduction section of this paper, compared with vehicles driven by internal combustion engines, EVs' motor torque can be measured easily with much better accuracy. Using this newly added parameter, i.e. the driving torque, the slip level could be more accurately estimated. According to Equ. 1-Equ. 5, the wheel acceleration and the vehicle acceleration can

be calculated as:

$$\dot{V}_w = \frac{r \cdot T - \dot{V} M r^2}{J_w} \quad (8)$$

$$\dot{V} = \frac{F_d}{M} \quad (9)$$

The ratio of vehicle acceleration to wheel acceleration is defined as  $\alpha$ :

$$\alpha = \frac{\dot{V}}{\dot{V}_w} \quad (10)$$

From Equ. 8 - Equ. 10, a new parameter can be defined as  $R_{at}$ :

$$R_{at} \triangleq \frac{\dot{V}_w}{T} = \frac{r}{J_w + \alpha M r^2} \quad (11)$$

Equ. 11 describes the relationship between wheel acceleration and motor torque. The ratio of them is defined as  $R_{at}$  here. Mathematically, the denominator of  $R_{at}$ ,  $J_w + \alpha M r^2$ , is the total equivalent inertia of vehicle body and wheel. In order to prevent vehicle skidding, the value of  $\alpha$  should be close to 1. And in the case of acceleration,  $\alpha$  should be taken less than 1 to avoid limiting the acceleration performance of the vehicle. According to Equ. 3,  $\alpha$  is approximately equal with  $1-\lambda$ . It is considered that the slip ratio ranging from 0.1 to 0.3 is safe, therefore the value of  $\alpha$  ranging from 0.7 to 0.9 can be defined as normal values. If  $\alpha$  is less than 0.7, the vehicle is considered being skid and if  $\alpha$  is higher 0.9, the acceleration performance will be limited. Since  $\alpha$  is related with  $R_{at}$ , as shown in Equ. 11, by calculating  $R_{at}$ , the vehicle slip level can be estimated and used for the anti-skid control. The relationship between  $R_{at}$  and  $\alpha$  is shown in Figure 3.

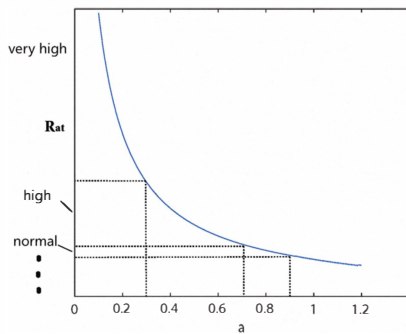


Fig. 3.  $\alpha$  Vs  $R_{at}$

### C. Controllers Design

In this subsection, two types of anti-skid controller, a rules-based controller and a fuzzy logic controller are designed. The control diagram of the system is shown in Figure 4.

As shown in Figure 4,  $R_{at}$  is calculated by dividing signals of the wheel acceleration and motor torque, and be used as the input of the controller. The output is a compensation torque calculated by the controller to adjust the motor torque for the prevention of vehicle skidding. To get a more stable result, the

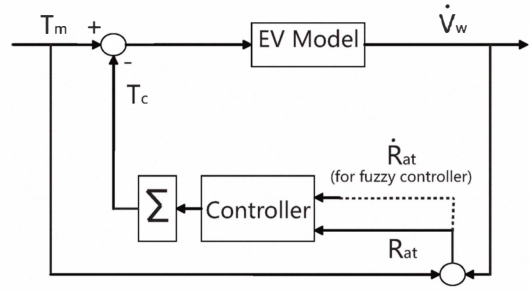


Fig. 4. The anti-skid control diagram

output of controller,  $\Delta T_c$ , is the variation of the compensation torque. The final motor torque can be calculated by Equ. 12.

$$T = T_m - \sum \Delta T_c \quad (12)$$

1) *Rule-based controller*: Firstly, the compensation torque  $T_c$  is determined by rules. Because the relationship between  $\alpha$  and  $R_{at}$  is close to the inversely proportional function as shown in Figure 3. When  $R_{at}$  is very high, it will be a very dangerous situation that may lead to vehicle skidding. Therefore a big increment of output compensation torque need to be generated. And when  $R_{at}$  is too low, the vehicle's acceleration performance will be adversely limited. However, because this situation is not dangerous, a relatively small decrement of the output compensation torque can keep the vehicle being stable. Therefore, the rules can be described as follows:

- 1) If  $R_{at}$  is very high, the compensation torque increases a lot
- 2) If  $R_{at}$  is high, the compensation torque increases a little
- 3) If  $R_{at}$  is normal, the compensation torque keeps no change
- 4) If  $R_{at}$  is low, the compensation torque decreases a little
- 5) If  $R_{at}$  is very low, the compensation torque decreases a lot

The relationship between  $R_{at}$  and the controller output is shown as Figure 5.

