Control of Transient Response via Polynomial Method ⋆

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Abstract:
In this paper, a general discussion on the controller design for transient response control is first conducted based on the polynomial method. It is shown for general all-pole close-loop systems, their step responses have zero or nearly zero overshoot under the nominal characteristic ratio assignment $[2.5, 2, 2, \ldots]$; while the time constant τ determines the speed of response. The control of the classical benchmark two-mass system is introduced as a case study, in which the m-IPD control configuration is adopted. It is found the time constant τ cannot be arbitrarily specified under the m-IPD control configuration and the nominal characteristic ratio assignment. The designed m-IPD controller demonstrates a sufficient damping performance. Meanwhile, the large and negative derivative gain designed under the nominal characteristic ratio assignment leads to a poor robustness. Through the complementary sensitivity function analysis, the robustness of the polynomial-based m-IPD controller design is found to depend on the term of $(K_d^* s^{*2} + K_p^* s^* + K_i^*)$; while the characteristic ratio assignment and the time constant determine the nominal time response of the close-loop control system.

Keywords: Polynomial method, transient response, two-mass system, damping, robustness.

1. INTRODUCTION

It is well-known in the most control system problems, the time responses are the final evaluation of the performance of the control system. However, for the classical and modern control design, they are difficult to directly deal with the transient response. On the other hand, there exists an alternative approach called algebraic design using polynomial expressions (i.e., the polynomial method), in which controller is designed based on the characteristic polynomial of the closed-loop control system. It was reported that the assignment of so-called characteristic ratios in the polynomial method had a strong co-relationship with the damping (i.e. the overshoot) of a close-loop system; while the speed of response relates with the time constant [Kim et al. (2003)][Kim et al. (2002)]. Namely. the transient response can be addressed from the characteristic ratios and the time constant (the definitions can be found in following sections). Naslin empirically observed these relationships in 1960s [Naslin (1969)]. Kessler developed the polynomial method before Naslin and recommended that the characteristic ratios should all be two [Kessler (1960)]. Manabe proposed the Coefficient Diagram Method (CDM) based on Naslin's findings and the Lipatov-Sokolov stability criterion [Manabe (1998)][Manabe (2003)].

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Since the polynomial method is based on the closed-loop characteristic polynomial, a general controller design and discussion are possible. In addition, the configuration of the controller is defined at the beginning. The coefficients of the polynomial are then determined under certain design criteria. Therefore, the polynomial method could be applied in designing low-order controllers for various control problems.

This paper firstly uses general all-pole close-loop systems as examples to discuss the nominal characteristic ratio assignment for the nonovershooting step response and time constant selection for rising time adjustment. Then, the benchmark two-mass system is introduced as a case study for applying the polynomial method in a challenging engineering problem, the vibration control of the two-mass systems. The m-IPD controller is designed under the nominal characteristic ratio assignment, in which the time constant cannot be independently specified. Besides the nominal time responses, the robustness of the controller design is also discussed in order to confirm and improve the practicability of the polynomial-method-based controller design.

2. GENERAL ALL-POLE CLOSE-LOOP SYSTEMS

2.1 Overshoot

For a general all-pole close-loop system as

$$G(s) = \frac{a_0}{a_n s^n + a_{n-1}s^{n-1} + ... + a_1 s + a_0} \quad (1)$$

its characteristic equation

$$a_n s^n + a_{n-1}s^{n-1} + ... + a_1 s + a_0 = 0 \quad (2)$$

can be rewritten as an equation of characteristic ratios γ_i $(i = 1, \ldots, n-1)$ and time constant τ

$$\frac{1}{\gamma_{n-1}\gamma_{n-2}^2 \cdots \gamma_1^{n-1}}(\tau s)^n + ... + \frac{1}{\gamma_1}(\tau s)^2 + (\tau s) + 1 = 0 \quad (3)$$

where the so-called characteristic ratios are defined as

$$\gamma_1 = \frac{a_1^2}{a_0 a_2},\ \gamma_2 = \frac{a_2^2}{a_3 a_1}, \ldots, \gamma_{n-1} = \frac{a_{n-1}^2}{a_{n-2}a_n} \quad (4)$$

and τ is defined as

$$\tau = \frac{a_1}{a_0} \quad (5)$$

respectively. As shown in (3), the time response of the all-pole close-loop system is sped up by a factor of $1/\tau$, which is straightforward from the Laplace transform of time-scaled functions. Meanwhile, the other control design criteria are solely determined by the characteristic ratios.

