Output Power Improvement by Impedance Matching Networks for a Class E Power Amplifier Driven Wireless Power Transfer Systems

Minfan Fu, Zefan Tang, Ming Liu, Shuangke Liu, Xinen Zhu, Chengbin Ma*, University of Michigan and Shanghai Jiao Tong University Joint Institute, Shanghai, China Email: fuminfan@sjtu.edu.cn

Abstract—Wireless power transfer working at several Megahertz has been continuously developed for charging small/medium-power devices during these years. A class E power amplifier can be used to drive the system with high efficiency. This paper uses the conventional class E power amplifier in a MHz system. A system-level analysis is carried out to show that the system efficiency can be maximized for different coupling coefficient. However, an output power is observed to decrease with the coupling coefficient. This drop is caused by the rectifier's input reactance and the load sensitivity of the power amplifier. Then two different impedance matching networks are used to compensate this drop. In the experiment, the impedance matching networks can successfully enhance to output capability and provide a stable output power without tuning the input voltage. For the best case, the output power can be improved from 5 W to 10 W with an overall system efficiency about 80%.

Index Terms—Wireless power transfer, class E power amplifier, rectifier input reactance, impedance matching networks, maximum efficiency tracking

I. INTRODUCTION

Wireless power transfer (WPT) has experienced a continuous development over the past years. Various WPT-charged devices have been proposed and designed, which include but not limited to small sensors, wearable devices, mobile phones, and electrical vehicles [1]-[5]. Usually, the WPT systems consist of similar power conversion blocks, such as ac sources, compensated coupling coils, rectifiers and output voltage regulators. When a WPT system is designed, it should select proper frequency and circuit topologies according to the requirements of each specific application [6]. It is attractive to increase the resonance frequency to several MHz for large spatial freedom [7]. Such a MHz system is particularly suitable for small-power portable devices due to its excellent user experience. Based on the success of kHz WPT, MHz WPT has also attracted wide interest during these years. Many system analysis, design, optimization and control techniques, although some are validated at kHz, can be directly used in a MHz system. Such examples include high-efficiency power amplifiers [7], [8], coils' design and analysis [9], [10], novel rectifiers [11], [12], impedance matching networks [13], and power flow control [14].

In order to drive a MHz system, it requires a stable and high-efficiency dc/ac conversion topology. The class E power amplifier (PA) is a promising candidate due to its simple structure and high efficiency [7], [8], [15]. It can reach a theoretical 100% efficiency by zero-voltage switching (ZVS) and zero-voltage derivative switching (ZVDS). However, significant variations can cause the output deviation from its original design, and even permanently damage the transistor. These undesirable variations are mainly introduced by the coupling and load variation in a WPT system. Therefore, it is necessary to satisfy PA's optimal load requirement. This optimal operation can be achieved by several approaches, such as frequency tuning, dynamic impedance matching, and PA's parameters tuning [7], [13], [16]. Some papers have also used the dc/dc converter after the rectifier to optimize the system efficiency [17]-[20]. These works can be directly introduced in a class E PA driven system to achieve maximum efficiency tracking. Before any control approaches are applied, it is necessary to carry out a comprehensive analysis for a class E PA driven system. This paper uses a classical class E PA to drive a 6.78 MHz WPT system. It analyzes the PA's load sensitivity and derive its load requirements. Then a systemlevel simulation is used to investigate the system performance under different load and coupling coefficient. The output power drop phenomena is observed for large coupling coefficient if the maximum efficiency is tracked. This phenomena is caused by the rectifier input reactance and PA's load requirement. In order to compensate this drop, two different impedance matching networks, placed at transmitter side or receiver side, are used to enhance output power capability without increasing the input voltage.

This paper is organized as follows. In Section II, the system configuration is given with efficiency definition for different subsystems. It first briefly reviews the characteristics for the power amplifier and discusses the overall system performance under different coupling and load condition. It shows that the output power decreases with the coupling coefficient if the maximum efficiency is tracked for different coupling conditions. In Section III, the rectifier input impedance is analyzed individually and shown to have significant influence for the output power. Two different impedance matching networks are proposed to compensate the output power drop. Then in section IV, the theoretical analysis and system design are validated by experimental results. Finally the conclusion is drawn in Section V.



Fig. 1. Circuit model for wireless power transfer system.

