Kriging Assisted On-line Torque Calculation for Brushless DC Motors Used in Electric Vehicles

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Abstract- **Torque is one of the most important control factors for a vehicle's motion. Compared with internal combustion engine, electric motors can have a more accurate torque feedback. In electric vehicles, direct torque control of the permanent magnet synchronous motor has been studied. However, because of non-ideal back EMF phenomenon, direct torque control of brushless DC motors has not been widely used. In this paper, a new method using kriging to calculate back EMF in a real-time on-line fashion is presented. Kriging prediction is used to approximate the back EMF of the motor based on information from sampled points. With motor speed and rotor position as inputs, kriging predicts back EMF as the output that is used to calculate the motor torque with three phase currents. Using this novel method, motor torque can be accurately calculated and implement in micro control units of vehicles, even when facing extremely high/low temperature and aging conditions.**

I. INTRODUCTION

Electric vehicles (EVs), especially for those using in-wheel motors, can have better motion control performance than vehicles only using internal combustion engines [1]. For example, Antilock Braking System (ABS) and Traction Control System (TCS) can be realized more accurately without additional hardware for EVs [2]. However, all those functions require accurate calculation of the torque generated by motors. Torque is one of the most important control factors for a vehicle's motion since the vehicle's acceleration essentially is a direct response of the torque. For internal combustion engine vehicles (ICEVs), torque is provided by the combustion in cylinders. However, it is difficult to estimate the torque accurately for ICEVs due to the chemical reaction of the combustion. On the other hand, in EVs, the torque is generated by motors and in general proportional to the motor current [10]. It is possible to estimate the torque more accurately for motors than that from internal combustion engines. In this regard, how to accurately calculate the motor torque is critical for motion control, driving and safety performance of EVs.

Several torque control methods for electric motors have been proposed in the literature [3~9], in which torque calculation is the most important part. A typical one of them is the vector control method [9, 10]. In this method, it is assumed that the motor torque is always linear to the motor current. However, the actual relationship of the torque and current is not ideally linear, especially when the current reaches a certain large value. Recently, lots of research work focuses on direct torque control (DTC) methods [3~5]. Using those DTC methods, accurate torque values can be obtained with reduced torque ripple and better torque control performance [3~8].

Generally, different types of motors (e.g., permanent magnet synchronous motors (PMSMs) and brushless DC (BLDC) motors) could use different torque control methods. Although PMSMs are widely used in modern electric vehicles, BLDC motors are still very common for electric cars and bicycles. As a matter of fact, there are millions of electric bicycles with BLDC motors being used everyday in China. Compared to PMSMs, BLDC motors are cheaper, simpler, but more reliable. They can work properly even under extreme working conditions (e.g., raining, rough road, etc.). Their outstanding reliability has already been approved by the significant amount of electric bicycles for many years. Moreover, the control methods for BLDC motors are much simpler compared to those of PMSMs or other types of motors [6]. For small and low cost electric vehicles which may become popular in the future, BLDC motors can certainly be one of the reasonable technical options.

Several approaches have been proposed to calculate the torque in motor control methods for different types of motors. For PMSMs with sinusoidal back electromotive force (EMF), researchers proposed different methods to calculate the torque and to reduce torque ripple [5]. For BLDC motors with trapezoidal back EMF, Liu proposed a direct torque control method based on the torque calculation from flux-linkages [3]. However, this method has differential terms which are difficult to be calculated or implemented in the motor controllers. In Yang's paper [4], the torque was calculated using three phase currents and the back EMF where the back EMF was obtained by assuming that the top value of the back EMF is linear to the motor speed. However, the actual max back EMF value is not ideally linear to the speed, and the transient procedure from the min to the max EMF cannot be demonstrated in this relationship. Since the back EMF of these BLDC motors is not ideal trapezoidal, the transient procedure of the back EMF is very important for torque calculation [11].

