Optimization Based Energy Control for Battery/Super-capacitor Hybrid Energy Storage Systems

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***Abstract*—Batteries have been widely used as electrical energy storage units nowadays. However, due to their low power-density, it is usually necessary to combine batteries with other energy storage units, such as super-capacitors, in hybrid energy systems. In this paper, an optimization based control strategy is proposed to improve the energy efficiency as well as battery life time for battery semi-active hybrid systems. Sharing the similar idea as average current strategy but without any predefined driving cycle, this strategy aims to converge the current of the battery pack to the average current of the test trip by mainly enforcing the energy left in the super-capacitor pack to be same as its initial state while to maintain the current variation as small as possible. To achieve those two objectives, a two-objective optimization problem, whose objectives represents the preferences of the battery and super-capacitor packs, is formed and easily solved by using KKT conditions at any control point. The simulation results of this strategy are compared with those from average current strategy, and show that the proposed strategy can achieve comparable performance.**

Keywords—batteries; supercapacitors; hybrid energy storage systems; optimization; energy management; utility functions; muli-agent systems.

I. INTRODUCTION

Energy management in electric vehicles, smart grids and renewable energy systems has become an important research topic recently [1-5]. Among all energy storage units, batteries are considered to be the most basic ones and have been widely used nowadays, even though they still have limitations such as relatively low power-density. This shortcoming can be leveraged to certain degree by combining them with super-capacitors which have relatively higher power-density but lower energy-density. Due to this complementariness, battery/super-capacitor hybrid energy storage systems (HESSs) are becoming more and more attractive for applications with highly cost-efficient energy storage units.

Current battery/super-capacitor HESSs have different structures [6], which can be generally classified into two types, passive and active. Power circulations can happen in passive systems since they do not use any DC-DC converter, while active systems can achieve the same end voltages for battery and super-capacitor packs by using one or two DC-DC converters. This paper focuses on the control strategy for the HESS with a single DC-DC converter on the battery side, which is also known as the battery semi-active hybrid system (BSHS) [6].

For BSHSs or similar systems, people have developed different control strategies [8-16]. However, since they do not directly control the current of the battery pack, large current variation is inevitable, which can significantly affect the life time of the battery pack. To address this important issue, a common strategy, average current strategy (ACS) [17], has been developed recently to maintain the output current of the battery pack at the average current of the entire driving cycle (refer to the Appendix for the detailed definition of the average current). This strategy is known so far to be the best one because two objectives, i.e., the output current of the battery pack should go to the average current of the driving cycle and the current variation should be as small as possible, can be achieved in this strategy simultaneously. Since the battery pack provides an almost constant current to take care of the average load demand and the super-capacitor pack takes care of the high-frequency demand, high energy efficiencies of BSHSs and long life time of battery packs can be achieved by using ACS. However, this strategy can only be applicable when the entire driving cycle is clearly defined in advance and exactly followed during the experiments, which is very difficult for the real-world applications if it is not impossible. This shortcoming has severely limited the applicability of ACS for real-world applications. In order to increase the life time of battery packs, an optimization based strategy has been proposed in the literature [20, 21], whose main objective was to control the current variation of the battery pack and, in addition, to reduce the energy loss on super-capacitors with the consideration of predicted load profiles. However, they did not maintain the current level of the battery pack.

To achieve similar effects as in ACS, the current of the battery pack has to be converged to the average current (which, actually, cannot be known in advance for real-world applications). One possible mechanism to achieve this is to control the rest energy in the super-capacitor pack at the end of the test trip to be the same as its initial state [19]. In this way, from the energy point of view, the super-capacitor pack will not make any contribution to satisfying the average energy demand, but will take care of the high-frequency load supply instead, which is actually equivalent to pushing the current of the battery pack to be close to the average current of the test trip.

