Subsystem-Level Efficiency Analysis of a Wireless Power Transfer System

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Abstract—In this paper, a mid-power 13.56 MHz wireless power transfer (WPT) system is used as an example to discuss the relationship between the overall system and each subsystem. There are four subsystems discussed, including a power amplifier, a pair of resonant coils, a rectifier, and an electronic load. The paper evaluates the system performance under different mutual inductances in two cases. One is a simple WPT system without a rectifier, and the other with the rectifier. It is found that the existence of rectifier introduces more power reflection, especially when strong coupling occurs. The analysis in this paper can guide the design of an optimal load and input impedance matching network. Finally, the theoretical analysis is validated by experimental results.

Keywords-wireless power transfer system; magnetic resonance; subsystem efficiency; impedance matching; optimal load control

I. INTRODUCTION

Various wireless power transfer (WPT) technologies have been developed in recent years, such as inductive coupling, magnetic resonance coupling, and far-field method [1] [2]. Among these technologies, magnetic resonance coupling becomes a promising candidate for mid-range and high-efficiency power transfer [3]. In order to realize WPT using magnetic resonance coupling, especially MHz WPT system, researchers with different backgrounds are dedicating on improving efficiencies of power amplifier, coupling system, and rectifier [4] - [6]. In addition, an optimal load control and input impedance matching network can minimize the power loss caused by the power reflection [7] [8]. Meanwhile, when designing a complete WPT system, an optimized design of a subsystem does not guarantee an optimal efficiency in overall system level. It is required to provide a systematic analysis on the interaction among the the subsystems. Many factors can affect subsystem efficiencies, such as mutual inductance coefficient, load variation, and power level. This paper focuses on the influence of the mutual inductance coefficient and load variation. The analysis is important to guide an optimized design and control of a WPT system, in which each subsystem works collaboratively.

The objective of this paper is to investigate the relationship between the system efficiency and subsystem efficiencies in a 13.56 MHz WPT system for mid-power application. The results provide a guideline for the system design, the optimal load control and input impedance matching. This paper defines the system efficiency and efficiencies of each subsystem in Section II. Based on the definitions, section III and IV discusses the power transfer characteristics for a simple WPT system without rectifier and a general WPT system with rectifier, respectively. The optimal efficiency curves for both the overall system and subsystems are shown and compared. It is found that the existence of the rectifier enlarges the transfer power reflection.

II. DEFINITION OF EFFICIENCY

A typical configuration of a WPT system is shown in Fig. 1. The system consists of a power amplifier, a coupling system, a rectifier and a load. Z_{in} and Z_l are the input impedances at the input ports of the coupling system and the rectifier, respectively. R_l is the load. Γ_{in} is the reflection coefficient at the output port of the power amplifier. The system efficiency is defined as

$$\eta_{sys} = \frac{P_l}{P_i},\tag{1}$$

where P_l is the power absorbed by the load, and P_i is the incident power from the power amplifier. The system efficiency can also be represented as a product of the following efficiencies,

$$\eta_{sys} = \eta_{tran} * \eta_{coil} * \eta_{rec}, \tag{2}$$

where η_{coil} and η_{rec} are the conversion efficiencies of coupling system and rectifier, respectively. η_{tran} is related to the power reflection condition. The following sections report on a detailed investigation into each efficiency term.



Figure 1: System configuration.



Figure 2: Equivalent circuit model of a WPT system.

III. COUPLING-ONLY SYSTEM

Fig. 2 shows the circuit model of a WPT system. The subsystems after the coupling system [refer to Fig. 1] is represented by an equivalent impedance Z_l . In this model, Z_S is the equivalent internal impedance of the AC source (i.e. the power amplifier). The coupling system is modeled by a series resistor-inductor-capacitor circuit. The magnetic coupling is described by a mutual inductance L_m . Here $L_m = k * L$ and k is the mutual inductance coefficient.