It has been reported that the set of characteristic ratios are co-related with the damping of a close-loop system [Kim et al. (2003)]. $G(s)$ can be approved to have monotonically decreasing magnitude of frequency response and thus small overshoot in step response under the condition of $\gamma_i > 2$ for all $i = 1, \ldots, n-1$ (refer to Appendix A). The influence of each characteristic ratio can be evaluated by using the system sensitivities to varying characteristic ratios, which are defined as

$$S_{\gamma_i}(s) = \frac{\partial G(s)/G(s)}{\partial \gamma_i/\gamma_i} \text{ for } i = 1, \ldots, n-1 \quad (6)$$

Taking a 5th-order system with the initial characteristic ratio assignment of $\gamma_i = 2$ (i=1, …, 4) as an example, it can be seen that the characteristic ratios with lower indices have a more dominant influence on the overall performance (see Fig. 1).

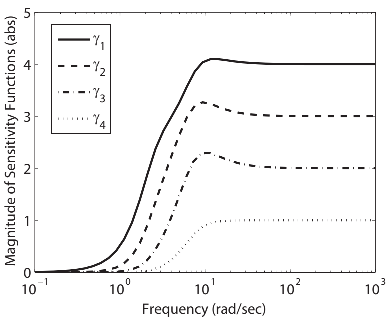


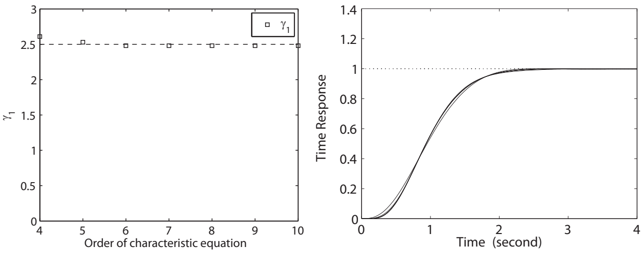
Fig. 1. Magnitude of sensitivity functions to the variation of γ_i's

The design of nonovershooting controllers is of interest in order to establish a general baseline for the controller design. Considering 1) the characteristic ratio assignment of $\gamma_i > 2$ $(i = 1, \ldots, n-1)$ as a starting point; 2) the dominant influence of low-index characteristic ratios, it is straightforward to adjust the single characteristic ratio γ_1 $(\gamma_1 \geq 2)$ while keep all other higher-index characteristic ratios fixed and equal with two. Due to the difficulty in finding exact analytical solutions for high-order systems, the values of γ_1 are searched by numerical simulation that enable nonovershooting step responses of the all-pole close-loop systems (1). For the sake of simplicity, the time constant τ is taken as 1 in the simulation.

As shown in Fig. 2, it is interesting to find all the γ_1's are close to 2.5 despite various orders of the systems. Therefore, a general and easy-to-remember baseline for the controller design could be considered as having the following characteristic ratio assignment

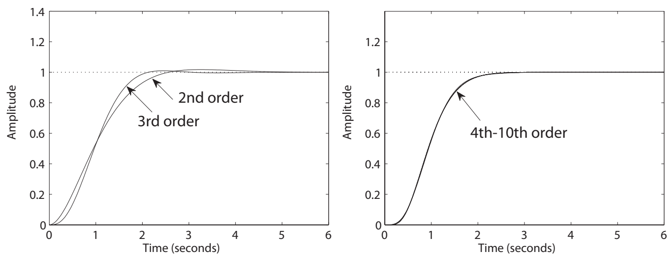
$$\gamma_1 = 2.5 \text{ and } \gamma_i = 2 \text{ for } i = 2, ..., n-1 \quad (7)$$

This nominal characteristic ratio assignment is actually identical with and explains the standard form of the CDM method, which was based on intensive experimental studies [Kim et al. (2003)][Manabe (2003)]. The step responses under nominal characteristic ratio assignment [2.5 2 2 …] demonstrate good transient responses with zero or nearly zero overshoot, as shown in Fig. 3.



(a) (b)

Fig. 2. The values of γ_1 for having nonovershooting step responses of the general all-pole close-loop systems (from the 4th-order to 10th-order). (a) γ_1. (b) Step responses.



(a) (b)

Fig. 3. Step responses of the general all-pole close-loop systems (from the 2nd-order to 10th-order) under nominal characteristic ratio assignment [2.5 2 2 …].

2.2 Speed of Response

For the all-pole close-loop systems, the overshoot and speed of response (i.e., the rising time) can be independently specified, as explained in (3). The step responses of the 5th-order all-pole close-loop system are shown in Fig. 4, which are under the nominal characteristic ratio assignment [2.5 2 2 2] and a time constant τ varying from 0.25 to 2.

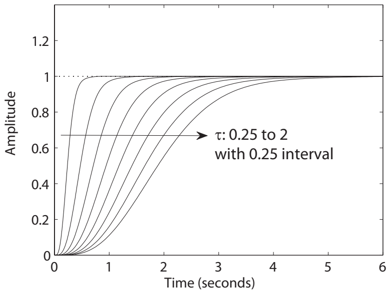


Fig. 4. Step responses of the 5th-order all-pole close-loop system with various τ.

