

Efficiency Optimization and Power Distribution Design of a Megahertz Multi-receiver Wireless Power Transfer System

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Abstract—It is attractive to achieve multi-receiver wireless power transfer (WPT) through a megahertz operating frequency. However, the power distribution among receivers can be a difficult task due to the different load characteristics, i.e., power requirement, receiver size, and coupling coefficient. This paper proposes a general design methodology of efficiency optimization and power distribution of a MHz multi-receiver WPT System. A Class E rectifier is introduced in the MHz multi-receiver WPT system to provide the design freedom for the power distribution and efficiency optimization. Based on the fundamental analysis and analytical derivations of the Class E rectifier and coupling coils, a numerical optimization problem is formulated to achieve the power distribution and maximized efficiency simultaneously. Finally, an example 6.78-MHz three-receiver WPT system is built to verify the proposed design methodology. The experimental results show that the proposed design can meet the power requirement of loads within 5% error and achieve system efficiency of 90% at a power level of 25W.

Index Terms—multi-receiver, megahertz Wireless power transfer, Class E rectifier, power distribution

I. INTRODUCTION

Megahertz (MHz) wireless power transfer (WPT) is now being considered a promising candidate for the mid-range transfer of a medium amount of power [1], [2]. A higher operating frequency (such as 6.78 and 13.56 MHz) is desirable for a more compact and lighter WPT system with a longer transfer distance and make it possible to charge multiple receivers simultaneously. Lots of researches have been done on the design and optimization of one-receiver WPT systems [3]–[7], however, there are still many unsolved problems to apply these techniques for a multiple-receiver system due to the different power requirement and charging condition among loads. For the the multi-receiver WPT system, most of works focus on efficiency, resonance frequency, or cross coupling analysis [8]–[10]. However, there are few works addressing power distribution among receivers. Further research work is needed to address the most critical problem in the multi-receiver WPT system for real applications.

Initial discussions about the power distribution for a two-receiver WPT system can be found in [11], where the equivalent series resistance (ESR) of coils are ignored and the proposed model is too complicated to analyze the WPT system with more receivers. In [12] a power distribution control method is discussed by adjusting the load impedance of the receivers. However, an additional hardware is required for the

load impedance adjustment. Ref. [13] proposed an impedance matching and power division method utilizing impedance inverters at the receiver sides. Generally, the additional control circuits or the impedance inverters will increase the system complexity and power loss in the real application. In [14], [15], a frequency-shifting based method was investigated to realize the power control in a multi-receiver WPT system. Band pass filters or certain resonance circuits were used to implement frequency selection among the multiple receivers. However, the complicated power transmitter and filtering circuits of the frequency shifting method make it difficult to charging multiple receivers simultaneously. Furthermore, frequency shifting or frequency tuning is impractical in megahertz WPT applications due to the ISM band limitation.

Based on power distribution and efficiency optimization, this paper proposes a novel design method for a multi-receiver WPT system with simple topology and fixed components. Firstly, the input impedance and efficiency of the Class E rectifiers is given and serve as the basis to analyze the coupling coils. Then the efficiency of the coupling coils is derived and formulated. Then the basic principals of power distribution among receivers and the overall system efficiency is analyzed and formulated. Based on the analytical derivations, the whole system design is formulated as an optimization problem to achieve the power distribution and efficiency optimization. Finally, a 6.78-MHz WPT system with three receivers is built to validate the proposed design method. The experimental results shows that the WPT system using the proposed optimization design method can meet the power requirement of each load and achieve the system efficiency of 90%.