II. SYSTEM ANALYSIS AND DISCUSSION

A. Circuit model

Fig. 1 shows the circuit model for the proposed WPT system. It includes a class E power amplifier, two compensated coupling coils and a rectifier. The overall system efficiency η_{sys} is defined as

$$\eta_{sys} = \frac{P_{load}}{P_{pa}},\tag{1}$$

where P_{load} is the power received by R_{load} , and P_{pa} is the input power from the dc source. The system efficiency can also be represented as a product form,

$$\eta_{sys} = \eta_{pa} \cdot \eta_{coil} \cdot \eta_{rec},\tag{2}$$

where η_{pa} , η_{coil} , and η_{rec} are the efficiencies of the PA, the coupling coils, and the rectifier respectively.

B. Class E Power Amplifier

As shown in Fig. 1, a conventional class E PA consists of a RF choke L_f , a transistor S_1 , a shunt capacitor C_s , and a series resonant circuit L_0 , C_0 with net reactance jX. V_{pa} and I_{pa} are the input voltage and current from the dc source. The efficiency for the power amplifier is

$$\eta_{pa} = \frac{P_{coil}}{P_{pa}},\tag{3}$$

where P_{coil} is the PA output power. In order to achieve high η_{pa} , the PA's parameters, i.e., L_f , C_s , L_0 and C_0 , should be properly designed to achieve ZVS and ZVDS for S_1 . For an ideal PA, it has infinite L_f , high Q for the resonant circuit, and lossless S_1 . The PA efficiency can be optimized for any fixed resistive load Z_{opt} by Raab's equations,

$$\begin{cases} C_{s} = \frac{8}{\omega \pi (\pi^{2} + 4)Z_{opt}} \approx \frac{0.184}{\omega Z_{opt}} \\ X = \frac{\pi (\pi^{2} - 4)}{16} Z_{opt} \approx 1.15 Z_{opt} \\ P_{coil} = \frac{8}{\pi^{2} + 4} \frac{V_{pa}^{2}}{Z_{opt}} \approx 0.587 \frac{V_{pa}^{2}}{Z_{opt}} \\ j\omega L_{0} + \frac{1}{j\omega C_{0}} = jX \end{cases}$$
(4)

where ω is the working frequency [21]. In practice, Z_{coil} should be controlled close to Z_{opt} for the success of PA, and $Z_{coil} = R_{coil} + j * X_{coil}$. Ψ_{coil} is the phase of Z_{coil} and $\Psi_{coil} = \tan^{-1}(X_{coil}/R_{coil})$. In a WPT system, Z_{coil} is actually the input impedance of the coupling coils, and its variation will cause significant output deviation from the original design. For a normalized class E PA $[V_{pa} = 1 V]$ and $Z_{opt} = 1 \Omega$], the circuit parameters can be derived based on (4). The PA's output characteristics are shown in Fig. 2 for different Z_{coil} . Theoretically, η_{pa} can reach 100% at the optimal point Z_{opt} , where $R_{coil} = 1 \ \Omega$ and $\Psi_{coil} = 0^{\circ}$. It is also should be noticed that there is a region around Z_{opt} for a high η_{pa} . For better illustration, the PA's performance is further evaluated in Fig. 3 when either R_{coil} or Ψ_{coil} are fixed at its optimal value. η_{pa} of Fig. 3 (a) has a narrower peak compared to that of Fig. 3 (b), which means η_{pa} is more sensitive to R_{coil} . While P_{coil} are sensitive to both R_{coil} and Ψ_{coil} .



Fig. 2. Efficiency of PA for different Z_{coil} .



Fig. 3. PA's performance around Z_{opt} . (a) Sweeping R_{coil} when $\Psi_{coil} = 0^{\circ}$. (b) Sweeping Ψ_{coil} when $R_{coil} = 1\Omega$.