For the above 5th all-pole close-loop system with a unity time constant τ=1, its rising time is 1.0962. From (3), it is straightforward to determine a time constant τ that enables a step response with specific rising time t_r as

$$\tau = \frac{t_r}{1.0962} \tag{8}$$

The similarity of the step responses for various order systems is shown in Fig. 3 under the nominal characteristic ratio assignment [2.5 2 2 . . .]; therefore, (8) can be a general approximation to estimate the required time constant for the other all-pole close-loop systems with various orders.

3. EXAMPLE: BENCHMARK TWO-MASS SYSTEM

3.1 Normalization of Two-mass Model

As shown in Fig. 5(a), the benchmark two-mass system is usually modeled by two masses connected with a non-stiff coupling shaft, where K_s is spring coefficient, J_m and J_l are the inertias of the drive and load sides, respectively. In the typical benchmark two-mass control problem, only the velocity of the drive side is assumed measurable, whereas driving torque, load torque and the velocity of the load side are not measurable [Sugiura et al. (1996)][Lee et al. (2007)]. The controller design needs to be able to control the velocity of the load side within well-suppressed vibrations by using only the velocity feedback of the drive side. Various approaches have been proposed for the two-mass system control during the past decade such as the feedback of imperfect derivative of the torsional torque estimated by a disturbance observer [Sugiura et al. (1996)], a two-degree-of-freedom control structure using an observer-based state feedback compensator [Dhaouadi et al. (1993)], μ-synthesis based on a descriptor form representation [Hirata et al. (1996)], a series anti-resonance finite-impulse response compensator [Vukosavić et al. (1998)], the intelligent control using fuzzy and neural network [Lee et al. (2007)][Wang et al. (2002)][Orlowska-Kowalska et al. (2007-1)][Orlowska-Kowalska et al. (2007-2)][Orlowska-Kowalska et al. (2010)], etc.

Meanwhile, the low-order PID controllers and their modifications are predominant in industry. Improvement on the low-order controller design would significantly contribute to real engineering applications. As aforementioned, the polynomial method can be applied in the design of the low-order controller for the benchmark two-mass system.

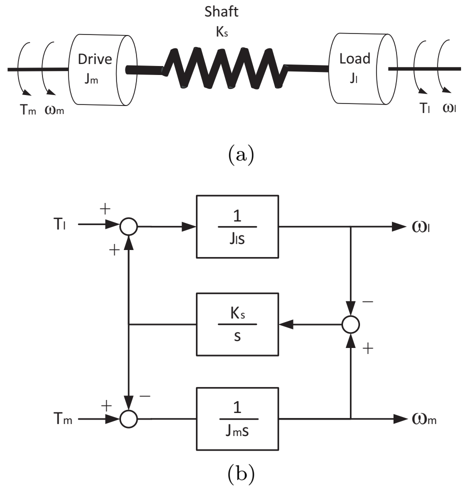


Fig. 5. The modeling of the benchmark two-mass system. (a) Model. (b) Block diagram.

As shown in Fig. 5(b), the transfer function of the two-mass system between driving torque T_m and angular velocity of drive side ω_m is derived as

$$P(s) = \frac{s^2 + \omega_a^2}{J_m s(s^2 + \omega_r^2)} \tag{9}$$

where ω_r and ω_a are the resonance frequency and anti-resonance frequency

$$\omega_r = \sqrt{K_s\left(\frac{1}{J_m} + \frac{1}{J_l}\right)},\ \omega_a = \sqrt{\frac{K_s}{J_l}} \tag{10}$$

respectively. The transfer function (9) can be normalized by replacing the Laplace operator s with s^*=s/ω_a

$$P(s^*) = \frac{q}{J_m}\frac{1}{\omega_a}\frac{s^{*2}+1}{qs^{*3}+s^*} \tag{11}$$

where q is the inertia ratio defined as the ratio of drive inertia to total inertia

$$q = \frac{J_m}{J_m + J_l} \tag{12}$$

For the sake of simplicity, the normalized two-mass system model can be taken as

$$P_n(s^*) = \frac{s^{*2}+1}{qs^{*3}+s^*} \tag{13}$$

in which the normalized resonance frequency and anti-resonance frequency are

$$\omega_r^* = \frac{1}{\sqrt{q}},\ \omega_a^* = 1 \tag{14}$$

respectively. Similarly, it is straightforward from the Laplace transform of time-scaled functions that the real response is sped up by a factor of ω_a.