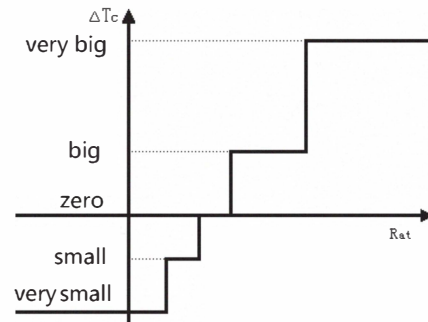


Fig. 5. The output of the controller

2) *Fuzzy logic controller* : Fuzzy controller has satisfied performance for nonlinear systems. Actual vehicle dynamics is highly nonlinear and there are many uncertainties included. Due to this reason, fuzzy logic controller has been applied to vehicle control in recent years and shown promising results. Here, a fuzzy logic controller is designed to prevent vehicle skidding. The control configuration is same with the rule-based control, as shown in Figure 4, but the controller is replaced by a fuzzy logic controller. And the change rate of  $R_{at}$  is selected as another input of the controller. The membership function of  $R_{at}$ , its change rate and the output is shown as Figure 6.

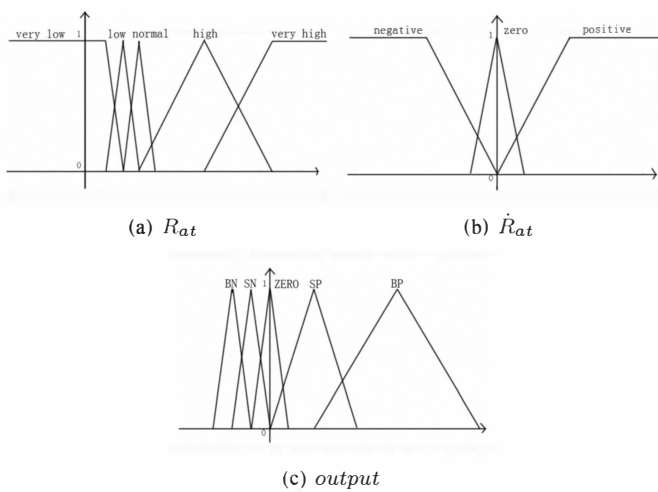


Fig. 6. Membership functions of inputs and output

The fuzzy rules are described as Table II.

TABLE II  
FUZZY LOGIC RULES

$R_{at}$ \ $\dot{R}_{at}$	negative	zero	positive
very high	SP	BP	BP
high	zero	SP	SP
normal	SN	zero	SP
low	SN	SN	zero
very low	BN	BN	SN

As shown in the above fuzzy rules, if  $R_{at}$  is very high and  $\dot{R}_{at}$  is negative, the compensation torque increases a little. If  $R_{at}$  is very high and  $\dot{R}_{at}$  is zero or positive, the compensation torque increases a lot. The other rules are decided following this basic consideration. The same as rules-based control proposed above, the increment of compensation torque is relatively big when the output state of the controller is BP to prevent the vehicle from skidding as soon as possible. And the decrement is relatively small when the output state of the controller is BN or SN in order to avoid adverse effect on vehicle's acceleration performance.

### III. SIMULATION RESULTS

Simulations are carried out to verify the anti-skid control based on  $R_{at}$ , the ratio of wheel acceleration to motor torque. In the simulations, the mass of vehicle is chosen as  $500kg$ , the radius of wheel  $0.25m$  and the wheel inertia  $1.1kg \cdot m^2$ . Firstly, a constant motor torque command from driver is chosen as  $500 N \cdot m$  to make it easy for vehicle to skid. And the wheel model coefficients is chosen as  $B=5, C=2, D=0.3, E=1$ , which is the data set for the snow road. Figure 7 shows the simulation results of the velocity of the wheel and vehicle and the slip ratio without control.