Given the discussion above, an optimization based control strategy for BSHSs is proposed in this paper to provide com-

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parable control results as in ACS, but without any predefined driving cycle. To extend the life time of the battery pack [23] and to improve the energy efficiency of the BSHS [19], two objectives are identified in this work: i) to minimize the current variation and ii) to minimize the difference between the rest energy in the super-capacitor pack and its initial state. This control strategy is achieved by solving an optimization problem using Karush-Kuhn-Tucker (KKT) conditions at any control point [25]. This optimization problem will take the present situation of the BSHS as its inputs and calculate the corresponding required output current for the battery pack. Then, the DC-DC converter will control the current of the battery pack in an on-line fashion. The proposed strategy control the input side current of the DC-DC converter which is not like in ACS where the out side current is controlled. Since the power split ratio for the battery and super-capacitor packs, which is determined based on the optimal solution from the optimization procedure, will be updated at the current control point based on the current state of the system, thus it is not necessary to predefine any driving cycle. The simulation results show that the controlled current of the battery pack converges to the average current of the test trip with the energy left in the super-capacitor pack remaining similar to its initial state. Compared to ACS, this strategy can converge the current of the battery pack to that in ACS and the current variation in our strategy is also small enough. Since the BSHS considered in this work consists of multiple energy units connected together, multi-agent based simulation software, named Netlogo [7], is applied in this work to demonstrate the developed control strategy.

The remaining of this paper is organized as follows. Section II discusses the structure of the BSHS used in this work and the corresponding simulation model. The details of the proposed control strategy are provided in Section III, with the definitions of utility functions, objectives, constrains, and weight coefficients used in the optimization problem. Section IV provides the simulation results and corresponding analysis, followed by the conclusion of this work in Section V.

II. SYSTEM STRUCTURES AND SIMULATION MODELING

The BSHS considered in this paper and the corresponding simulation model are introduced in details in this section.

The battery semi-active hybrid system consists of a single DC-DC converter, a battery pack (two in series for one set and four sets in parallel in this work), and a super-capacitor pack (eight in series in this work), as showed in Fig.1.

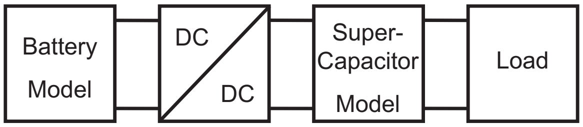


Fig. 1. Structure of Battery Semi-active Hybrid System

To simulate the BSHS shown above, a multi-agent based programming tool and modeling environment, Netlogo, is applied [7]. The battery pack and the super-capacitor pack are treated here as two agents, who can collaboratively and mutually control their own energy contributions to the entire system determined by the proposed control strategy. Figure 2 shows the interface of the simulation model.

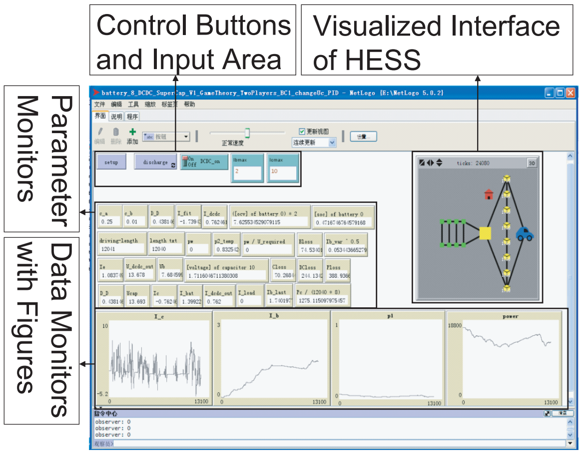
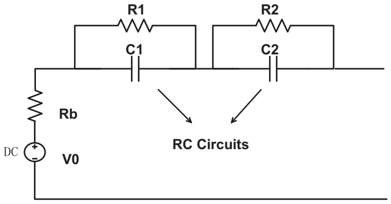
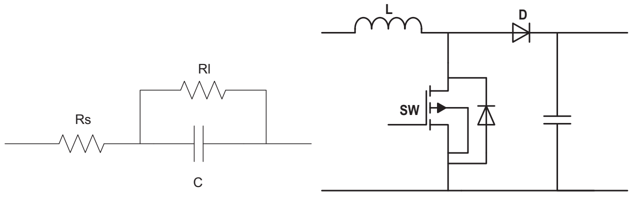


