Power Distribution of a Multiple-Receiver Wireless Power Transfer System: A Game Theoretic Approach

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Abstract—Wireless power transfer (WPT) has shown its potential over conventional charging systems in recent years. However, it is still challenging to determine the power distribution of a multiple-receiver WPT system for its high sensitivity, complex coupling and load relationships. This paper discusses a game theory based control approach for the power distribution of a multiple-receiver WPT system. The power receiver systems (i.e., a receiving coil, a DC-AC rectifier, a DC-DC converter, and an ultracapacitor pack in this paper.) are modelled as independent agents with different preferences under the Matlab simulation environment where the preferences of each agent are represented by utility functions. Then a non-cooperative power distribution game is set up and a generalized Nash equilibrium is found which is used as the reference solution formula to be updated at every control instant. Meanwhile, the generalized Nash equilibrium is found through finding out the variational equilibrium by Karush Kuhn-Tucker conditions (KKT) conditions. The simulation results show that the game theory based control is comparable to the highest efficiency impedance distribution approach in a three-receiver WPT system.

Index Terms—Game Theory; Wireless Power Transfer; Multiple-receiver System; Ultracapacitor; Generalized Nash equilibrium.

I. Introduction

Wireless power transfer (WPT) is becoming a popular research area due to the rising interests in charging various electronic devices (e.g., cellphones, laptops, electric vehicles, etc.). WPT provides a convenient and safe non-contacting approach for charging electronic devices which also allows flexible power management for multiple-energy systems. Among various WPT systems, the one-receiver WPT systems have been widely studied for all WPT technology [1]–[11]. Moreover, charging multiple loads with a single charging platform is one of the most attractive advantages over the traditional charging approaches. However, since the characteristics and state of charge of each load may be complex, the optimized management and control of a multiple-receiver WPT system is still a challenging task.

This paper discusses the modeling and control of a multiple-receivers WPT system. Since each receiver is independent but related in the multiple-receiver WPT system, the agent based modeling and decentralized control is applied to fully respect the performance and requirements of each load. The load in this paper uses ultracapacitor (UC) packs as an example. UCs provide fast and efficient energy delivery without chemical reaction involved [12], [13]. Meanwhile, the state of charge (SOC) is easier to be estimated for an UC because the SOC of an UC is proportional to the cell voltage. Thus, for the WPT system, a fast and reliability energy storage source, i.e., the UC pack, is preferred.

Besides the hardware aspects, several power management approaches have been proposed for one-receiver WPT system. The resonance frequency tracking approach is widely used in KHz WPT systems [14]. A coil distance adjusting approach is applied by using four symmetric coils [15]. Impedance matching is a straightforward approach to reduce the reflected power and reach a high system efficiency [16], [17]. Meanwhile, a dynamic impedance tuning with perturbation-and-observation-based tracking approach is studied under coil moving situation [18]. For multiple-receiver WPT system, a multiple primary coils analysis is done to reduce the inductance [19]. Besides, a highest efficiency impedance distribution approach (HEIDA) uses triple-receivers system as an example to achieve the highest system level efficiency in multiple receiver system [20]. However, these approaches only consider the WPT system itself but not the characteristics of the energy storage system. Since the basic purpose of WPT systems is to charge energy storage systems, taking the characteristics of energy storage systems into consideration could rise the entire system efficiency and flexibility.

This paper develops a power distribution control approach for the multiple-receiver WPT system considering both the characteristics of WPT system and the power receiving systems (PRSs). PRSs are first modeled an independent but

related agent. The preference for each agent is to achieve their most preferred charging power according the remaining energy in the UC pack. Moreover, the preferences are represented as utility functions which quantify the satisfaction level of an agent for a certain power distribution. Game theory (GT) is a powerful tool for modelling interactions among self-interested players and predicting their choices of strategies [21]–[23]. A GT based control is applied to achieve a balanced power distribution in the multiple-receiver WPT system. The game settles a generalized Nash equilibrium (GNE) by finding the variational equilibrium with KKT conditions at each control instant and the GNE among agents balances the different preferences of each PRSs. Finally, the GT based control is compared with the HEIDA under simulation and achieves a comparable result.