II. MODELING AND ANALYSIS

The entire configuration of a MHz multi-receiver WPT system is demonstrated in Fig. 1 (a), including a power transmitter TX and n receivers $RX_i (i \in [1, \dots, n])$. i is always used to denote different receivers. The TX consists of a MHz power source and a transmitting coil. The transmitting coil is fully resonant with the compensation capacitor C_{tx} to achieve an unit power factor. Each RX_i consists of a receiving coil, a current driven Class E rectifier and a DC load. The receiving coils are also compensated by a series-connected capacitor $C_{rx,i}$. In the following subsection, the efficiencies of the Class E rectifier and coupling coils are formulated

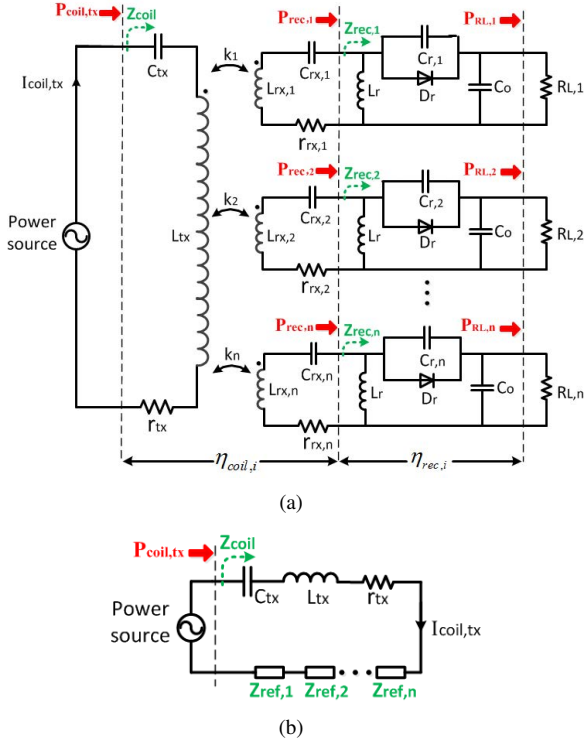


Fig. 1. Circuit configuration (a) Typical multi-receiver WPT system configuration. (b) Equivalent circuit of the TX.

analytically considering the power loss in the self-resistances of coupling coils (r_{tx} and r_{rx}), on-resistance of the rectifying diode (r_{D_r}), and ESR of the filter inductor (r_{L_r}). The overall system efficiency and power distribution principals are also analyzed and formulated to serve as the basis of the systematic parameter design.

A. Class E rectifier analysis

The circuit of the current-driven Class E rectifier in Fig. 1 (a) consists of a DC filter inductor L_r , a rectifying diode D_r , a parallel capacitor $C_{r,i}$, and a filter capacitor C_o . $C_{r,i}$ is the only design parameter in the Class E rectifier, which can be uniquely determined by the duty cycle of the diode D_i and final DC load $R_{L,i}$. The parameters of L_r , D_r and C_o are assumed to be the same among all the rectifiers. Based on the equivalent circuit analysis [16], the efficiency and input impedance of the rectifier, $\eta_{rec,i}$ and $Z_{rec,i} (=R_{rec,i} + jX_{rec,i})$ can be obtained.

$$\eta_{rec,i} = \frac{P_{RL,i}}{P_{rec,i}} = \frac{R_{L,i}}{R_{L,i} + r_{L_r} + \frac{c_i r_{D_r}}{\sin^2 \phi_{rec}}} \quad (1)$$

$$c_i = \frac{D_i}{2} + D_i \sin^2 \phi_{rec,i} - \frac{1}{\pi} \sin \phi_{rec,i} \cos(\phi_{rec,i} - 2\pi D_i) + \frac{1}{8\pi} \sin(2\phi_{rec,i} - 4\pi D_i) + \frac{3}{8\pi} \sin 2\phi_{rec,i} \quad (2)$$

$$\phi_{rec,i} = \arctan \left[\frac{1 - \cos 2\pi D_i}{\sin(2\pi D_i) + 2\pi(1 - D_i)} \right] \quad (3)$$

$$R_{rec,i} = 2(R_{L,i} + r_{L_r}) \sin^2 \phi_{rec,i} + 2c_i r_{D_r},$$

$$X_{rec,i} = -\frac{1}{\pi} \left[\frac{e_i}{\omega C_{r,i}} + r_{D_r} f_i \right] \quad (4)$$