Another critical problem for motor control is motor parameters can vary a lot when motor working condition changes. Motors used in EVs face more severe problems under extreme environment situations, such as temperature variation, vibration, and aging. In a hot summer day, the temperature of a running motor in EVs can reach above 100°C. During the winter in some cold districts, motors need to be started at a temperature far below zero centigrade. Motor flux can be 20%

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smaller in the extremely high temperature and 6% larger in the extremely low temperature [12, 13]. At the same time, the motor resistance becomes 48% larger in the hot season and 25% smaller under cold environment [12, 13]. Moreover, the motors in electric bicycles and vehicles are designed for a long life time, i.e., about or more than 10 years, comparable to ICEVs nowadays. Considering the aging phenomenon during the lifetime of motors, magnet capability remains only 70% of their original value at the end of their lives for permanent magnet motors [14]. Thus, the static back EMF calculation is not enough for obtaining an accurate torque value. A dynamic on-line back EMF prediction method with certain learning capability is definitely necessary and important for real-time motor controllers in EVs.

In this paper, a new and efficient kriging assisted on-line torque (KAOT) estimation method for BLDC motors is proposed, which can be easily implemented in micro control units (MCUs). In this approach, the back EMF is estimated on-line in a dynamic manner. The motor torque can be accurately calculated with large temperature variation. Moreover, aging effect is automatically taken care of within the proposed method. This approach is developed with the help of kriging prediction which estimates the response of a function at unobserved points based on the function values at sampled points. There are several methods that can be used for prediction, such as kriging, neural networks, radial basis function, and so on [15]. These prediction methods can be off-line or on-line. For on-line approximation which is more flexible for complex control systems, neural networks have been widely used as an intelligent control method. However, neural networks require a great amount of samplings and calculation, which consumes more computational resources of MCUs [15, 16]. For torque calculation of BLDC motors in this paper, the motor speed and rotor position are the inputs while the back EMF is the output. In this case, kriging is more appropriate in terms of computational efficiency. With this help of this method, we could accurately estimate the back EMF and then the motor torque in the MCU of a vehicle without the assumption on the linearity of the back EMF and the motor speed. More importantly, the effect of large variation of motor parameters due to aging and temperature is taken care of in this method, which makes the proposed method more realistic and applicable for BLDC motors used in EVs.

In this paper, the background of motor torque calculation and kriging is provided in Sec. Ⅱ, followed by the discussion on our new KAOT calculation method in Sec. Ⅲ. The simulation and experiment results are presented in Sec. Ⅳ and Ⅴ. Conclusions and possible further work are discussed in Sec. Ⅵ.

II. BACKGROUND

A. Torque Calculation for Permanent Magnet Motors

For PMSMs with sinusoidal back EMF, the torque can be calculated as in (1):

$$T = \frac{3}{2} \cdot \frac{p}{2} (\varphi_{s\alpha} i_{s\beta} - \varphi_{s\beta} i_{s\alpha}) \tag{1}$$

where T is the torque of the motor, p is the number of poles, $\varphi_{s\alpha}$ and $\varphi_{s\beta}$ are α and β axis's stator flux linkage respectively, and $i_{s\beta}$ and $i_{s\alpha}$ are β and α axis's stator currents respectively [3, 5].

For PMSMs, their voltage space vectors need only three PWM switching signals for three phases. The states of the upper switch and the lower switch should be opposite. For example, in phase a, if the upper switch is 1, the lower switch must be 0. Thus, in order to control six switches, only three switching signals are necessary [3]. However, BLDC motors that have non-sinusoidal back EMF use a different control mechanism. The torque equation for BLDC motors has a more complex form as shown in (2):

$$T = \frac{3}{2} \cdot \frac{p}{2} \left(\frac{d\varphi_{r\alpha}}{d\theta} i_{s\alpha} + \frac{d\varphi_{r\beta}}{d\theta} i_{s\beta} \right) \tag{2}$$

where θ is the electrical angle and $\varphi_{r\alpha}$, $\varphi_{r\beta}$, $i_{s\alpha}$, $i_{s\beta}$ are rotor flux linkages and stator currents [3, 7, 8]. The difficulty here is that (2) has differential terms which can be a serious problem for real-time calculation and control.