The rest of this paper is organized as follows. Section II describes the system configuration and modelling. Then the non-cooperative power distribution game is introduced in Section III. In Section IV, the simulation results are given comparing with the HEIDA. Finally, conclusions are drawn in Section V.

II. SYSTEM CONFIGURATION AND MODELLING

The multiple-receiver WPT system (MRWPTS) configuration and the modelling in this paper are based on a reference 13.56MHz MRWPTS [20]. The simulation environment used in this paper is the Matlab. The entire MRWPTS circuit model is shown in Fig. 1. The power transferring system (PTS) contains an ideal AC power source (i.e., constant current source.) to represent the power amplifier (PA) and a transfer coil, which consists a self inductance L_t, a capacitor C_t, and a resistance R_t. Meanwhile, the ith PRS contains a self inductance L_i, a self capacitor C_i, a self resistance R_i, a full bridge DC-AC rectifier (i.e., treated as ideal device here.), a unidirectional DC-DC converter and an UC pack. The DC-DC converter is used to tune the impedance of the PRS. The model of a single UC unit (i.e., $700F$ in this paper.) contains a series resistance R_{si} and a parallel resistance R_{li}, while the UC pack with capacitance C_{uci}, applies the series connection. Since the position of the coils is assumed to be fixed in this paper, the mutual inductances between the transferring coil and receiving coils are constant values M_i, shown in Fig. 1. Note that the mutual inductances between receiving coils are ignored for simplicity in this paper.

Based on the circuit model of the MRWPTS shown in Fig.1, a resonance circuit model is built for clear meaning and easy understanding, shown in Fig. 2. In the PTS, the $Z_{R,i}$ represents the equivalent impedance for the ith power receiving system. While, in the PRS, the AC power source jwM_1I, stands for the equivalent PA from the point view of the PRS, while the $Z_{L,i}$ means the equivalent impedance for the DC-DC converter and the UC pack. Based on the resonance circuit model, using the Kirchhoff laws, $Z_{R,i}$ can be written in (1) and the power received by $Z_{L,i}$ is written in (2) [20]. Note that the $Z_{L,i}$ can be tuned by controlling the DC-DC converter and the range
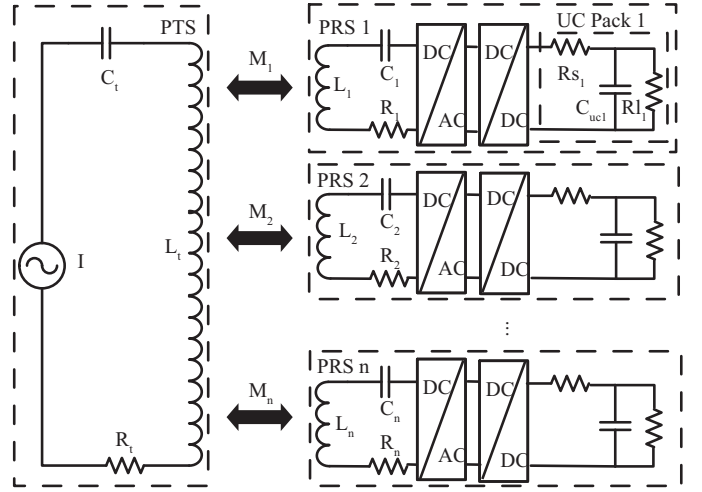

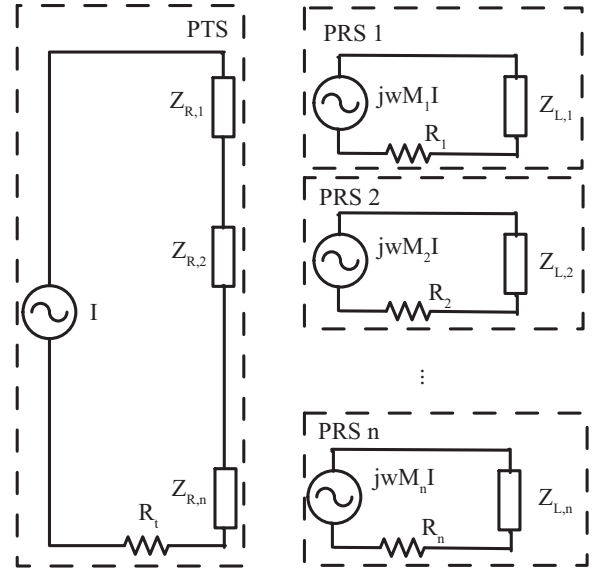Fig. 1. The multiple-receiver WPT system configuration.