C. Overall System Performance

In the WPT system [see Fig. 1], all the circuits looking into the coils serve as the load for the power amplifier. It is necessary to ensure a reasonable Z_{coil} for high η_{pa} , which is a necessity for a high η_{sys} . There are two subsystems after the PA, namely the coupling coils and the rectifier. For the coupling coils, L, C and R with different subscripts (t and r) represent the coils inductors, compensated capacitors and parasitic resistors. The resonance is achieved by

$$j\omega L_t + \frac{1}{j\omega C_t} = 0 \text{ and } j\omega L_r + \frac{1}{j\omega C_r} = 0.$$
 (5)

Under resonance, Z_{coil} can be derived based on the circuit model of the coupling coils as

$$Z_{coil} = R_t + \frac{\omega^2 k^2 L_t L_r}{R_r + Z_{rec}},\tag{6}$$

where Z_{rec} is the input impedance of the rectifier and k is the coupling coefficient. Here Z_{rec} is no longer a pure resister due to the obvious parasitic parameters of the rectifier, especially the shunt capacitors of the diodes. It is impractical to obtain Z_{rec} in terms of R_{load} analytically. In this paper, a system-level simulation is carried out to evaluate the overall system performance for different k and R_{load} .

A WPT system of Fig. 1 is built in Advanced Design System (ADS), a famous high frequency simulation tool. The system

simulation parameters are given in Table I, which are also used in the final experiment. Fig. 4 gives the system performance when k = 0.2 for different R_{load} . η_{sys} is maximized at $R_{load,opt}$. The maximum efficiency point is a compromise of η_{pa} , η_{coil} , and η_{rec} . For each k, there exists a $R_{load,opt}$ to achieve maximum η_{sus} . All these maximum η_{sus} 's and their corresponding output power P_{load} can be obtained and summarized in Fig. 5. It shows the maximum available η_{sys} increases with k. In a WPT system, a dc/dc converter is usually used after the rectifier to regulate the final output voltage. And the control of dc/dc converter can also be used to keep track with the maximum system efficiency as reported in [17]-[20]. However, in this system, when the maximum η_{sys} is tracked, its output power capability will decrease with k. It equally means a receiving device will get less power when it is placed closer to the transmitting coil.

TABLE I System parameters

Z_{opt}	V_{pa}	L_{f}		C_s	L_0	I	S_1
15 Ω	20 V	$68~\mu H$	l	287 pF	1.47 μH		SUD06N10
Co	L_t, L_r	C_t, C_r	Į	R_t, R_r	C_f		$D_1 - D_4$
523 pF	3.34 μH	165 pF		0.7 Ω	10 uF		STPSC406



Fig. 4. WPT system efficiencies and output power when k = 0.2.

The drop of P_{load} is because of the impedance mismatch. Fig. 6 gives the port impedances, Z_{rec} and Z_{coil} , when $R_{load,opt}$ is applied for different k. $R_{load,opt}$ increases with k to ensure a maximum η_{pa} . The increase of $R_{load,opt}$ leads to an increase of R_{rec} . Finally a stable R_{coil} (about 10 Ω) is obtained to ensure a high-efficiency PA [refer to (6)]. However, a negative Ψ_{rec} is obtained and decreases with k, and this negative and decreasing Ψ_{rec} will cause a positive and increasing Ψ_{coil} due to impedance inversion effect of the coupling coils [refer to (6)]. According to Fig. 3 (b), the increase of Ψ_{rec} will naturally cause a drop of P_{coil} .



Fig. 5. η_{sys} and P_{load} for different k when $R_{load,opt}$ is applied.



Fig. 6. Impedance at different ports.

III. IMPEDANCE MATCHING

The drop of P_{load} is because of a nonzero Ψ_{coil} , which is essentially caused by the rectifier input reactance X_{rec} . A straightforward method is to use impedance matching networks (IMN) to eliminate the reactance effects. In practice, the IMNs can be added at the transmitter (TX) side, the receiver (RX) side or both sides as shown in Fig. 7. This paper will compare the system performance when the IMN is added at the TX side and the RX side respectively.



Fig. 7. System with impedance matching networks.

When an IMN is added at the TX side, its objective is to ensure $\Psi_{coil} = 0$, which equally means a zero X_{coil} . X_{coil} can be derived as

$$X_{coil} = \frac{-\omega^2 k L_t L_r X_{REC}}{(R_r + R_{REC})^2 + (X_{REC})^2} + j\omega L_t + \frac{1}{j\omega C_t}.$$
 (7)

Therefore, a zero X_{coil} can be achieved by varying C_t . When IMN is added at the RX side, a zero Ψ_{coil} can also be achieved by tuning C_r . According to (6), a pure resistive Z_{rec} can ensure a pure resistive Z_{coil} . Therefore, C_r can be used to compensate X_{rec} to provide a zero X_{rec} . And C_r can be tuned according to

$$j\omega L_r + \frac{1}{j\omega C_r} + jX_{rec} = 0.$$
(8)

These two IMNs are compared in simulation as shown in Fig. 8. Both IMNs can improve the system overall efficiency and output power when V_{pa} is fixed at 20 V. The system efficiency is slightly improved, because PA's efficiency is insensitive to Ψ_{coil} variation [refer to Fig. 3]. Moreover, the output power is significantly improved when IMNs are applied. This simulation result is well consistent with the system analysis.