Fig. 2. Interface of Netlogo

The battery pack model used here is a down-scaled representation of the battery pack used in I-MiEV, manufactured by Mitsubishi for Japanese market. As showed in Fig.3(a), one battery cell in the pack contains an ideal voltage source, a series resistance, and two resistance-capacitor oscillation circuits. Parameters of those components can be represented by six-ordered polynomial functions of the SOC (State of Charge) [18]. In addition, the model of a single super-capacitor in the pack is showed in Fig.3(b), including an ideal capacitor, a series resistance, and a parallel resistance. The DC-DC converter on the battery side has a typical boost structure as shown in Fig.3(c) [24].



(a) Battery Model



(b) Super-capacitor Model (c) DC-DC Converter Model

Fig. 3. Models

The detailed parameters of the super-capacitor, and the DC-DC converter are listed in Table.I.

III. OPTIMIZATION BASED CONTROL STRATEGY

Similar to ACS, there are two objectives in the proposed control strategy for the BSHS. The first objective is to minimize the current variation in order to extend the life time

TABLE I. PARAMETERS OF BSHS

DC-DC converter			
R_{sw} (Ω)	0.005	R_L (Ω)	0.006
L (uH)	32	U_d (V)	0.7
Single super-capacitor			
R_s (Ω)	0.0025	R_l ($k\Omega$)	1.23
C (F)	1760	I_{cmax} (A)	20
U_{cmax} (V)	2.5	U_{emp} (V)	0.77

of the battery pack. The second objective is to minimize the difference between the energy left in the super-capacitor pack and its initial state in order to improve the energy efficiency. In this regard, optimization techniques should be applied. Since the battery and the super-capacitor packs are collaboratively working together as the agents in the simulation, the energy based objectives of the optimization based control strategy will be presented by the preferences of the agents, that is, the utility functions of the battery and the super-capacitor packs will be used as the objectives. The details of the optimization are discussed in the following.

A. Utility Function

The preferences of the battery and the super-capacitor packs can be expressed by utility functions [22], which quantify how much benefit one energy source could obtain if it provides certain energy at a particular time to the system. For example, $u_a > u_b$ if and only if the agent prefers outputting a units of the energy more than b units.

In this work, the preference of the battery pack focuses on its life time, which is related to two factors, the current amplitude and discharging current rate $\frac{\mathrm{d}I_{bat}}{\mathrm{d}t}$. The preference of the super-capacitor pack, on the other hand, focuses on the difference between the rest energy in the super-capacitor pack and its initial state. In this case, quadratic functions are used to represent those utility functions which can achieve their maximum values when the currents can meet the corresponding load demands [22]. The utility functions of two packs are defined as follows.

1) Utility Function of Battery Pack: The utility function of the battery pack, u_{bat} is equivalent to the utility of the battery life time, u_{life}, which contains two parts, u_{ave} and u_{dif}, as shown in (1). The aim of u_{ave} is to minimize the current amplitude of the battery pack while the aim of u_{dif} is to minimize the current variation $\frac{\mathrm{d}I_{bat}}{\mathrm{d}t}$, as shown in (2) and (3).

$$u_{bat} = u_{life} = w_{ave}u_{ave} + w_{dif}u_{dif} \tag{1}$$

$$u_{ave} = 1 - a(I_{bat} - I_{ave})^2 \tag{2}$$

$$u_{dif} = 1 - b(I_{bat} - I_{lbat})^2 \tag{3}$$

In (1), w_{ave} and w_{dif} are weight coefficients for u_{ave} and u_{dif}, respectively. How to determine the values of those two coefficients will be discussed later in Section III-D. I_{ave} in (2) is the average current of the test trip so far (i.e., from the beginning to the current control point). In (3), I_{lbat} is the current of the battery pack at the last control point, which is used to obtain $\frac{\mathrm{d}I_{bat}}{\mathrm{d}t}$, since $\frac{\mathrm{d}I_{bat}}{\mathrm{d}t}$ is equal to $\frac{I_{bat}-I_{lbat}}{\mathrm{d}t}$. The coefficient a in (2) can be calculated using (4) which is designed to normalize the value of u_{ave} to be zero when I_{bat} comes to its maximum value I_{bmax} (i.e., 10 A in this work). The coefficient b in (3) is used to normalize the value of u_{dif} to be zero when $\frac{\mathrm{d}I_{bat}}{\mathrm{d}t}$ comes to its maximum threshold. Since the maximum value of $\frac{\mathrm{d}I_{bat}}{\mathrm{d}t}$ determines the maximum current variation that the system could take, the designer can specify this threshold based on design requirements of the system. In this work, this threshold is defined as 0.1, as shown in (5), and it can be changed for different requirements.