Fig. 2. The multiple-receiver WPT system resonance model.

of the $Z_{L,i}$ is $[R_i, \infty)$ if it is possible.

$$Z_{R,i} = \frac{\omega^2 M_{t,i}^2}{R_i + Z_{L,i}} \tag{1}$$

$$P_{L,i} = I^2 \frac{\omega^2 M_{t,i}^2 Z_{L,i}}{(R_i + Z_{L,i})^2} \tag{2}$$

Therefore, for each single PRS, $P_{L,i}$ can be determined though tuning $Z_{L,i}$ (i.e., controlling the DC-DC converter.). Thus, under a constant I (i.e., the input current of PA.), each PRS could control its own receiving power independently under a common constrain:

$$I^2(R_t + \Sigma Z_{R,i}) \le P_{in,max}, \tag{3}$$

where this constrain is limited by the input power of the PA, $P_{in,max}$.

For the sake of representing the different characteristics of each PRS, the agent based simulation is used in this paper and each PRS is modeled as an independent but related agent, as shown in Fig. 3. At each control instant, the agents evaluate their preferences by their own utility function and physical model. The utility function quantifies the satisfaction level of each individual agent for a certain power distribution. Then agents communicate with each other through the environment to determine a power distribution by the GT based control. Note that the number of the agent can be easily extend to describe a more complex multiple-receiver WPT system.
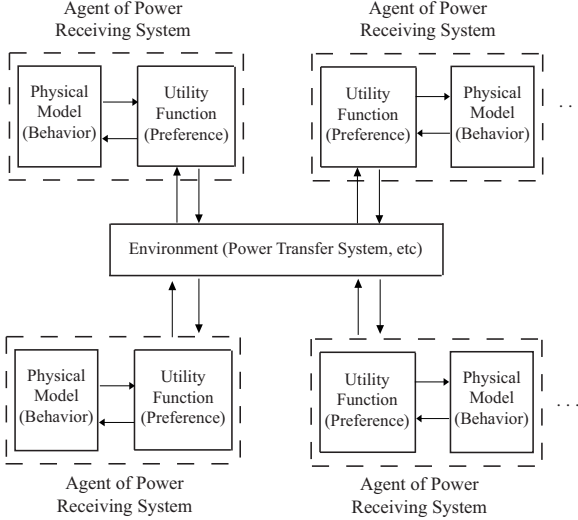

Fig. 3. The agent based simulation model of a multiple-receiver WPT system.

III. NON-COOPERATIVE POWER DISTRIBUTION GAME

A. Game Formulation

To study the interactions among PRSs, a non-cooperative power distribution game is set up at each control instant (i.e., the control instant is defined as one second for simplicity.). In this game, the players (i.e., the PRSs.) set up a GNE according to their own preferences (i.e., utility functions.). Meanwhile, since each PRS could control $P_{L,i}$ independently, the players are treated as selfish and independent. Note that all PRSs are also related because they have a common constrain, the limited input power $P_{in,max}$. Let $G = [N, P_i, U_i]$ represent the non-cooperative power distribution game with complete information at each control instant, where the total number of players is N, the strategy set, P_i, shows the strategy set for the ith player, and U_i means the utility function set for the ith player. Note that, for the ith player, the p_i can be controlled by tuning the $Z_{L,i}$ according to (2) and has a common constrain by the physical limitation, shown in (3). This constrain can be recalculated as following form:

$$I^2 R_t + \Sigma p_i \frac{R_i + Z_{L,i}}{Z_{L,i}} \leq P_{in,max}, \tag{4}$$

$$\Sigma P_i \leq \frac{P_{in,max} - I^2 R_t}{2}, \tag{5}$$

where this inequality is hold when $Z_{L,i}$ is larger than R_i.