$$e_i = \pi(1 - D_i)[1 + 2 \sin \phi_{rec,i} \sin(\phi_{rec,i} - 2\pi D_i)] + \frac{1}{4} [\sin(2\phi_{rec,i} - 4\pi D_i) - \sin(2\phi_{rec,i})] + \sin(2\pi D_i) \quad (5)$$

$$f_i = \frac{1}{2} - \frac{\cos(2\phi_{rec,i})}{4} - \frac{\cos(2\phi_{rec,i} - 4\pi D_i)}{4} - \sin \phi_{rec,i} \sin(\phi_{rec,i} - 2\pi D_i) \quad (6)$$

In the above equations $\phi_{rec,i}$ is the initial phase of the input current to the rectifier and c_i , e_i , f_i are all intermediate variables. The duty cycle of the i -th diode, D_i , can be implicitly expressed as

$$C_{r,i} = \frac{1 + \frac{[\sin(2\pi D_i) + 2\pi(1 - D_i)]^2}{1 - \cos(2\pi D_i)} - 2\pi^2(1 - D_i)^2 - \cos(2\pi D_i)}{2\pi\omega(R_{L,i} + r_{L_r} + r_{D_r})} \quad (7)$$

where r_{L_r} and r_{D_r} are the ESR of the inductor and the diode. From (1)–(6), it can be seen that the $\eta_{rec,i}$ and $Z_{rec,i}$ are uniquely determined by the DC load $R_{L,i}$ and duty cycle of the diode D_i . For the convenience of formulation, D_i is considered as a design parameter in the system design approach, then $C_{r,i}$ can be calculated by the uniquely determined D_i and the final DC load $R_{L,i}$. Therefore, the design variable of the rectifier is finalized as D_i and $R_{L,i}$ is seen as a constant parameter.

B. Coupling coils analysis

In the coupling system shown in Fig. 1 (a), L_{tx} , r_{tx} , C_{tx} , $L_{rx,i}$, $r_{rx,i}$ and $C_{rx,i}$ are the self-inductances, self-resistances and in-series compensation capacitors of the transmitting coil and receiving coils. The $L_{rx,i}$ and $r_{rx,i}$ can be different for each receiving coil due to the different size or power requirements. M_i is the mutual inductance between the transmitting coil and the i -th receiving coil. C_{tx} is well designed to fully resonant with the transmitting coil and $C_{rx,i}$ serves as the design variable to realize the power distribution among RX_i . As shown in Fig. 1 (b), the loading effect of the RX_i on the TX can be represented by the reflected impedance ($Z_{ref,i}$) connecting in series at the transmitting side. The reflected impedance $Z_{ref,i} (=R_{ref,i} + jX_{ref,i})$ can be expressed in (8) and (9). The derived input impedance of rectifier in (4) is taken into consideration for a more accurate modeling.

$$R_{ref,i} = \frac{\omega^2 M_i^2 (R_{rec,i} + r_{rx,i})}{(R_{rec,i} + r_{rx,i})^2 + \left(\omega L_{rx,i} - \frac{1}{\omega C_{rx,i}} + X_{rec,i} \right)^2} \quad (8)$$

$$X_{ref,i} = -\frac{\omega^2 M_i^2 \left(\omega L_{rx,i} - \frac{1}{\omega C_{rx,i}} + X_{rec,i} \right)}{(R_{rec,i} + r_{rx,i})^2 + \left(\omega L_{rx,i} - \frac{1}{\omega C_{rx,i}} + X_{rec,i} \right)^2} \quad (9)$$