Fig. 8. System performance comparison under different IMNs.

IV. EXPERIMENTAL VERIFICATION

A. Experiment Setup

A 6.78 MHz wireless power transfer system is implemented to verify the proposed approaches. As shown in Fig. 9, the system consists of a class E power amplifier, two coupling coils with IMNs, a rectifier and an electronic load. A dc supply is used to provide a constant V_{pa} (20 V) during the measurements. The input power and output power can be directly read from the dc supply and the electronic load, which be used to obtain the overall system efficiency. Two coupling coils are face to face with a vertical distance d. During the experiment, IMNs at TX or RX side is achieved by tuning C_t or C_r until a zero Ψ_{coil} is obtained.

B. Experiment results

The original system performance is first measured for different d when no IMNs are used, i.e., C_t and C_r are designed according to (5). In Fig. 10, three typical distances are selected to show system output characteristics for different coupling coefficient. When d is 45 mm, the maximum system efficiency can be tracked if R_{load} is controlled at about 30 Ω [refer to





Fig. 9. Experiment setup.

Fig. 10 (a)]. With this load, the system output power is about 8 W [refer to Fig. 10 (b)]. When d get smaller, although a higher maximum system efficiency can be obtained, P_{load} actually decreases as the analysis predicts.

Fig. 11 compares the system performance for three conditions, no IMN, IMN at TX side, and IMN at RX side. At first, more positions are chose to obtain the system performance like Fig. 10. During the measurement, a d varying from 16 mm to 50 mm corresponds to a k varying from 0.095 to 0.3. For each d, it can give the maximum η_{sys} , and its corresponding $R_{load,opt}$ and P_{load} . Then these maximum efficiencies are abstracted and summarised in Fig. 11 for no IMN case. Then two IMNs are tested individually for each k to achieve a zero Ψ_{coil} by tuning the capacitors manually. As shown in Fig. 11, both IMNs have similar effects. The system efficiency is slightly improved and be stable at about 80%. The output power can be significantly enhanced when IMNs are applied. The power drop in original system has been successfully avoided. For large k (= 0.3), P_{load} is even been doubled from 5 W to about 10 W. All these curves are well consistent with the simulation results in Fig. 8.

V. CONCLUSION

In this paper, a classical class E power amplifier is briefly reviewed. It analyzes the PA's optimal load requirement and discusses its load sensitivity. Then a class E driven WPT system is given and analyzed in simulation. It shows that the system output power will decrease significantly with the coupling coefficient when the maximum system efficiency is



Fig. 10. Original performance of WPT system under different positions. (a) Overall system efficiency. (b) System output power.

tracked. This output power drop is caused by the rectifier input reactance for a MHz system. Then two kinds of impedance matching networks are used to eliminate the rectifier input reactance's effects. In the final experiment, the results show both IMNs can enhance the output power significantly with a stable system efficiency at about 80%. The best case is to improve the output power from 5 W to 10 W when a fixed 20 V dc input is applied.

REFERENCES

- S. Y. R. Hui, W. Zhong, and C. K. Lee, "A critical review of recent progress in mid-range wireless power transfer," *IEEE Trans. Power Electron.*, vol. 29, no. 9, pp. 4500–4511, 2014.
- [2] D. Ahn and S. Hong, "Wireless power transmission with self-regulated output voltage for biomedical implant," *IEEE Trans. Ind. Electron.*, vol. 61, no. 5, pp. 2225–2235, 2014.
- [3] Y. Zhang, Z. Zhao, and T. Lu, "Quantitative analysis of system efficiency and output power of four-coil resonant wireless power transfer," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 3, no. 1, pp. 184–190, 2015.
- [4] S. Li and C. C. Mi, "Wireless power transfer for electric vehicle applications," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 3, no. 1, pp. 4–17, 2015.
- [5] S. Y. Choi, B. W. Gu, S. Y. Jeong, and C. T. Rim, "Advances in wireless power transfer systems for roadway-powered electric vehicles," *IEEE*



Fig. 11. System performance comparisons. (a) Overall system efficiency. (b) System output power.