B. Utility Functions

The utility function for each player, u_i, shows the preference for this player when the player gives its own strategy, p_i. The higher the value of the utility function one player has reach, the higher preference this player achieves. The utility function of the ith player, u_i, can be expressed as a function of power received by PRS and a state parameter p'_i, as follows,

$$u_i = 1 - (p_i - p'_i)^2, \tag{6}$$

where

$$p'_i = \frac{P_{in,max}}{N} + (1 - SOC_i)\frac{P_{in,max}(N-1)}{N}. \tag{7}$$

In (7), SOC_i means the state of charge of the UC pack in the ith PRS. In this function, a quadratic function is used as the utility function because that the utility function defined in this paper should have a peak value at $p_i = p'_i$. Thus, the closer the variable p_i to p'_i the higher the utility of the player becomes. Besides, the quadratic function is easy to calculate for a real time control. Meanwhile, the state parameter, p'_i, represents the satisfaction of each player which means that charging the UC pack at p'_i satisfies the ith player most. In (7), p'_i is defined as a combination of two parts in which the first part is a constant power (i.e., the equally divided power of the PA.) and the second part is dynamic power (i.e., expectation for the rest of the power of the PA depending on the SOC.). Note that the basic idea of designing p'_i is that the minimum preferred charging power is to equally divide the $P_{in,max}$ while the maximum preferred charging power is to be charged at the $P_{in,max}$.

C. Generalized Nash Equilibrium

The Nash equilibrium is a classical equilibrium in the GT. When this equilibrium is achieved, no player can improve its own preference by unilaterally changing its strategy. Since the utility function for a single agent contains only his own single control variable (i.e., p_i.) while each utility function shares a common constrain (i.e., (5).), the non-cooperative power distribution game is actually a generalized Nash equilibrium problem (GNEP) [24]. Then the Nash equilibrium in this problem is a GNE which could be found through finding the variational equilibrium by the KKT condition [24]. Note that, since the strategy set, P_i, is a convex set, the GNEP is a jointly convex GNEP [24]. Thus, the solution of the variational equilibrium is guaranteed to be one of the solution of the jointly convex GNEP with the utility function set (i.e., the objective function set.), U_i, to be first order continuous. Meanwhile, the solution found by the variational equilibrium is treated as the solution to be updated because it is more socially stable than any other GNE [25]. Thus, by the KKT condition, the GNEP is reformulated as a Lagrangian function

for each player with a common constrain shown as following:

$$L(p_i, \lambda_i) = u_i + \lambda G(p_i), \tag{8}$$

$$G(p_i) = \Sigma p_i + \frac{I^2 R_t - P_{in,max}}{2}, \tag{9}$$

$$0 \leq \lambda_i \perp -G(p_i) \geq 0, \tag{10}$$

where λ_i is the Lagrange multipliers for the Lagrangian function of each player. Since this problem is a convex problem and the constrain is a convex set, the KKT condition is also sufficient condition for this problem. By taking partial derivative with p_i of the Lagrangian function in (8), the variational equilibrium can be solved by (11)(12).

$$\frac{dL_i}{dp_i} = 2(p_i - p_i') + 2\lambda_i = 0, \tag{11}$$

$$\lambda_1 = \lambda_2 = ... = \lambda_N. \tag{12}$$

There are two kinds of solution for (11). The first kind of solution is that when $\lambda_i = 0$ (i.e., the equality constrain does not hold.), the solution can be reach by (13). Under this kind of situation, each player could reach his highest preference. However, this kind of solution is nearly impossible to happen because the $\Sigma p_i'$ is defined to be larger than $P_{in,max}$. If the p_i' is redefined, this kind of solution may be achieved. The second kind of solution is that when $\lambda_i > 0$ (i.e., the equality constrain holds.), the solution can be reach by (14). In this case, each player could not satisfy their preference partly at the same time. Thus they reach the solution in (14) by reducing some of their preference but reach a equilibrium solution. Since the solution formulation is quite simple, these two kinds of solutions for the GNE can be updated at each control instant.