In the above equations, M_i is the mutual inductance between the transmitting coil and i -th receiving coil, which can be represented as

$$M_i = k_i \sqrt{L_{tx} L_{rx,i}} \quad (10)$$

According to the Kirchhoff voltage law, the efficiency of the transmitting coil and i -th receiving coil, $\eta_{coil,tx}$ and $\eta_{coil,rx,i}$, can be derived as

$$\eta_{coil,tx} = \frac{\sum_{i=1}^n R_{ref,i}}{\sum_{i=1}^n R_{ref,i} + r_{tx}} \quad (11)$$

$$\eta_{coil,rx,i} = \frac{R_{rec,i}}{R_{rec,i} + r_{rx,i}} \quad (12)$$

Then efficiency of the overall system is defined as

$$\eta_{sys} = \eta_{coil,tx} \sum_{i=1}^n \eta_{coil,rx,i} \eta_{rec,i} \quad (13)$$

Note that the overall system efficiency defined in (13) doesn't consider the efficiency of the power source activating the transmitting coil.

C. Power distribution analysis

Since the loading effect of the RX_i on the TX can be represented by the reflected impedance ($Z_{ref,i}$) connecting in series at the transmitting side, the power received by each RX_i is equal to the power dissipation on the reflected impedance. Thus the resistance of the reflected impedance can be properly designed to realize the power distribution among each load. From (8) it can be seen that $C_{rx,i}$ and $Z_{rec,i}$ can be properly designed to get the desired $R_{ref,i}$. Here we define the coil efficiency from TX to RX_i as

$$\eta_{coil,i} = \frac{P_{rec,i}}{P_{coil,tx}} \quad (14)$$

where $P_{rec,i}$ is the input power of the i -th rectifier and $P_{coil,tx}$ is the input power of the transmitting coil. Then $\eta_{coil,i}$ can be derived as follows.

$$\eta_{coil,i} = \frac{R_{ref,i}}{\sum_{i=1}^n R_{ref,i} + r_{tx}} \frac{R_{rec,i}}{R_{rec,i} + r_{rx,i}} \quad (15)$$

From (12) we know that a higher $R_{rec,i}$ can achieve a higher $\eta_{coil,rx,i}$, and may lead a higher $\eta_{coil,i}$. While a higher $R_{rec,i}$ may also lead to a lower $R_{ref,i}$ and thus a lower $\eta_{coil,i}$. Therefore, there exist an optimum $R_{rec,i}$ for each receiver to achieve the highest overall system efficiency. The power of the final DC load can be derived based on the coil efficiency as

$$P_{RL,i} = P_{coil,tx} \eta_{coil,i} \eta_{rec,i} \quad (16)$$

In combination with section II-A, it's obvious that D_i can serve as the design parameter in the rectifier to achieve the desired $\eta_{rec,i}$ and $R_{rec,i}$, while $C_{rx,i}$ serves as the design variables to achieve the desired $R_{ref,i}$. Then the well designed

$\eta_{rec,i}$, $R_{rec,i}$ and $R_{ref,i}$ can achieve the desired power distribution and optimum overall system efficiency. Therefore, in the multi-receiver WPT system, the design parameters are finalized as D_i and $C_{rx,i}$.

III. OPTIMIZATION PROBLEM

Based on the aforementioned analysis and derivations of the multi-receiver WPT system, a systematic parameter design approach is developed below that maximizes the system efficiency while the power requirement of each load is met. The systematic parameter design can be formulate as an optimization problem that aims to achieve the maximum system efficiency while the power requirements of each load serve as the constrains. Firstly, the constant parameters \mathbf{P} and the design variables \mathbf{X} are defined in vector as

$$\mathbf{P} = (\omega, L_{tx}, r_{tx}, k_i, L_{rx,i}, r_{rx,i}, r_{L_r}, r_{D_r}, R_{L,i})$$

$$\mathbf{X} = (C_{rx,i}, D_i) \quad (17)$$

Here ω is the operating frequency of the MHz WPT system, which is usually 6.78MHz or 13.56MHz. L_{tx} , r_{tx} and $L_{rx,i}$, $r_{rx,i}$ denote the inductance and ESR of the transmitting coil and receiving coils. k_i is the coupling coefficient between the transmitting coil and i -th receiving coil. r_{L_r} and r_{D_r} are the ESR of the dc filter inductor and the on-resistance of the rectifying diode. $R_{L,i}$ is the equivalent final DC load of the i -th rectifier. The feasible range of the design variable \mathbf{X} can be defined as