$$a = (I_{bmax} - I_{ave})^{-2} \tag{4}$$

$$b = 0.1^{-2} \tag{5}$$

2) Utility Function of Super-capacitor Pack: The utility function of the super-capacitor pack u_{cap} can also be expressed as an utility of the energy performance of the super-capacitor pack u_e with a weight coefficient w_e, as shown in (6). In this function, the larger the utility values, the closer the current of the super-capacitor pack I_{cap} to a designed current value I_{fit}. As shown in (7), c can be calculated in the same way as a and b in (4) and (5), and I_{cmax} is defined in Table.I. As shown in (8), I_{fit} is determined in a way to make sure when I_{cap} is closer to I_{fit}, the energy left in the super-capacitor pack is closer to its initial state (i.e., 14.8V in this work). In this function, I_{fit} should be greater than zero if the current energy in the super-capacitor pack is larger than the initial state. Thus, when the current of the super-capacitor pack is close to I_{fit}, the super-capacitor pack should be discharged at the next control point and its energy level should become closer to its initial state. The same effect can be observed if the present energy in the super-capacitor pack is smaller than its initial state when the super-capacitor pack should be charged (The larger the difference between the energy left in the super-capacitor pack and its initial state is, the larger $\mid I_{cap} \mid$ should be). The range of I_{cap} is from $-I_{cmax}$ to I_{cmax}. In (8), U_{cmax} and U_{emp} are the maximum and minimum voltages of a single super-capacitor whose values are listed in Table.I.

$$u_{cap} = w_e u_e = w_e[1 - c(I_{cap} - I_{fit})^2] \tag{6}$$

$$c = (I_{cmax} - I_{fit})^{-2} \tag{7}$$

$$I_{fit} = \Big(2\frac{U_{cap}^2 - U_{emp}^2}{U_{cmax}^2 - U_{emp}^2} - 1\Big)I_{cmax} \tag{8}$$

B. Problem Formulation

The utility functions of the battery and super-capacitor packs can be directly used as objective functions of the optimization problem, as shown in (9).

$$\begin{aligned} OBJ_1 &: f_{min} = -u_{bat} \\ OBJ_2 &: f_{min} = -u_{cap} \end{aligned} \tag{9}$$

To transform this bi-objective optimization problem into a single-objective optimization problem, the weighted-sum approach is used. The entire objective function can be formulated as in (10), where w_{ave}, w_{dif} and w_e are three weight coefficients. The selection of those coefficients will be discussed later in Section III-D.

$$\begin{gathered} OBJ : f_{min} = -w_{ave}u_{ave} - w_{dif}u_{dif} - w_e u_e \\ w_{ave} + w_{dif} + w_e = 1 \\ 0 \le w_{ave}, w_{dif}, w_e \le 1 \end{gathered} \tag{10}$$

Constraints are also necessary for this optimization problem to make sure the system is physically feasible. One important constraint is to guarantee that the sum of the currents from the battery and super-capacitor packs is equal to the load demand I_{load}. Others are physical box constraints on the upper and lower bounds of the currents of the battery and super-capacitor packs, as shown in (11),

$$\begin{aligned} I_{cap} + I_{bat}(1-D) - I_{load} &= 0 \\ -I_{bat} &\leq 0 \\ I_{bat} - 10 &\leq 0 \\ -I_{cap} - 20 &\leq 0 \\ I_{cap} - 20 &\leq 0 \end{aligned} \tag{11}$$

where D is the duty cycle of the DC-DC converter.