$$p_i = p_i'. \tag{13}$$

$$p_i = \frac{P_{in,max} - I^2 R_t}{N} + \frac{Np_i' - \Sigma p_i'}{N}. \tag{14}$$

The existence of the GNE discussed in this paper can be proved by proving the existence of the variational equilibrium [24], [25]. In order to prove the existence of a variational equilibrium in a variational inequality problem (i.e., $VI(P,F)$, where P means all possible power set satisfying (9) and F is defined as (15).), the Jacobian of F (JF) is calculated as (16). It is obvious that the JF is positive definite on P, and therefore, F is strictly monotone. Thus, this GNEP has a unique global variational equilibrium solution [24].

$$F = \frac{dU_i}{dp_i} = \begin{pmatrix} 2p_1 - 2p_1' \\ 2p_2 - 2p_2' \\ \vdots \\ 2p_i - 2p_i' \end{pmatrix} \tag{15}$$

$$JF = \begin{pmatrix} 2 & 0 & \cdots & 0 \\ 0 & 2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2 \end{pmatrix}. \tag{16}$$

TABLE I
PARAMETERS FOR SIMULATION ENVIRONMENT

Agent	N	$M_{t,i}$	R_i	V_{max}	V_{ini}	C
PRS one	3	0.13e-6 H	1.04 Ω	7.5 V	1.5 V	700 F
PRS two	3	0.232e-6 H	2.05 Ω	7.5 V	2.5 V	700 F
PRS three	3	0.215e-6 H	2.05 Ω	7.5 V	5 V	700 F

IV. SIMULATION RESULTS

In the simulation, a comparing approach, the HEIDA is applied. The HEIDA tries to maximize the system efficiency at any time instant according to the calculated impedance distribution and is treated as the most popular method currently. A three-receiver WPT system is used as an example system to show the performance of the GT based control with an eleven minutes load profile. This load profile is chosen because it shows the entire charging procedure with all possible combinations of multi-receiver WPT system. Note that the proposed GT based control could be applied on any multiple-receiver WPT system with any load profile. As shown in Table. I, the information for the UC packs are assumed by the authors which could be any value. When a PRS is full (i.e., the SOC of the UC pack reaches 100%.), it will be turned off. As shown in Fig.4, the load profile contains eleven minutes with three different coil combinations (i.e., no receiving coil, two receiving coils, and three receiving coils.) which covers all possible multiple receiver WPT systems with three receivers. The blue, red, and black lines show the on/off situation of different receiving coils, respectively and for the last five minutes, it continues the same situation with the fifth minute. The last six minutes are used to charge the UC pack to 100% SOC.
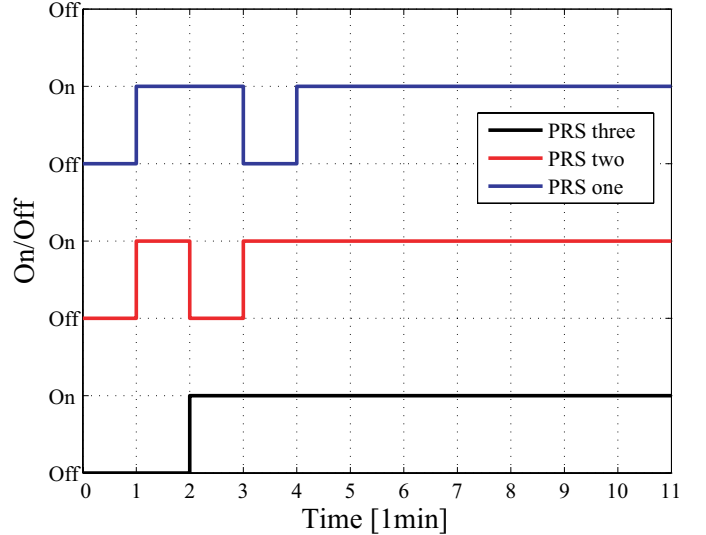

Fig. 4. The load profile.