$$\mathbf{X} \in (\mathbf{X}^{lower}, \mathbf{X}^{upper}) \quad (18)$$

where \mathbf{X}^{lower} and \mathbf{X}^{upper} are the lower and upper bounds of the design parameters \mathbf{X} , respectively. The feasible range of \mathbf{X} can be finalized according to the power level of the system as well as the commercially availability of the devices. The objective function $f(\mathbf{X}, \mathbf{P})$ is selected as the system efficiency defined in (13).

$$f(\mathbf{X}, \mathbf{P}) = \eta_{sys} \quad (19)$$

The load power derived in (16) have to meet the target power requirements $P_{RL,i}^{target}$, which serve as the constraints of this optimization problem. Note that the input power of the transmitting coil, i.e., $P_{coil,tx}$, doesn't act as a design parameter in this optimization problem, because when this system is designed to achieve its maximum efficiency and the required power output, the input power of the system, namely, $P_{coil,tx}$, is uniquely determined. The constraints of the optimization problem is given as follows.

$$P_{RL,i} = P_{RL,i}^{target}, i \in (1, \dots, n) \quad (20)$$

Based on the aforementioned analytical derivations, the optimization problem is formulated to achieve the maximum overall system efficiency while the power requirements of each load serve as the constraints.

$$\max_{\mathbf{X}} f(\mathbf{X}, \mathbf{P}) = \eta_{sys}, \mathbf{X} \in (\mathbf{X}^{lower}, \mathbf{X}^{upper})$$

$$s.t. P_{RL,i} = P_{RL,i}^{target}, i \in (1, \dots, n) \quad (21)$$

Given the nature of this optimization problem, a popular population-based heuristic approach, Genetic algorithm (GA), can be used to address the global or at least near to global optimal solution. When solved as an optimization problem, the equality constraints is difficult to deal with and it is better to define minimum allowable error of the load power and transfer the equality constraints in (21) into inequality constraints.

IV. EXPERIMENTAL VERIFICATION

As shown in Fig. 2, an example 6.78-MHz WPT system with three receivers is built up to verify the above analytical derivations and parameter design based on power distribution and efficiency optimization. This experimental system shares the same circuit configuration with the one in Fig. 1 (a). Note that a 6.78-MHz current output power amplifier is used as the power source and the electronic loads serve as the final dc loads in this system. All the three receiving coils used in this system are the same. The vertical distances between the transmitting coil and receiving coils are 2cm and the coupling coefficients are about 0.1. All the constant parameters and the target load power are given in Table I.

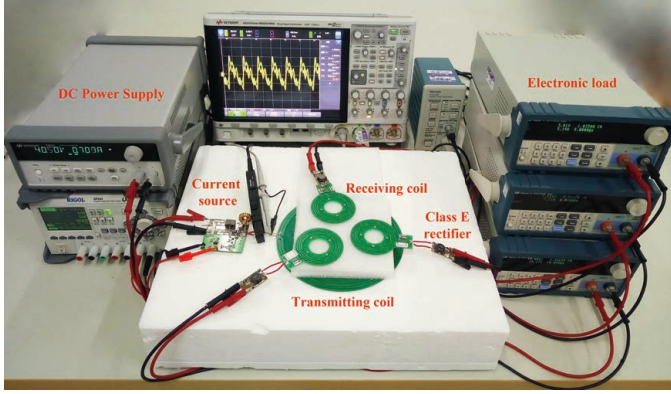


Fig. 2. The experimental multi-receiver WPT system

TABLE I
PARAMETER WITH CONSTANT VALUE

ω	L_{tx}	r_{tx}	$L_{rx,i}$	$r_{rx,i}$
6.78 MHz	4.23 μ H	0.95 Ω	1.59 μ H	0.45 Ω
r_{Lr}	k	r_{Dr}	$R_{L,1}$	$R_{L,2}$
0.2 Ω	0.1	0.2 Ω	5 Ω	8 Ω
$R_{L,3}$	$P_{RL,1}^{target}$	$P_{RL,2}^{target}$	$P_{RL,3}^{target}$	—
10 Ω	5 W	8 W	10 W	—