Two design variables of the optimization problems are $x_1 = I_{bat}$ and $x_2 = I_{cap}$ and the entire optimization problem can be formulated as (12).

$$\begin{aligned} OBJ: f_{min(x_1,x_2)} = &-w_{ave}[1 - a(x_1 - I_{ave})^2] \\ &-w_{dif}[1 - b(x_1 - I_{lbat})^2] \\ &-w_e[1 - c(x_2 - I_{fit})^2] \\ S.T.: x_2 + x_1(1-D) - I_{load} &= 0 \\ -x_1 &\leq 0 \\ x_1 - 10 &\leq 0 \\ -x_2 - 20 &\leq 0 \\ x_2 - 20 &\leq 0 \\ w_{ave} + w_{dif} + w_e &= 1 \\ 0 \leq w_{ave}, w_{dif}, w_e &\leq 1 \end{aligned} \tag{12}$$

C. KKT Conditions

Due to the simplicity of the problem in (12), KKT conditions are used to solve this problem [25]. Since the box constrains defined in (12) are relative loose, actually the optimal solution does lie within the feasible domain instead of on the boundaries. Therefore, only the first constraint is shown in the KKT conditions. It can be easily proved that both necessary and sufficient KKT conditions hold for this problem and the optimal solution is given in (13) and (14). The lagrangian function, L, in (13) and (14) are used to find the optimal value of the current of battery pack and super-capacitor pack. This optimal solution determines the current split between the battery and super-capacitor packs and will be updated at any control point based on the current status of the system (i.e., the values of the parameters in (13) will be updated at any control point).

$$\begin{aligned} L = &- w_{ave}[1 - a(x_1 - I_{ave})^2] - w_{dif}[1 - b(x_1 - I_{lbat})^2] \\ &- w_e[1 - c(x_2 - I_{fit})^2] + v(x_2 + x_1(1-D) - I_{load}) \\ \frac{\mathrm{d}L}{\mathrm{d}x_1} =& 2aw_{ave}(x_1 - I_{ave}) \\ &+ 2bw_{dif}(x_1 - I_{lbat}) + (1-D)v = 0 \\ \frac{\mathrm{d}L}{\mathrm{d}x_2} =& 2w_e c(x_2 - I_{fit}) + v = 0 \\ \frac{\mathrm{d}L}{\mathrm{d}v} =& x_1(1-D) + x_2 - I_{load} = 0 \\ KKT \quad point&: \\ x_1 =& \frac{aI_{ave}w_{ave} + bI_{lbat}w_{dif}}{aw_{ave} + bw_{dif} + c(1-D)^2 w_e} \\ x_2 =& I_{load} - (1-D)x_1 \\ v =& 2cw_e[(1-D)x_1 + I_{fit} - I_{load}] \end{aligned} \tag{13}$$

$$Sufficient \quad Condition: \quad \nabla^2 L = \begin{pmatrix} 2aw_{ave} + 2bw_{dif} & 0 \\ 0 & 2cw_e \end{pmatrix} \tag{14}$$

D. Determination of Weight Coefficients

As shown above, the optimal solution of the problem in (13) can be represented symbolically as a function of weight coefficients. As long as the values of those coefficients can be determined, the actual solution can be obtained at any control point. In this problem, w_e can be first calculated using a linear function as shown in (15). This function is used to make sure that w_e goes to w_{min} when the energy left in super-capacitor pack converges to the initial state and it becomes one when the super-capacitor pack is full or empty. In (15), U_{ini} is the initial voltage of a super-capacitor unit, which is calculated by (16) (i.e., 1.85V in this work). w_{min} is the lowest value of w_e which is also an user-defined value. In this work, $w_{min} = 0.7$. With the calculated value of w_e, w_{ave} and w_{dif} can be determined with $\frac{w_{ave}}{w_{dif}} = \frac{1}{4}$, which is an user-defined proportion to make sure the discharge curve of the battery pack can be appropriately shown.