The input impedance of the coupling coil Z_{in} can be directly derived based on this circuit model,

$$Z_{in} = \frac{(R + j\omega L + \frac{1}{j\omega C})(Z_l + R + j\omega L + \frac{1}{j\omega C}) + \omega^2 L_m^2}{Z_l + R + j\omega L + \frac{1}{j\omega C}}.$$
 (3)

Under resonance, i.e. $j\omega L + \frac{1}{i\omega C} = 0$, Z_{in} equals to

$$R + \frac{\omega^2 L_m^2}{Z_l + R}.$$
(4)

The power transmission efficiency can be defined as

$$\eta_{tran} = \frac{P_i - P_r}{P_i} = \frac{P_{in}}{P_i} = 1 - \Gamma_{in}^2,$$
(5)

where

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0},\tag{6}$$

and P_r is the reflected power to the power amplifier. In order to eliminate the power reflection, it requires $Z_{in} = Z_S^*$. Accordingly, based on (4), the corresponding Z_l for a minimum power reflection can be derived as

$$Z_{l}^{'} = \frac{\omega^{2} L_{m}^{2}}{Z_{S}^{*} - R} - R.$$
⁽⁷⁾

The other efficiency term is calculated as

$$\eta_{coil} = \frac{P_{Z_l}}{P_{in}} = \frac{\omega^2 L_m^2 Z_l}{(R^2 + RZ_l + \omega^2 L_m^2)(R + Z_l)},$$
(8)

where P_{Z_l} is the power absorbed by Z_l . And the overall efficiency η'_{sys} for this simplified system is

$$\eta_{sys}^{'} = \frac{P_{Z_l}}{P_i} = \eta_{tran} * \eta_{coil}.$$
(9)



Figure 3: Efficiencies comparison by sweeping Z_l when k = 0.12.



Figure 4: Maximum efficiency and optimal Z_l under various k for the coupling-only system.

Fig. 3 shows the three efficiency curves for η_{tran} , η_{coil} , and η'_{sys} with a fixed k. The system parameters used in simulation is shown in Table I. The parameters are from a real coupling system used in the final experiment. Three points m_1 , m_2 , and m_3 are used to show the maximum efficiency points for η'_{sys} , η_{tran} and η_{coil} , respectively. It is clear that η'_{sys} is jointly determined by η_{tran} and η_{coil} .

The system efficiency η'_{sys} for different k's when sweeping Z_l is shown in Fig. 4(a). The maximum points and corresponding optimal Z_l are summarized in Fig. 4(b). Since k is negatively related to the transfer distance, the result shows that a high system efficiency can be maintained within a reasonable distance, and it quickly drops when the distance keeps increasing.

IV. COUPLING SYSTEM WITH RECTIFIER

In the previous section, the maximum power transfer can be achieved by tuning Z_l [refer to Fig. 4(b)] under various mutual

TABLE I: PARAMETERS OF THE TWO RESONANCE COILS

Coil diameter	320 mm	Resonant frequency	13.56 MHz
Inductance L	7.8 µH	Capacitance C	17.6 pF
Internal resistance R	3.52 Ω	Source impedance Z_s	50 Ω



Figure 5: System configuration with a rectifier.



Figure 6: Efficiency curve under various R_l when k = 0.12.

inductances. This impedance is the equivalent input impedance of the following circuits, among which at least a rectifier should be included. For a conventional full-bridge rectifier, such as shown in Fig. 5, there exists a corresponding optimal load resistance R_l to maximize the system efficiency. It means the power received by R_l is maximized for a given P_i . The exact analytical expression of R_l is difficult due to the nonlinear nature of rectifiers and complex model of Schottky diodes. Instead, a nonlinear circuit simulation tool, the Advanced Design System (ADS) from Agilent, is used. In the ADS-based simulation, the resonance coils are built with the same resistor-inductor-capacitor model as in the previous subsection, and the rectifier are built with the diode Spice model (STPSC406).