As shown in Fig. 6, with a larger q, the two normalized resonance frequencies, ω_r^* and ω_a^*, become close, i.e., a stronger tendency of pole/zero cancellation. It is known the pole/zero cancellation leads to a poor robustness of a closed-loop control system [Folly (2008)]. On the other hand, a lower resonance frequency ω_r^* correspondingly

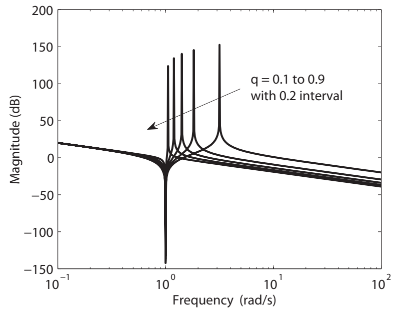


Fig. 6. Bode magnitude plots for the normalized two-mass model

requires a larger damping; therefore, additional phase lag needs to be provided by the controller, which eventually sacrifices robust stability margin. Namely, the performances of damping and robustness in the two-mass system control essentially contradict each other, especially when the inertia ratio q is large. A fundamental issue in the controller design of the two-mass system is to achieve a proper tradeoff between the two performances. The polynomial-method-based controller design would be desirable because of its capability to explicitly incorporate the factor of damping.

3.2 m-IPD Controller Design

As shown in Fig. 7, the m-IPD controller configuration is applied in the velocity control of two-mass systems. Unlike the classical PID controllers, an alternative configuration called setpoint-on-I-only configuration is adopted to smooth the discontinuity of the reference command ω_r by the integral (i.e., the I controller) [Chen (1993)]. This special PID controller is usually named IPD controller in order to distinguish from the classical PID configurations. And the prefix "m" (i.e., modified) refers to the additional first-order low-pass filter $1/(T_d s + 1)$ in the m-IPD controller. As shown in (14), a larger inertia ratio q causes a lower resonant frequency $\omega_r^* (= 1/\sqrt{q})$ that requires a larger damping for vibration suppression. With the additional low-pass filter and the D controller, m-IPD controller is expected to provide a more sufficient damping than IP controller. By comparing (13) with (11), the controller parameters K_p, K_i, K_d and T_d can be calculated as

$$K_p = K_p^* \frac{J_m \omega_a}{q} \tag{15}$$

$$K_i = K_i^* \frac{J_m \omega_a^2}{q} \tag{16}$$

$$K_d = K_d^* \frac{J_m}{q} \tag{17}$$

$$T_d = \frac{T_d^*}{\omega_a} \tag{18}$$

where K_p^*, K_i^*, K_d^* and T_d^* are the controller parameters designed by using the normalized model (13).

Similarly, the closed-loop transfer function of the m-IPD control loop is

$$G_n(s^*) = \frac{K_i^*(s^{*2} + 1)}{a_5 s^{*5} + a_4 s^{*4} + a_3 s^{*3} + a_2 s^{*2} + a_1 s^* + a_0} \tag{19}$$

where

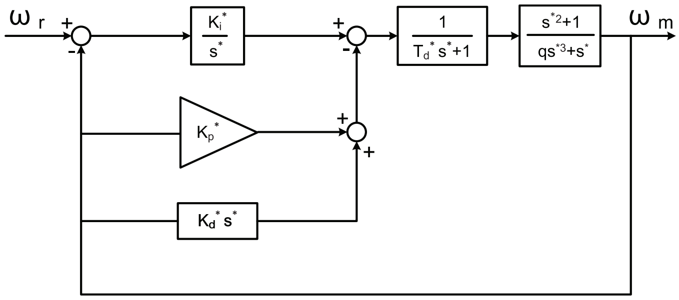


Fig. 7. Block diagram of m-IPD velocity control of the normalized two-mass system.

$$a_5 = qT_d^* = \frac{1}{\gamma_4 \gamma_3^2 \gamma_2^3 \gamma_1^4} \tau^5 a_0 \tag{20}$$

$$a_4 = q + K_d^* = \frac{1}{\gamma_3 \gamma_2^2 \gamma_1^3} \tau^4 a_0 \tag{21}$$

$$a_3 = T_d^* + K_p^* = \frac{1}{\gamma_2 \gamma_1^2} \tau^3 a_0 \tag{22}$$

$$a_2 = 1 + K_i^* + K_d^* = \frac{1}{\gamma_1} \tau^2 a_0 \tag{23}$$

$$a_1 = K_p^* = \tau a_0 \tag{24}$$

$$a_0 = K_i^* \tag{25}$$

The relationship between the coefficients and the controller parameters is described as

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 - 1 \\ a_3 \\ a_4 - q \\ a_5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & q \end{bmatrix} \cdot \begin{bmatrix} K_p^* \\ K_i^* \\ K_d^* \\ T_d^* \end{bmatrix} \tag{26}$$