As shown in Fig. 5, the overall power responses mainly contain three kinds of results. For the first case, all the receiving coils are under off situation which means the system is off. Meanwhile, for the second and third case (i.e., the

two and three receiving coils cases.), there are more than one receiving coils in which each coil gives their preferred charging power, p_i', and reach a balanced solution power p_i. For example, in the second minutes, the p_1' and p_2' give a high value. Due to the power limited by the PTS (i.e., $P_{in,max} = 30W$ and $I = 1.414A$.), the p_1 and p_2 are relative low. With the increasing SOC of UC pack in PRS one, the p_1' becomes lower according to (14). For the rest time instant, similar conditions can be analysed. The generalized Nash equilibrium points are shown in Fig. 6. The Nash equilibrium points in all kinds of cases will move with the different SOC according to (14).

The power responses comparing with HEIDA are shown in Fig. 7. he HEIDA results are shown in dash lines and the GT based control are shown in solid lines. Comparing the results of the GT based control and the HEIDA, the GT based control gives a higher power in two receiving coils case than HEIDA. While for three coils case, the GT based control gives a power distribution according to their preferences other than system efficiency.

The UC SOC responses of these two methods, shown in Fig. 8, show that the GT based control has a better performance. For example, the PRS one (i.e., the blue line.) gets more power and reach a higher SOC for the sake of its low initial voltage. While for the PRS three (i.e., the black line.) gets less power because it has a high initial SOC. Beside, for the GT based control, all UC packs finial reach above 95% SOC while for the HEIDA, PRS one has only 75% SOC at the end of the test. Note that the p_i is changing in real time with their SOC changing, e.g., when the SOC of one PRS is nearly full the charging power p_i' will drop automatically and vice versa.

Meanwhile, as shown in Fig. 9, the overall system efficiency (i.e., from the PA to the DC-DC converter.) for the GT based control (i.e., the blue line.) is a little bit lower than that of the HEIDA (i.e., the red line.) because the HEIDA is designed for the highest system efficiency. However, the GT based control has still reached a high system efficiency over 85%. The entire simulation results show that the GT based control gives a better power performance with a small sacrifice on the system efficiency.

V. CONCLUSIONS

In this paper, a GT based control approach is developed to determine the power distribution of multiple receiver WPT systems. This control approach emphasizes the characteristics of the load, i.e., the power requirement of each UC pack as well as the system efficiency. The PRSs are modelled as independent but related agent under the Matlab simulation environment. Moreover, the preference of each PRS is modeled by utility functions to fully show their characteristics, respectively. After that, to balance the power requirement among each PRS, a non-cooperative power distribution game is set up at every control instant and the balanced solution is reach by the GNE. Meanwhile, the GNE is calculated by finding the variational equilibrium through KKT conditions.
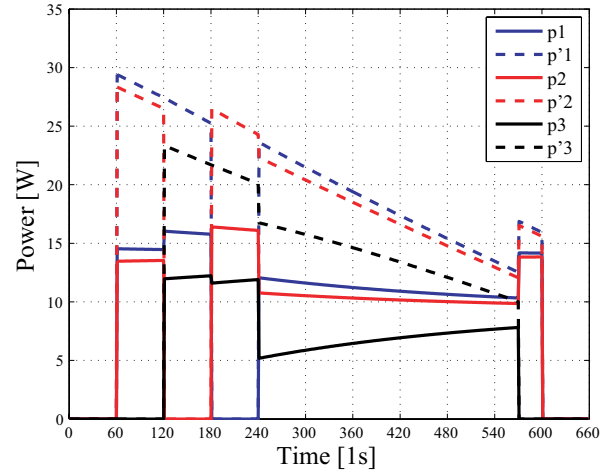