Different from PMSMs, BLDC motors' voltage space vectors require all six PWM switching signals for the three phases. There are at most one upper switch and one lower switch being "ON" at the same time, and others are all "OFF". That is, there is only one route for the current to pass through coils and all six switching signals are required to determine which route is open [3].

Another way to calculate the motor torque was proposed by Yang for BLDC motors [4]. The torque is estimated using the following formula:

$$T = \frac{e_a i_a + e_b i_b + e_c i_c}{\omega} \tag{3}$$

where e_a, e_b, e_c are the three phase back EMF, i_a, i_b, i_c are the three phase current, and ω is the motor speed. In order to obtain the torque value, three phase currents, three phase back EMF, and the motor speed are necessary. For BLDC motors, the motor speed can be calculated based on the feedback signals from Hall position sensors, while the current signal can be obtained from Hall current sensors.

B. Kriging Prediction

Kriging is a method to predict the response of unobserved points according to the response value sampled at the observed points [15, 16, 18]. It has been widely used as an approximation model recently for predicting simulation's responses at different points, especially in the field of system design [15, 17, 19]. The simulation model that will be approximated using kriging should be deterministic generally, which means that repeated runs of the model with same inputs should have the same responses [18]. Also, kriging has the advantage that kriging provides actual response values at the observed point, which may not always be realized in other response surface methods [19].

Suppose the deterministic response from the simulation model is represented as $y(x)$, which is estimated by a known polynomial $p(x)$ and a random process $Z(x)$ with zero mean and non-zero variance:

$$y(x) = p(x) + Z(x) \quad (4)$$

The following is the covariance matrix of $Z(x)$:

$$cov[Z(\omega), Z(x)] = \sigma^2 R(\theta, \omega, x) \quad (5)$$

where R is the correlation model between two observed points ω and x. The parameter θ is unknown and will be determined by the degree of the correlation among responses. The larger the θ value, the faster the decrease speed of R. In this paper, the correlation function R is defined as:

$$R(\theta, \omega, x) = exp\,[-\sum_{n=1}^{N} \theta_n |\omega_n - x_n|] \quad (6)$$

where ω_n and x_n are *n-th* components of the observed points ω and x. The estimated $y(x_0)$ at an unobserved point x_0 can be given by:

$$y(x_0) = \beta + r^T(x_0) R^{-1}(y - p\beta) \quad (7)$$

where:

$$\beta = (p^T R^{-1} p)^{-1} p^T R^{-1} y \quad (8)$$

$$r^T(x_0) = [R(x_0, x_1), R(x_0, x_2), \ldots, R(x_0, x_{n_0})) \quad (9)$$

where x_{n_0} is the n_0-*th* observed points. In this paper, we choose a zero-order polynomial regression model, that is p=1 in (8).

III. Kriging Assisted On-line Torque (KAOT) Calculation

A. A novel approach of back EMF and torque calculation

An essential problem for Eq. (3) is how to calculate the back EMF. In Yang's work, three phase back EMF are obtained by using the shape function, which assumes that the amplitude of three phase back EMF is linear to the motor speed [4]. Using the shape function, the back EMF as well as the torque can be calculated. However, based on the experiments and surveys of actual motors, this relationship is not ideally linear and using this relationship to obtain the back EMF may not be accurate. More importantly, Yang's work cannot estimate the changes of motor parameters due to the temperature variation and aging.

In this paper, kriging is used for the first time in the field of motor control to estimate the back EMF. The back EMF is estimated based on kriging prediction with the motor speed and rotor position signals being inputs and motor parameters being changed to demonstrate the effect of temperature variation and aging. After that, the predicted back EMF can be used for the calculation of the motor torque. The original simulation of calculating the back EMF, the setup of kriging prediction model, and the analysis model of our KAOT for BLDC motors are discussed in the following three sub-sections respectively.