For the above nonhomogeneous matrix equations (26), it has a unique solution provided the rank of its coefficient matrix is as equal as the rank of its augmented matrix; in addition, the two ranks should also be equal with four, i.e., the number of unknown parameters K_p^*, K_i^*, K_d^* and T_d^*. Therefore, it can be found the coefficients of the close-loop characteristic equation must satisfy the following relationship

$$a_2 - 1 - a_0 - (a_4 - q) = 0 \tag{27}$$

$$a_3 - a_1 - a_5/q = 0 \tag{28}$$

With (21)(23)(25)(27), a_0 is solved as

$$a_0 = \frac{1 - q}{\frac{1}{\gamma_1}\tau^2 - \frac{1}{\gamma_3 \gamma_2^2 \gamma_1^3}\tau^4 - 1} \tag{29}$$

In order to have a positive a_0 (i.e., a positive K_i^*), the upper and lower limits of τ are

$$\tau_{max} = \gamma_1 \gamma_2 \sqrt{\frac{1 + \sqrt{1 - \frac{4}{\gamma_3 \gamma_2^2 \gamma_1}}}{2} \gamma_3} \tag{30}$$

$$\tau_{min} = \gamma_1 \gamma_2 \sqrt{\frac{1 - \sqrt{1 - \frac{4}{\gamma_3 \gamma_2^2 \gamma_1}}}{2} \gamma_3} \tag{31}$$

respectively.

Similarly, τ can be determined using (20)(22)(24)(28)

$$\frac{1/q}{\gamma_4\gamma_3^2\gamma_2^3\gamma_1^4}(\tau^2)^2 - \frac{1}{\gamma_2\gamma_1^2}(\tau^2) + 1 = 0 \tag{32}$$

It can be seen the value of τ cannot be arbitrarily specified if the requirement on damping (i.e., the nominal characteristic ratio assignment) must be satisfied. Under the m-IPD control configuration, the overshoot and speed of response in the two-mass system control cannot be independently adjusted. τ^2 has real and positive solutions under the nominal characteristic ratio assignment [2.5 2 2 2] if and only if

$$\Delta = b^2 - 4ac \geq 0 \tag{33}$$

where a, b and c are the quadratic coefficient, the linear coefficient and the constant term of the quadratic equation (32), respectively. From (33), it can be found

$$q \geq \frac{1}{4} \tag{34}$$

For the two-mass systems with small q's, the IP controller has been approved to be sufficient [Ma et al. (2012)]. Therefore, in this paper, the m-IPD controller design is mainly discussed for large q's. The two positive solutions of τ with various q are shown in Fig. 8. The m-IPD-based controller design has two solutions of the time constant, τ_1 and τ_2, to satisfy the nominal characteristic ratio assignment. Combining with the limits of τ in (30)(31), the smaller time constant τ_2 can meet the requirement for all the q's.

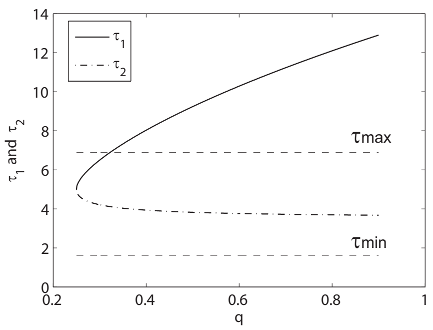


Fig. 8. Solutions of τ versus q under nominal characteristic ratio assignment.

After specifying the time constant τ, the coefficients of the characteristic equation can be determined using the nominal characteristic ratio assignment (refer (20)-(25)). The unique solution of the m-IPD controller parameters is then calculated by solving the nonhomogeneous matrix equations, whose the coefficient matrix and the augmented matrix have same rank of four under the time constant τ. The step velocity responses of the nominal two-mass systems are shown in Fig. 9 with various inertia ratio q. All the step responses demonstrate an excellent damping performance, i.e. nearly zero overshoots. The speed of response is actually accelerated with a large q, which is explained by the smaller τ_2 for larger q's in Fig. 8.

3.3 Robustness Analysis

As shown in Fig. 6, the stronger tendency of pole/zero cancellation with a larger inertia ratio q leads to a poor robustness of a closed-loop control system. A proper tradeoff between the performances of damping and robustness must be discussed during the controller design, especially when the inertia ratio q is large. As aforementioned, the transient response (i.e., overshoot and rising time) can determined from the time constant τ and the characteristic ratios γ_i $(i = 1, \ldots, n)$ of the close-loop characteristic equation; while the robustness is also affected by the specific controller configuration. For the robustness analysis, the equivalent transformation of the m-IPD control blockdiagram is conducted, as shown in Fig. 10. It is interesting to find the m-IPD control configuration is actually equivalent with the general two-parameter control configuration.