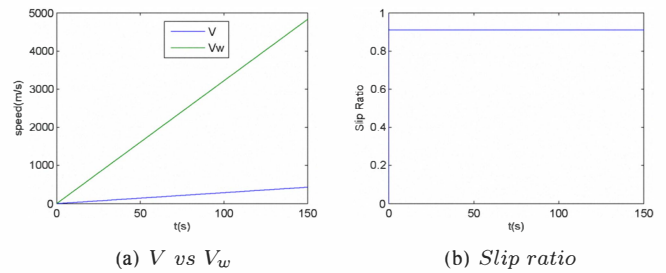


Fig. 7. Simulation results without control

As shown in Figure 7, the wheel velocity is much faster than the vehicle velocity and the slip ratio is almost 0.9, namely the vehicle is in a serious skidding. Then the rules-based controller and the fuzzy logic controller are applied. In the rules-based control, the variation of compensation torque “very big” is chosen  $50N \cdot m$ , and “big”  $12.5N \cdot m$ , “small”  $-5N \cdot m$ , “very small”  $-10N \cdot m$ . The membership function of fuzzy logic controller is determined based on the consideration of the rules-based controller. The simulation results are shown as Figure 8.

Figure 8 shows that for both rules-based controller and fuzzy logic controller, the velocities of wheel and vehicle are controlled within an acceptable range. The slip ratio is controlled in the range from 0.1 to 0.3 and the drive motor torque output is decreased to a safe level. In order to further verify the effectiveness of the proposed anti-skid controllers, the simulation data is changed, in which the motor torque signal is  $200 N \cdot m$ , and the vehicle is simulated running on the ice road surface. The simulation result is shown as Figure 9.

Simulation results show that the anti-skid control based on regulating the ratio  $R_{at}$  is also effective. If the vehicle skidding is detected, a corresponding compensation torque will be generated to stop the skidding. Both the rules-based control and fuzzy control have satisfied performance. The rules-based control has a more stable compensation torque because when the value of  $R_{at}$  drops into the normal range, the compensation torque will not change anymore. But the  $R_{at}$  with normal range is not specified to a certain value. For fuzzy logic controller, the compensation torque keeps changing until the value of  $R_{at}$  reaches the pre-described value. As shown in Figure 8(b) and Figure 9(c), it is interesting that the behavior of the controller is similar to the “cadence braking” technique performed by human driver, but here it is for the acceleration