B. Original simulation model of Back EMF

The original simulation of calculating the back EMF is built based on a BLDC motor model provided in Matlab/Simulink (see Fig. 1) [20]. In order to generate the training samples, the motor is driven by a dynamometer to accelerate to a certain rotation speed with a steady acceleration in this original simulation model, as shown in Fig. 2. In this case, the motor actually acts like an electrical generator and the back EMF can be considered as a function only related to the electrical angle and rotor speed. During this process, the electrical angle θ and back EMF are also changing, as given in Fig. 2. In each sample point, the rotor speed and electrical angle are obtained as inputs to the kriging model and the obtained back EMF being the response of the kriging model. Additionally, by changing the parameters in the original simulation model (e.g., motor flux and resistance), the effect of temperature changing and aging can be demonstrated clearly. With these observed values, a kriging prediction model will be established.

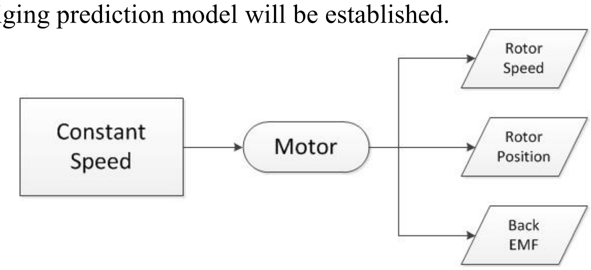


Fig. 1 Simulation Model

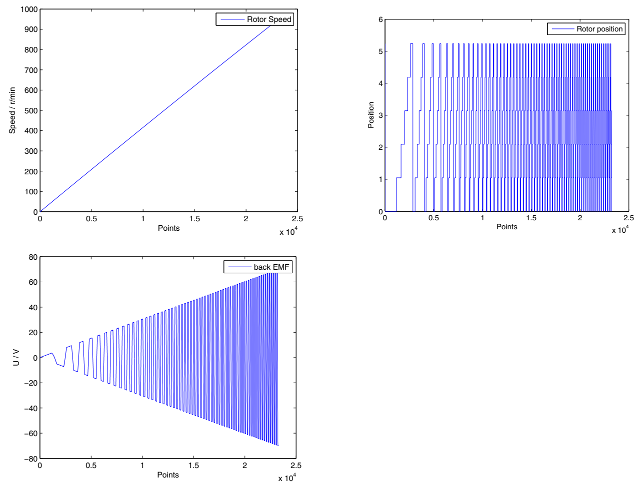


Fig. 2 Rotor Speed, Position and Back EMF

C. Sampling

Another important issue on building a prediction model is how to select samplings. The sampling technique used in this work is developed based on Latin hypercube sampling with special considerations about the property of the back EMF in BLDC motors.

Latin hypercube sampling (LHS) is a method to generate a sample distribution of parameters within a given space [21]. For a grid containing sample positions, if there is only one sample point in each row and each column, it is a Latin square. For a Latin hypercube, each sample is the only one in each hyper-plane. Sample points obtained by LHS contain points near the boundaries for each dimension, which ensures LHS to be a better representative sampling method than other random sampling techniques.

In an ideal case the back EMF changes significantly at six special electrical angle points, i.e., 0, 1/3 π, 2/3 π, π, 4/3 π, and 5/3 π. At each electrical angle point, LHS is applied to generate ten speed points. Sixty points in total are used as samples to build the kriging prediction model. The quantity of sample points used in the kriging prediction model is a key factor for the accuracy of the kriging model. However, more points do not always mean more accuracy, while fewer points may fail to set up an accurate model. As the best choice based on our experiments, sixty observed points are used in this paper to build the kriging prediction model with the linear correlation function and constant regression model being used. After the training process, the kriging prediction model can estimate the back EMF value when given a combination of motor speed and rotor position as inputs.