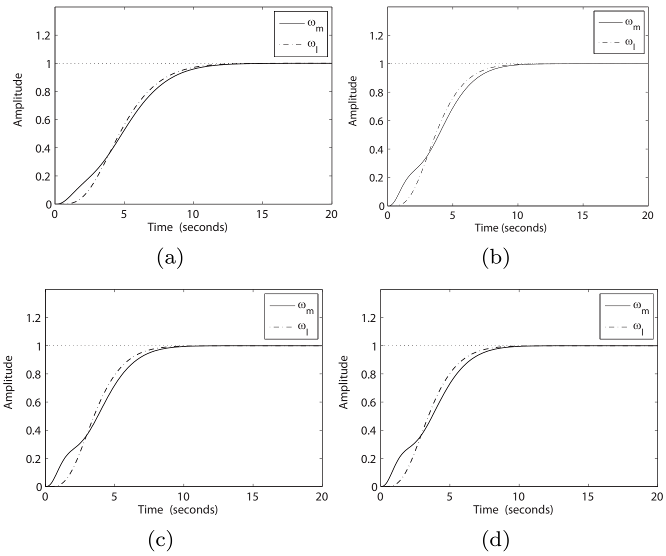


Fig. 9. Step responses of the drive and load velocities by the m-IPD control designed using τ_2 and the nominal characteristic ratio assignment. (a) q=1/4. (b) q=0.4. (c) q=0.6. (d) q=0.8.

For the equivalent control configuration in Fig. 10(b), its loop transfer function is

$$L(s) = \frac{s^{*2}+1}{qs^{*3}+s^*} \cdot \frac{K_d^* s^{*2} + K_p^* s^* + K_i^*}{T_d^* s^{*2} + s^*} \tag{35}$$

And its complementary sensitivity function is

$$T(s) = \frac{(s^{*2}+1)(K_d^* s^{*2} + K_p^* s^* + K_i^*)}{a_5 s^{*5} + a_4 s^{*4} + a_3 s^{*3} + a_2 s^{*2} + a_1 s^* + a_0} \tag{36}$$

where the coefficients a_i $(i = 0, \ldots, 5)$ are determined by τ_2 and the nominal characteristic ratio assignment (refer (20)-(25) and (32)). As shown by Bode plots of $L(s)$ and $T(s)$ in Fig. 11 and Fig. 12 respectively, the m-IPD control designed under the nominal characteristic ratio assignment [2.5, 2, 2, 2] has a poor robustness performance, especially when q is large. For example, when q is equal with 0.85 in the figures, the phase margin of $L(s)$ is nearly zero; the large magnitude of $T(s)$ in high-frequency range also indicts the poor robustness stability of the m-IPD controller design. As shown in Fig. 13, the step responses with variation of q further verify the deteriorated robustness when q is large.

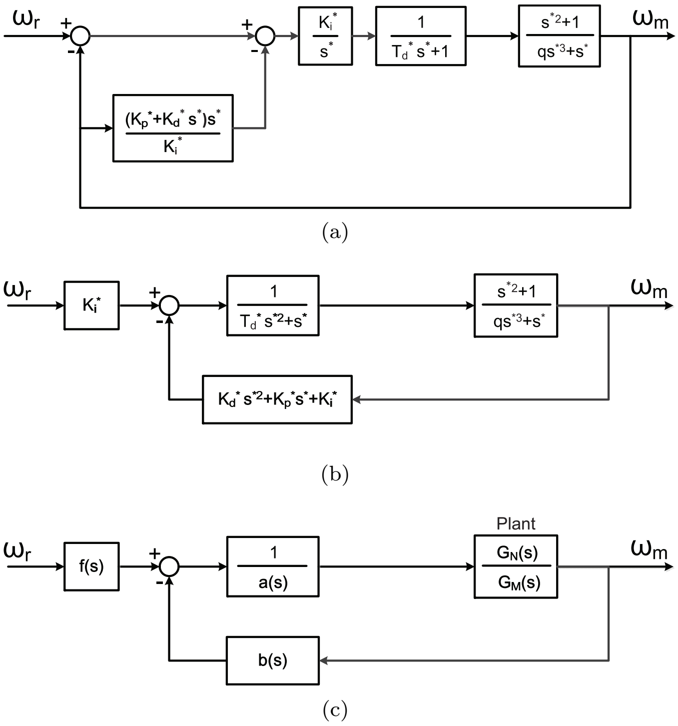


Fig. 10. Equivalent transform of the block diagram of m-IPD control. (a) Step#1. (b) Step#2. (c) General two-parameter control configuration

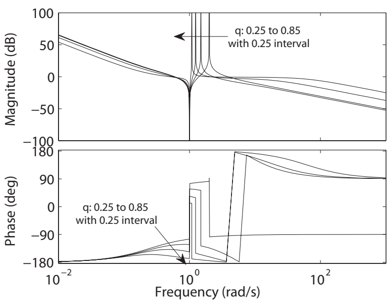


Fig. 11. The Bode plots of the loop transfer functions with various q.