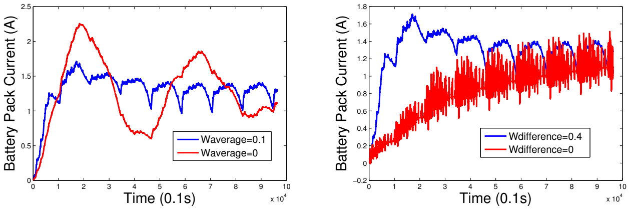
$$w_e = w_{min} + \frac{1 - w_{min}}{U_{ini}^2 - U_{emp}^2} \mid U_{ini}^2 - U_{cap}^2 \mid \tag{15}$$

$$U_{ini} = \sqrt{\frac{{U_{cmax}}^2 + {U_{emp}}^2}{2}} \tag{16}$$

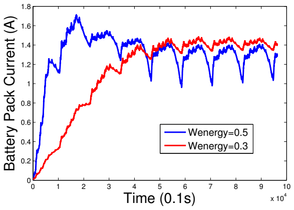
Further analysis on w_{ave}, w_{dif}, and w_e shows that they have special impacts on the discharge curve of the battery pack. Based on our analysis, w_{ave} focuses on the minimization of the difference between I_{bat} and I_{ave}, which is equivalent to controlling the amplitude of I_{bat}: the larger w_{ave}, the smaller the amplitude of I_{bat}. This observation can be verified by the simulation result shown in Fig.4(a).

Additionally, w_{dif} focuses on the minimization of the difference between I_{bat} and I_{lbat}, which controls the slope of I_{bat}. The simulation result shown in Fig.4(b) indicates that the larger w_{dif} , the smaller the slope of I_{bat}.

Finally, w_e is functioned to minimize the difference between I_{cap} and I_{fit}. Remember that the purpose of I_{fit} is to minimize the difference between the rest energy in the super-capacitor pack and its initial state in a very efficient way (i.e., the current of the battery pack should converge to the average current fast). Thus, a larger w_e will make I_{bat} converge to the average current of the test trip more quickly, as shown in Fig.4(c).



(a) Comparing w_{ave} (b) Comparing w_{dif}



(c) Comparing w_e

Fig. 4. Impacts of Weight Coefficients

IV. SIMULATION RESULTS AND ANALYSIS

The test driving trip used in the simulation is a combination of eight JC08 driving cycles (in order to make sure that SOC of the battery pack can drop from 50% to 15% and JC08 driving cycle is a new chassis dynamometer test cycle for light vechicles) [26]. Notice here although we use a specific driving cycle, we do not take any advantage of the properties of the driving cycle in advance or use the average current of this driving cycle in our approach. The reason of using a driving cycle here is just to provide a test trip for the comparison between the proposed strategy and ACS. The proposed strategy can be applicable for any other random driving cycle.

Considering the real world situation where the vehicle can stop in the middle of the trip, the average current value of the last trip will be recorded and used at the beginning of the next trip in the simulation.

A. Evaluation Criteria

As defined in (17-19), the average battery current (I_{bave}), the variation of I_{bat} (I_{bvar}), and the average energy in the super-capacitor pack (E_{cap}) are the quantified measures used as the evaluation criteria for the comparison between the proposed strategy and ACS. The detailed calculation of the average current in ACS is given in the Appendix. In (17), N is the total number of control points. In (19), C is the capacitance value of the super-capacitor pack listed in Table.I, and T is the total time of the entire test trip (i.e., $9632s$ in this simulation).

$$I_{bave} = \frac{\sum I_{bat}}{N} \tag{17}$$

$$I_{bvar} = \sqrt{\sum (I_{bat} - I_{lbat})^2} \tag{18}$$

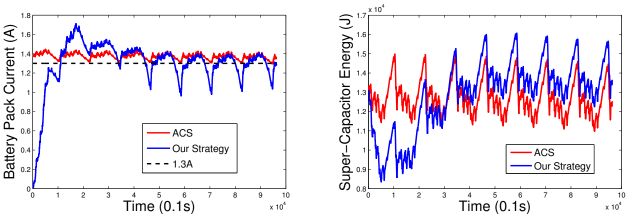
$$E_{cap} = \frac{\Sigma_{C(U_{cap}-U_{min})^2}}{2T} \tag{19}$$