Fig. 6 gives the simulation results by sweeping R_l under a fixed mutual inductance coefficient k = 0.12. Different to the result of Fig. 3, for maximizing the system efficiency, a new efficiency term η_{rec} should be considered beside η_{tran} and η_{coil} . Fig. 7(a) shows the system efficiency [refer to (2)] curve for different k, and the efficiency is the product of η_{tran} , η_{coil} and η_{rec} . The global maximum points and their corresponding $R_{l,opt}$ are summarized in Fig. 7(b). The efficiency curve shows that with the rectifier the system maximum efficiency points are not monotone anymore. Compared to the efficiency curve in Fig. 4(b), when k increases from zero to a certain value (around 0.1), the system efficiency increases as well; but the efficiency drops when k continues to increases. This result seems to be opposite to the empirical sense that a strong coupling of WPT will achieve a higher transfer



Figure 7: System efficiency and optimal load for the coupling system with the rectifier. (a) Efficiency curve under various k and R_l . (b) Maximum system efficiency and optimal R_l under various k.



Figure 8: Maximum efficiencies under various k.

efficiency.

In order to yield more insights into this observation, all the efficiency terms are compared in Fig. 8. In this figure, each curve consists of a series of maximum efficiency points under various mutual inductances. These curves illustrate the extreme condition of each efficiency term by tuning R_l . Some conclusions can be drawn as follows:

- 1) η_{coil} increases with respect to the mutual inductance coefficient k, and it is consistent with the empirical sense.
- 2) η_{rec} is almost constant. It means η_{rec} has a limited influence on η_{sys} .
- 3) η_{tran} has a similar trend within the range of small mutual inductance coefficients. However, when k is larger than some value (around 0.1), this efficiency term starts to drop gradually.

It can be seen that η_{sys} drops when k is large due to a worse η_{tran} . The input impedance of the coupling system Z_{in} is plotted by sweeping R_l when k = 0.01, 0.1 and 0.2, respectively in Fig. 9. Since the source impedance is 50 Ω (the origin O in the Smith chart), the power reflection is measured by the distance from any



Figure 9: Z_{in} under different R_l when k = 0.01, 0.1 or 0.2.

point of these curves to the origin. The larger the distance, the smaller η_{tran} is. It is observed that the smallest minimum distance corresponds to k=0.01. This observation is consistent with the curve of η_{tran} in Fig. 8. It can seen that Z_l' [refer to (7) for a 100% η_{tran}] is difficult to achieve by only controlling R_l . Controlling power reflection becomes crucial when strong coupling occurs in the WPT system.

V. EXPERIMENTAL VERIFICATION

The experimental setup is shown in Fig. 10. It includes a power amplifier, a bidirectional coupler (power detection), a coupling system, a full-bridge rectifier using SiC diodes and an electrical load. The incident power of P_i is controlled to be 40 W, and the electrical load is tunned to provide an optimal load. For different mutual inductance (by adjusting the transfer distance), the optimal load resistances are shown in Fig. 11(a), where five cases with different distances are measured. Fig. 11(b) summarizes the maximum efficiency points and corresponding optimal loads in Fig. 11(a) with various transfer distances (each distance is converted to its corresponding mutual inductance). The results show that the maximum system efficiency drops when strong coupling occurs, and again it is consistent with the simulation results in Fig. 8.

VI. CONCLUSION

This paper provides a subsystem-level discussion on a 13.56 MHz wireless power transfer system for mid-power application. Two systems, the coupling-only system and the system with rectifier, are discussed and compared. It is found a system with rectifier introduces more power reflection. This lowers the overall system when there is a strong coupling. All the analysis on the subsystem-level efficiencies can guide the design of an optimal load and input





Figure 11: System efficiency and optimal load in experiments. (a) Efficiency curve under various R_l and transfer distance. (b) Maximum system efficiency and optimal load under various k.

impedance matching network. Finally, the theoretical analysis is validated by the experimental results.

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