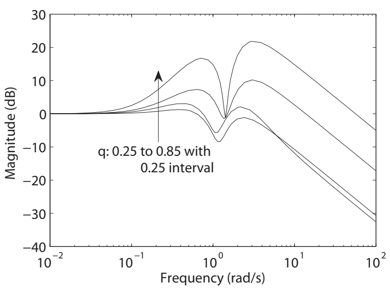


Fig. 12. The Bode magnitude plots of the complementary sensitivity functions with various q.

As shown in Fig. 10(c), the complementary sensitivity function of the general two-parameter control configuration is

$$T(s) = \frac{b(s)G_N(s)}{a(s)G_M(s) + b(s)G_N(s)} \tag{37}$$

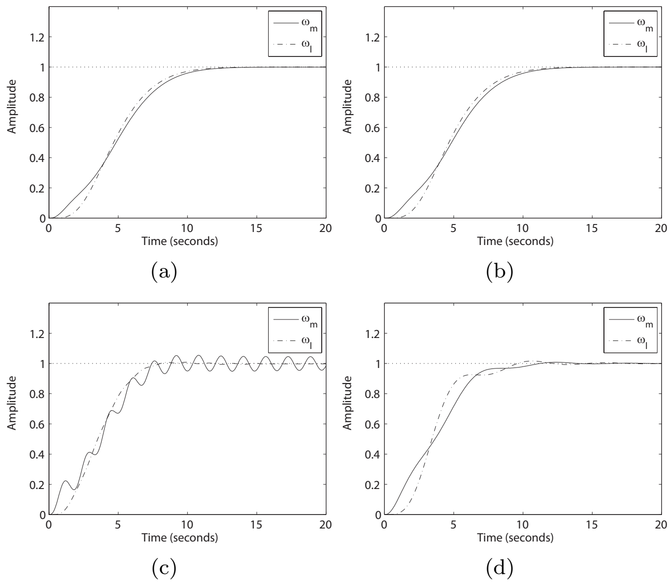


Fig. 13. Step responses with q variation. (a) -10% (Nominal q=0.25). (b) +10% (Nominal q=0.25). (c) -10% (Nominal q=0.8). (d) +10% (Nominal q=0.8).

The denominator of $T(s)$ is identical with the characteristic polynomial, which is determined by the nominal characteristic ratios. In its numerator, $G_N(s)$ is the numerator of the plant transfer function. Namely, the design of $b(s)$ determines the value of the complementary sensitivity function. In the m-IPD controller design, $b(s)$ is equal with the term of $(K_d^* s^{*2} + K_p^* s^* + K_i^*)$ in Fig. 10(b). For a good robustness, the magnitude of $(K_d^* s^{*2} + K_p^* s^* + K_i^*)$ should be small enough to suppress the complementary sensitivity function $T(s)$, especially in high-frequency range. Figure 14 shows the values of K_p^*, K_i^*, K_d^* and T_d^* designed under the nominal characteristic ratio assignment [2.5, 2, 2, 2] and the corresponding time constant τ. For a larger q, a stronger damping is required due to the lower resonance frequency ω_r^* (see Fig. 6). A large negative K_d^* is designed to provide sufficient damping, i.e., the introduction of phase lag rather than phase lead compared with the conventional derivative controllers.

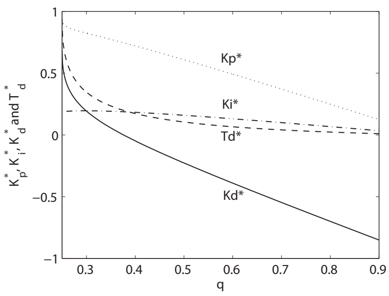


Fig. 14. The m-IPD controller parameters versus q under nominal characteristic ratio assignment.

However, the peak in the complementary sensitivity function $T(s)$ is caused by its large negative numerator term $K_d^* s^{*2}$. The large and negative derivative gains K_d^*'s also mean the positive feedback of the derivative control signal. As aforementioned, in the two-mass system control design, an essential issue is to achieve a proper tradeoff between damping and robustness performances. Although the nominal characteristic ratio assignment can provide a sufficient damping for large q's, the robustness performance is significantly sacrificed. In order to guarantee robustness, the damping performance must be lowered to some degree. Solutions as future works could be considered as 1) having a larger time constant τ to improve damping by slowing down of the response speed; 2) letting characteristic ratios with higher indices such as γ_4 or even γ_3 be determined by the specified τ; while keeping γ_1 and γ_2 as their nominal values. Namely, the robustness performance could be recovered by certain degree of sacrifice on the damping performance.