B. Simulation Results

As shown in Table.II, results from two strategies indicate that I_{bave} of the proposed strategy is almost the same as that in ACS (the difference is only 0.19%), while I_{bvar} is 0.0486 A^2, a little bit higher than that from ACS. Figure 5(a) shows the comparison with eight driving cycles in terms of I_{bvar} in which the black line represents the average current from ACS. As shown in Fig.5(b), E_{cap} from our strategy is 1.5% higher than that from ACS. The results show that our strategy provides almost the same current of the battery pack as that in ACS, and Ib_{bvar} and E_{cap} are comparable to those from ACS.

TABLE II. RESULT ANALYSIS

Strategies	I_{bave} (A)	I_{bvar} (A^2)	E_{cap} (J)
ACS	1.325	0.0255	12731.2
Our strategy	1.35	0.0486	12922.9



(a) Current of Battery Pack (b) Energy in Super-capacitor Pack

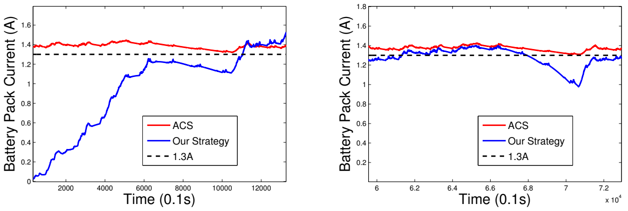
Fig. 5. Simulation Results

C. Results Analysis

Since I_{bave} from the proposed strategy is almost the same as that in ACS, u_e is verified to be effective. The battery utility u_{life} also makes sense since the I_{bvar} is only 0.0231 A^2 higher than that from ACS . Additionally, as in Fig.5(b), E_{cap} has the same characteristics as I_{bat}, implying that E_{cap} also converges to its initial state. Finally, the initial value of I_{ave} plays an important role here. As shown in Fig.6(a), the initial I_{ave} is set to be zero at the beginning of the simulation. Since the current of the battery pack needs to converge to the average current during the first cycle, the current variation in the battery pack is relatively large at the beginning. After that, I_{ave} is already the average current of the trip up to the present control point and the variation becomes relatively smaller as shown in Fig.6(b).

V. CONCLUSIONS

In this paper, an optimization based control strategy is proposed to solve the energy management problem for battery semi-active hybrid systems. This strategy aims to extend battery life time and simultaneously to improve the energy efficiency of the hybrid system by using optimization technique. To formulate the optimization problem, preferences of the battery and the super-capacitor packs are represented by utility



(a) Starting From Beginning

(b) Starting from Middle of Simulation

Fig. 6. The Initial Value of I_{ave}

functions. The corresponding utility functions are constructed in such a way that the output current from the battery pack should be as close to the average of the test trip as possible while maintaining the current variation as small as possible. Those two targets are achieved in this strategy by ensuring the energy left in the super-capacitor pack at the end of the trip to be the same as its initial state. The formulated optimization problem can be easily solved by using KKT conditions and the value of the optimal solution is updated at any control point. The results of this strategy are compared with those from ACS in terms of the average current of the battery pack, current variation, and average energy in the super-capacitor pack. The proposed strategy provides similar performance to ACS in terms of the average current and average energy in the super-capacitor pack but a little bit worse than ACS in terms of the current variation. Notice that those performances are achieved in the proposed strategy without pre-defining any driving cycle. The experiment verification has been done and will be presented in future works.

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APPENDIX

The average current strategy tries to keep the current output from the DC-DC converter to be a constant so that the battery pack will take care of the average current of the entire driving cycle while the super-capacitor pack will satisfy all dynamic current demands. The output current from the DC-DC converter in ACS can be calculated as (20).

$$I_{dc} = \frac{\int P_{DC}(t)\mathrm{d}t}{TU_{cap}} \tag{20}$$

where $P_{DC}(t)$ is the power demand of the driving cycle and U_{cap} is the end voltage of the super-capacitor pack, which is usually considered as a constant.