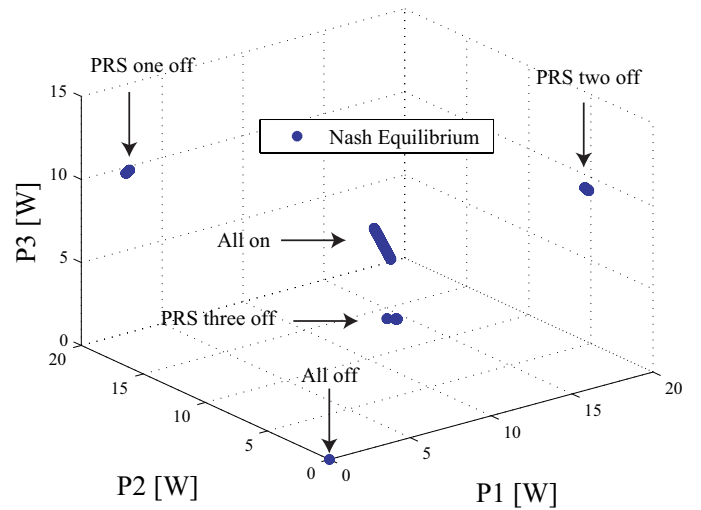Fig. 5.   The power response comparing with p_i'.

Fig. 6.   The generalized Nash equilibrium points.

The simulation results show that the GT based control has a better power performance with a small sacrifice on the system efficiency comparing with the HEIDA. In the future work, the proposed approach could be further developed to a general solution for multiple transfer to multiple receiver WPT system.

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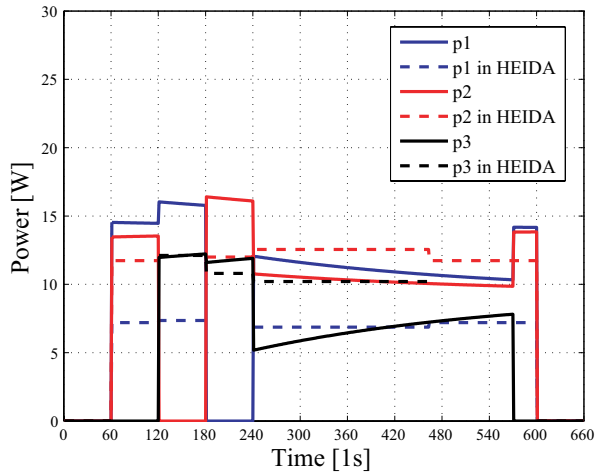
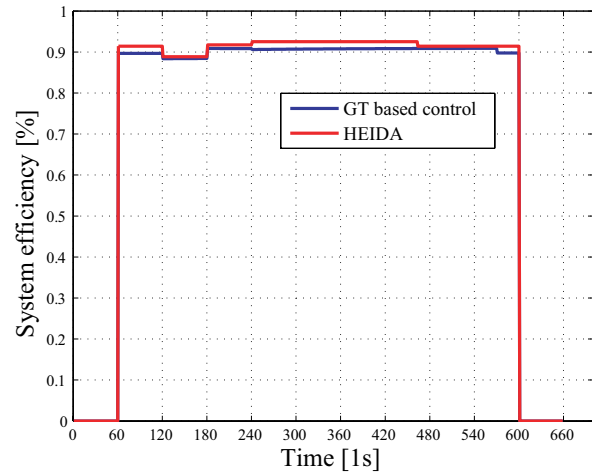Fig. 7. The power response comparing with the HEIDA.
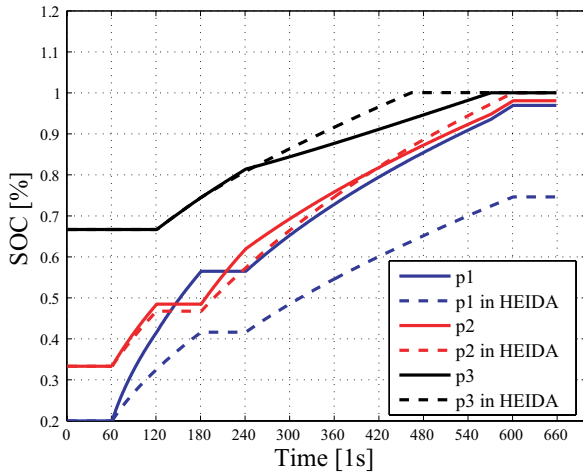

Fig. 9. The system efficiency response.

Fig. 8. The SOC response of UC packs.

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