4. CONCLUSION

In this paper, a general discussion on the transient response control is firstly conducted based on the polynomial method. It is shown for the general all-pole close-loop systems, their step responses have zero or nearly zero overshoot under the characteristic ratio assignment $[2.5, 2, 2, \ldots]$; while the time constant τ determines the speed of response, i.e., the rising time. The control of the classical benchmark two-mass system is introduced as a case study for the application of polynomial method in a challenging engineering problem. Due to the tendency of pole/zero cancellation in the normalized two-mass model, a tradeoff relationship exists between damping and robustness performances in the controller design, especially when the inertia ratio is large.

The m-IPD control configuration is adopted for two-mass system control. The characteristic ratio assignment [2.5 2 2 2] is used as a nominal set for the controller design due to its good damping performance. It is found the time constant τ cannot be arbitrarily specified under the m-IPD control configuration and the nominal characteristic ratio assignment. The designed m-IPD controller demonstrates a sufficient damping performance even when q is large. Meanwhile, the robustness analysis and step responses with q variation indict its poor robustness especially with large q's. The large and negative derivative gains designed under the nominal characteristic ratio assignment lead to a poor robustness. Through the complementary sensitivity function of the general two-parameter control configuration, it is found the robustness of the polynomial-based m-IPD controller design depends on the term of $(K_d^* s^{*2} + K_p^* s^* + K_i^*)$; while the characteristic ratio assignment and the time constant determine the nominal time response of the close-loop control systems.

It would be interesting to modify the choice of characteristic ratio assignment and the time constant in order to improve the robustness of the controller design. The other future works may include applying the polynomial method in the controller design of more complicated benchmark systems, such as the general inverted pendulum. Expanding the polynomial method to design multi-input-multi-output (MIMO) control systems would further extend its applications.

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Appendix A. RATIOS FOR NO RESONANT PEAK

For the all-pole transfer function $G(s)$ represented by (1), its squared magnitude in frequency domain is

$$|G(j\omega)|^2 = G(j\omega)G(-j\omega) = \frac{a_0^2}{Q(\omega)} \tag{A.1}$$

where

$$\begin{aligned} Q(\omega) = &[a_n(j\omega)^n + ... + a_1 j\omega + a_0] \\ &\cdot\ [a_n(-j\omega)^n + ... + a_1(-j\omega) + a_0] \end{aligned} \tag{A.2}$$

Let

$$Q(\omega) = \Sigma_{k=0}^{2n} A_k \omega^k \tag{A.3}$$

It can be found

$$A_{2k+1} = 0 \tag{A.4}$$

Therefore

$$Q(\omega) = \Sigma_{k=0}^{n} A_{2k} \omega^{2k} \tag{A.5}$$

where

$$\begin{aligned} A_{2k} = &(a_k^2 - 2a_{k-1}a_{k+1}) + 2(a_{k-2}a_{k+2} - a_{k-3}a_{k+3}) \\ &+ 2(a_{k-4}a_{k+4} - a_{k-5}a_{k+5})... \end{aligned} \tag{A.6}$$

With all the characteristic ratios $\gamma_i > 2$ $(i = 1, \ldots, n-1)$, it is obvious that

$$a_k^2 - 2a_{k-1}a_{k+1} > 0 \tag{A.7}$$

In addition, from (3) a_k can be represented as

$$a_k = \frac{\tau^k a_0}{\gamma_{k-1}\gamma_{k-2}^2\gamma_{k-3}^3 \cdots \gamma_1^{k-1}} \tag{A.8}$$

Then,

$$\frac{a_{k-m}a_{k+m}}{a_{k-m-1}a_{k+m+1}} = \gamma_{k+m}\gamma_{k+m-1}\cdots\gamma_{k-m} > 1 \tag{A.9}$$

i.e.

$$a_{k-m}a_{k+m} - a_{k-m-1}a_{k+m+1} > 0 \tag{A.10}$$

From (A.7) and (A.10), it can be concluded that all the terms in (A.6) are larger than 0, i.e. $A_{2k} > 0$. Namely, the value of $Q(\omega)$ monotonously increases with ω, which means monotonous decrease of $|G(j\omega)|^2$. Thus, $|G(j\omega)|$ has no resonant peak when all the characteristic ratios γ_i are larger than two. Therefore, the theorem is approved.