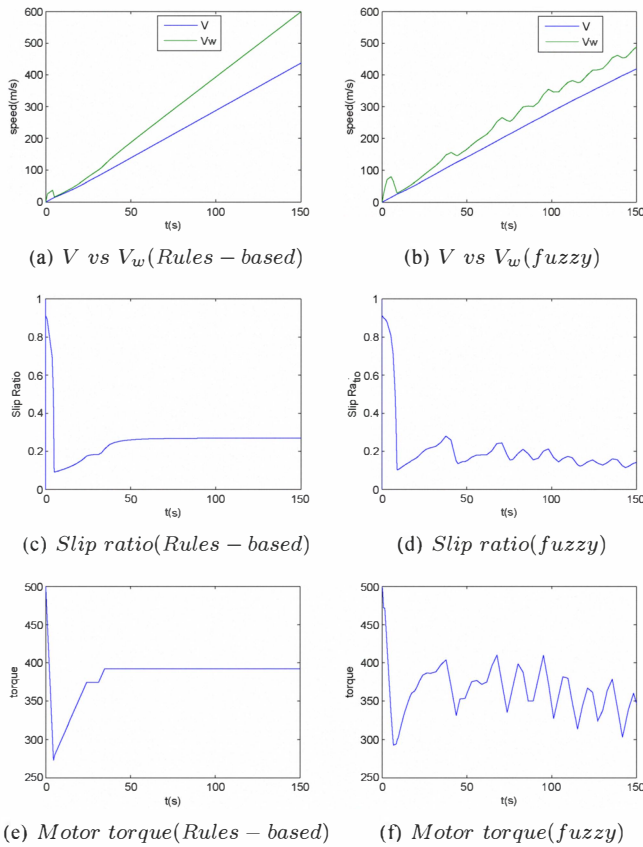


Fig. 8. Comparison of rules-based and fuzzy logic controllers

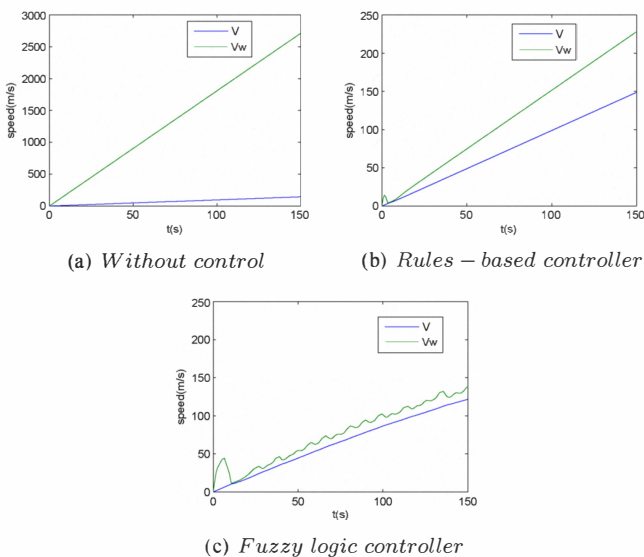


Fig. 9. The simulation results with modified data

case. It can be seen that the fuzzy control is relatively more accurate than the rules-based control.

#### IV. SUMMARY

In this paper, a new approach for EV anti-skid control without using the vehicle velocity is proposed. Two parameters of vehicles, the wheel velocity and drive motor torque, which are easy to measure for electric vehicles, are used to estimate the slip level in vehicle's longitudinal direction. A new parameter  $R_{at}$ , the ratio of wheel acceleration to motor torque, is introduced to estimate the vehicle slip level. If the vehicle skidding is detected, motor torque will be adjusted to stop the skidding. Two types of controllers, rules-based controller and fuzzy logic controller are designed based on this basic consideration. Simulation results show both two controllers have satisfied performance. But for real applications, there will be noise problems for measuring the wheel acceleration and driving motor torque. Necessary filtering of the signals is needed. Other further works may include applying the approach to EV test bench and verifying its feasibility for implementation in real EVs.

#### REFERENCES

- [1] Y. Hori. "Future Vehicle Driven by Electricity and Control - Research on Four-Wheel-Motored 'UOT Electric March II'". *IEEE Transactions on Industrial Electronics*, VOL. 51, NO. 5, October 2004, pp.954-962.
- [2] M. Bauer, M. Tomizuka. "Fuzzy Logic Traction Controllers And Their Effect On Longitudinal Vehicle Platoon Systems". *Vehicle System Dynamics*, VOL. 25, Issue 4 April 1996, pp.277 - 303.
- [3] V. D. Colli, G. Tomassi, M. Scarano. "Single Wheel Longitudinal Traction Control for Electric Vehicles". *IEEE Transaction on Power Electronics*, VOL. 21, NO. 3, May 2006, pp.799-808.
- [4] K. Fujii, H. Fujimoto. "Traction Control based on Slip Ratio Estimation Without Detecting Vehicle velocity for Electric Vehicle". Power Conversion Conference, 2007, Nagoya, Japan.
- [5] K. Hartani, M. Bourahla, Y. Miloud. "New Anti-skid Control for Electric Vehicle Using Behavior Model Control based on Energetic Macroscopic Representation", *Journal of Electrical Engineering*, VOL. 59, NO. 5, 2008, pp.225-233.
- [6] D. Yin, S. Oh, Y. Hori. "A Novel Traction Control for EV Based on Maximum Transmissible Torque Estimation". *International Journal of Intelligent Transportation Systems Research*, VOL 8, NO. 1, January 2010, pp.1-9.
- [7] P. Xu, Z. Hou, G.F. Guo, L.P. Zhang, B.G. Cao, H. Y. Long, X. Chen. "Anti-slip regulation of electric vehicle without speed sensor", IEEE International Symposium on Industrial Electronics, 2009, Seoul, Korea.
- [8] T. D. Gillespie, L. Q. Zhao, D. F. Jin. "Fundamentals Of Vehicle Dynamics (Chinese Vision)", Tsinghua University Press, 2006.
- [9] M. Short, M. J. Pont, Q. Huang. "Simulation of Vehicle Longitudinal Dynamics". Technical Report ESL 04/01, Embedded Systems Laboratory, 2004, University of Leicester.
- [10] U. Kiencke, L. Nielsen. "Automotive Control Systems for Engine, Driveline, and Vehicle (Second edition)", Springer, 2005.