D. Analysis model of KAOT

In our KAOT analysis model for BLDC motors (Fig. 3), a constant power source is used to supply the energy to the motor. In this case, however, three phase back EMF cannot be obtained simultaneously from the motor simulation since the motor now acts like a driving machine and its back EMF values are affected by many factors (e.g., power supply) [22, 23]. To overcome this difficulty, the estimated back EMF from kriging is used to calculate the torque which is the feedback to the motor controller. The entire analysis model is shown in Fig. 3 with the detailed back EMF estimation module being called out.

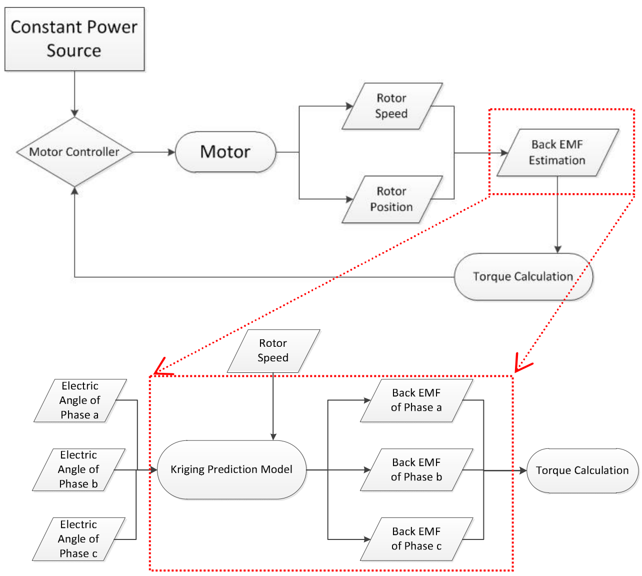


Fig. 3 KAOT Analysis Model

IV. RESULTS OF KAOT

As shown in Fig. 3, a predetermined torque demand is given to the motor controller. With the help of kriging prediction, the torque is directly calculated in a real-time fashion within the KAOT model.

First, with the kriging assisted back EMF estimation, the back EMF of the motor's *a* phase is estimated as shown in Fig. 4. Hereafter, back EMF and torque values based on the original motor model from Matlab/Simulink is referred as from simulation. As we can see from Fig.4, back EMF values from the simulation and KAOT are highly consistent to each other, which implies that the back EMF from kriging prediction is accurate enough and can be used in our KAOT calculation.

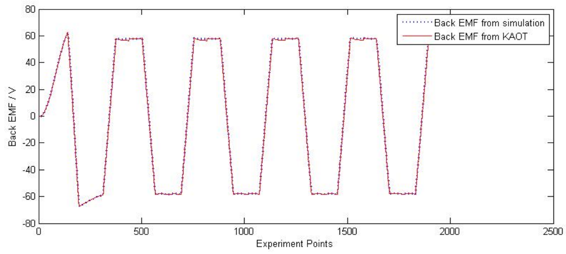


Fig. 4 Back EMF Estimation

Giving the electrical angle of *a* phase plus 2/3 π and 4/3 π as two new electrical angle inputs to the model, the new outputs can be the back EMF of *b* phase and *c* phase respectively. Thus, the motor's back EMF of *b* phase and *c* phase does not need another two prediction models.

Secondly, with three phases back EMF estimated, the calculated torque of the BLDC motor using KAOT is shown in Fig. 5, compared to the torque values generated from the Matlab simulation model.