Journal of Emerging and Selected Topics in Power Electronics, vol. 3, no. 1, pp. 18–46, 2015.

- [6] W. Zhang, S.-C. Wong, C. K. Tse, and Q. Chen, "Load-independent duality of current and voltage outputs of a series- or parallel-compensated inductive power transfer converter with optimized efficiency," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 3, no. 1, pp. 137–146, 2015.
- [7] S. Aldhaher, P. C. Luk, and J. F. Whidborne, "Tuning class e inverters applied in inductive links using saturable reactors," *IEEE Trans. Power Electron.*, vol. 29, no. 6, pp. 2969–2978, 2014.
- [8] S. Liu, M. Liu, M. Fu, C. Ma, and X. Zhu, "A high-efficiency class-e power amplifier with wide-range load in wpt systems," in *IEEE Wireless Power Transfer Conference (WPTC)*, May 13-15, 2015, Boulder, Colorado, USA.
- [9] V. Jiwariyavej, T. Imura, and Y. Hori, "Coupling coefficients estimation of wireless power transfer system via magnetic resonance coupling using information from either side of the system," *IEEE Journal of Emerging* and Selected Topics in Power Electronics, vol. 3, no. 1, pp. 191–200, 2015.
- [10] M. Fu, T. Zhang, C. Ma, and X. Zhu, "Efficiency and optimal loads analysis for multiple-receiver wireless power transfer systems," *IEEE Trans. Microw. Theory Tech.*, vol. 63, no. 3, pp. 801–812, 2015.
- [11] M. Liu, M. Fu, Z. Tang, and C. Ma, "A compact class e rectifier for megahertz wireless power transfer," in *IEEE PELS Workshop on Emerging Technologies: Wireless Power (WoW), June 5-6, 2015, Daejeon, Korea.*
- [12] S. Aldhaher, P. Luk, K. E. K. Drissi, and J. Whidborne, "High-inputvoltage high-frequency class e rectifiers for resonant inductive links," *IEEE Trans. Power Electron.*, vol. 30, no. 3, pp. 1328 – 1335, 2014.

- [13] T. C. Beh, M. Kato, T. Imura, S. Oh, and Y. Hori, "Automated impedance matching system for robust wireless power transfer via magnetic resonance coupling," *IEEE Trans. Ind. Electron.*, vol. 60, no. 9, pp. 3689–3698, 2013.
- [14] L. J. Chen, J. T. Boys, and G. A. Covic, "Power management for multiple-pickup ipt systems in materials handling applications," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 3, no. 1, pp. 163–176, 2015.
- [15] S. Aldhaher, P. Luk, A. Bati, and J. Whidborne, "Wireless power transfer using class e inverter with saturable dc-feed inductor," *IEEE Trans. Ind. Appl.*, vol. 50, no. 4, pp. 2710 – 2718, 2014.
- [16] N. Kim, K. Kim, J. Choi, and C.-W. Kim, "Adaptive frequency with power-level tracking system for efficient magnetic resonance wireless power transfer," *Electronics letters*, vol. 48, no. 8, pp. 452–454, 2012.
- [17] M. Fu, C. Ma, and X. Zhu, "A cascaded boost-buck converter for high efficiency wireless power transfer systems," *IEEE Trans. Ind. Informat.*, vol. 10, no. 3, pp. 1972–1980, 2014.
- [18] W. X. Zhong and S. Y. R. Hui, "Maximum energy efficiency tracking for wireless power transfer systems," *IEEE Tran. Power Electron.*, vol. 30, no. 7, pp. 4025–4034, 2015.
- [19] M. Fu, H. Yin, X. Zhu, and C. Ma, "Analysis and tracking of optimal load in wireless power transfer systems," *IEEE Trans. Power Electron.*, Accepted on Jul. 29, 2014.
- [20] H. Li, K. W. Jie Li, W. Chen, and X. Yang, "A maximum efficiency point tracking control scheme for wireless power transfer systems using magnetic resonant coupling," *IEEE Trans. Power Electron.*, vol. 30, no. 7, pp. 3998–4008, 2015.
- [21] F. H. Raab, "Effects of circuit variations on the class e tuned power amplifier," *IEEE J. Solid-State Circuits*, vol. 13, no. 2, pp. 239–247, 1978.