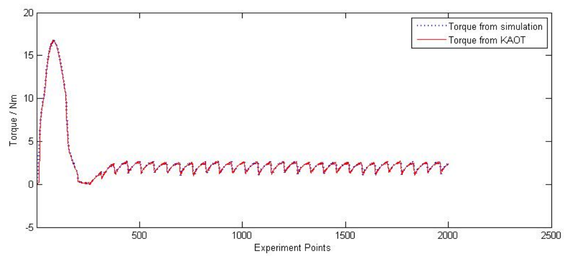


Fig. 5 Torque Calculation Result

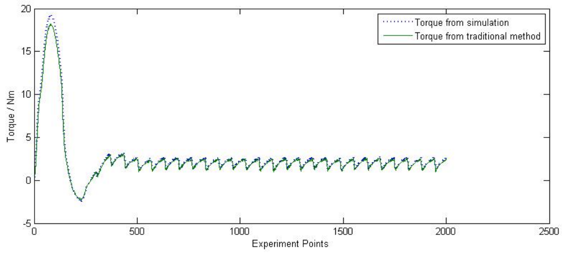
From the above results, it can be concluded that assisted by kriging prediction, the back EMF is accurately estimated to calculate a precise torque value. Since the number of points used in the kriging model is only sixty, this calculation can be implemented very fast and efficient in MCUs.

Note that the sophisticated simulation model developed based on Matlab/Simulink is of course capable of calculating the torque value. However, it is really difficult to implement such a complicated Matlab simulation model in the MCUs. The same as the traditional method [10], KAOT demonstrated here is a simple calculation of the torque value given the inputs of the electrical angle and rotor speed from Hall sensors in the motor, which can be performed in MCUs easily.

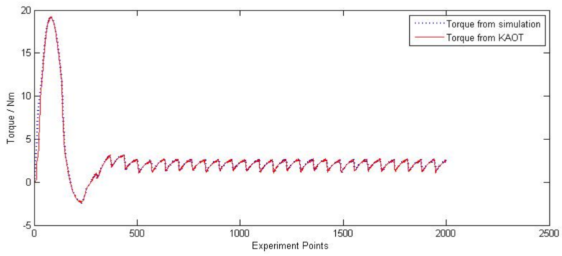
V. TEMPERATURE CHANGING AND AGING

The temperature changing and aging generally can affect the motor performance significantly, whose effect are also considered in this work. With the back EMF measured on-line, the back EMF estimating model can be revised to reflect the effect of temperature change and aging. Essentially, different back EMF values at the same speed and electrical angle points can be obtained with different temperature and aging conditions. In this regard, more accurate torque values during all working conditions of motors can be obtained.

At low temperature (i.e., the motor flux is 6% larger and the resistance is 25% smaller compared to the nominal temperature), the torque calculated using the traditional method is shown in Fig. 6 (a). Hereafter, the method that assumes torque is linear to the current is referred as the traditional method [10]. The largest error compared to the torque from the simulation at the top of the curve is 5.405%. On the contrary, the torque calculated from KAOT method is more accurate than that from the traditional method. The error at the top of the curse is only 0.002%, as shown in Fig. 6 (b).



(a)



(b)

Fig. 6 Torque Calculation Result at Low Temperature with (a) Traditional Method and (b) KAOT

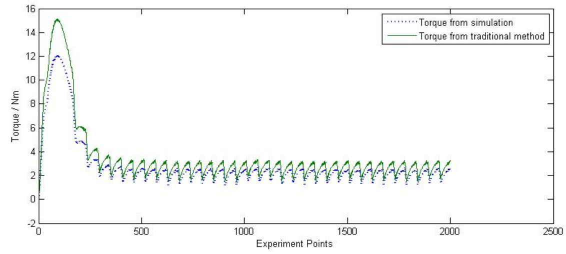
We also calculate the MSE of torque values after the motor reaches the stable region (i.e., the region between 500 and 2000 experiment points). When calculating the MSE, we consider the torque from the simulation as its standard (or nominal) value. Ten points randomly selected in the stable region are used to calculate the MSE value, as in Table I.

TABLE I

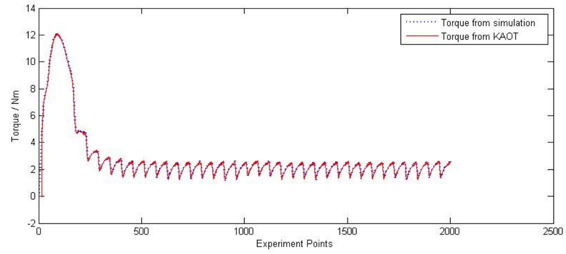
MSE COMPARISON UNDER DIFFERENT CONDITIONS

	Traditional Method	KAOT
Low temperature	0.018453	0.000225
High temperature	0.406558	0.000286
Aging	1.08247	0.000755

At high temperature (i.e., the motor flux is 20% smaller and the resistance is 48% larger compared to the nominal temperature), the error at the top of the curve from the traditional method reaches 25.000% (Fig. 7 (a)), compared to the results from KAOT in Fig. 7 (b) where the error remains very small at 0.041%.



(a)

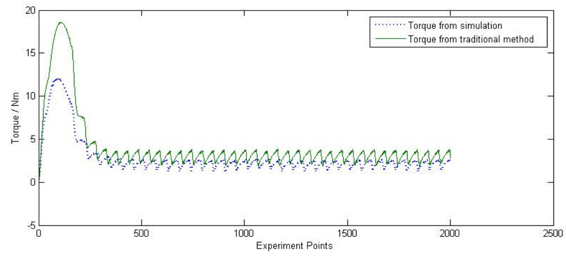


(b)

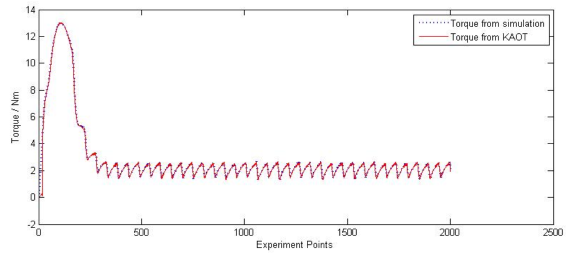
Fig. 7 Torque Calculation Result at High Temperature with (a) Traditional Method and (b) KAOT

The MSE value of high temperature results from the traditional method and KAOT in the stable region is shown in Table I too.

When consider aging, the error at the top of the curve reaches 42.857% for the traditional method (Fig. 8 (a)) while the error from KAOT is extremely small at 0.004% (Fig. 8 (b)), compared to the torque values from the simulation.



(a)



(b)

Fig. 8 Torque Calculation Result when aging with (a) Traditional Method and (b) KAOT

Again, the MSE comparison during the stable region for the aging case is given in Table I.

Based on the results shown above, it is very impressive that the temperature changing and aging can affect the performance of motor torque significantly. The traditional method cannot take those effects into account in a real-time fashion and the error can become very large, which leads to serious problems for vehicle torque control and motion stability. However, the proposed KAOT method can solve such problems and obtain accurate torque values even with the variation of temperature and aging effect in an on-line manner.

VI. CONCLUSION

BLDC motors can be a good choice for low cost EVs in the future. Accurate on-line torque calculation for BLDC motors used in EVs is important but has not been well studied. Estimating the back EMF is an essential problem for this type of problems. In this paper, a new on-line torque calculation method assisted by kriging prediction, KAOT, is proposed. Kriging prediction can estimate the back EMF value of unobserved combination of motor speeds and rotor positions based on sampled points. By applying the kriging model, the back EMF can be directly estimated in a real-time fashion and then used to calculate the motor torque as the feedback to the motor controller. Moreover, the temperature changing and aging are severe problems that we have to face for the motors used in EVs. In this regard, our new approach shows a huge improvement in terms of the accuracy in torque calculation, compared with the traditional method.

The new KAOT method for BLDC motors would be applied in MCUs to accurately implement the functions like ABS and TCS in EVs, in which accurately estimating the motor torque value is very critical. With a better torque feedback calculated, the performance of ABS and TCS for EVs could be improved significantly.

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