The feasible range of the design variables \mathbf{X} is finalized as

$$\mathbf{X}^{lower} = [100pF, 100pF, 100pF, 0.3, 0.3, 0.3]$$

$$\mathbf{X}^{upper} = [2000pF, 2000pF, 2000pF, 0.7, 0.7, 0.7] \quad (22)$$

The feasible range of the duty cycle of the diode D_i is determine considering the power output capacity of the rectifier as well as the voltage stress of the diode. Based on the proposed

optimization given in section III, the optimal results \mathbf{X}^* are calculated as follows.

$$[C_{rx,1}, C_{rx,2}, C_{rx,3}, D_1, D_2, D_3] = [424pF, 408pF, 389pF, 0.70, 0.63, 0.56] \quad (23)$$

Based on the optimization results, the shunt capacitors of the rectifying diode, $C_{r,i}$, can be calculated according to (7).

$$[C_{r,1}, C_{r,2}, C_{r,3}] = [443pF, 310pF, 705pF] \quad (24)$$

Fig. 3 shows the waveforms of the voltage across the diodes $V_{Dr,i}$ in the Class E rectifiers. The peak voltage across the diodes can be calculated to guide the selection of the diode during the system design. It can be seen that the peak value of the $V_{Dr,1}$, $V_{Dr,2}$ and $V_{Dr,3}$ are 24V, 39V and 45V, respectively. Schottky diode DFLS260 is used as the rectifying diode in the Class E rectifier. Actually, three different diode can be used in the three rectifiers to maximize the system efficiency and cost performance. The Class E rectifiers is well designed to achieve zero voltage switching (ZVS) and zero voltage derivative switching (ZVDS) in order to diminishing the switching losses, which can be well verified by the diode voltage waveforms $V_{Dr,i}$.

Fig. 4 shows the variation of load power $P_{RL,i}$ and overall system efficiency η_{sys} with respect to the input power of the transmitting coil $P_{coil,tx}$. It can be seen that as $P_{coil,tx}$ changes from 20W to 30W, the load power is proportional to growth, which is consistent with the theoretical analysis in (16). When the $P_{coil,tx}$ is between 25.25W and 26.84W, the errors of the load power are within 5%, which well verify the calculated results of GA (26.24W). When the $P_{coil,tx}$ changes from 25.25W to 26.84W, η_{sys} changes within 89.5% and 91.1%, which is quite close to the calculated result of GA (91.6%). Note that this system efficiency doesn't take the efficiency of power amplifier into account.

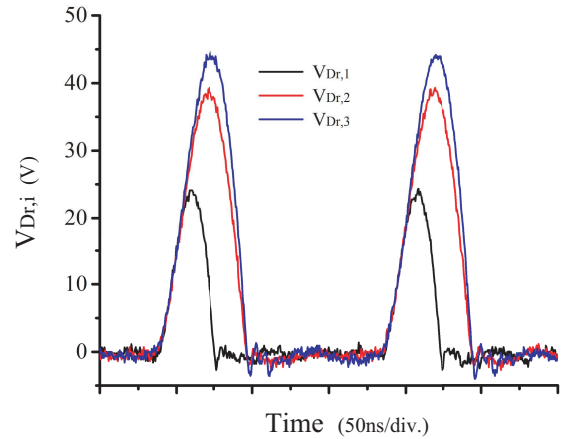


Fig. 3. Voltage waveforms across the diodes

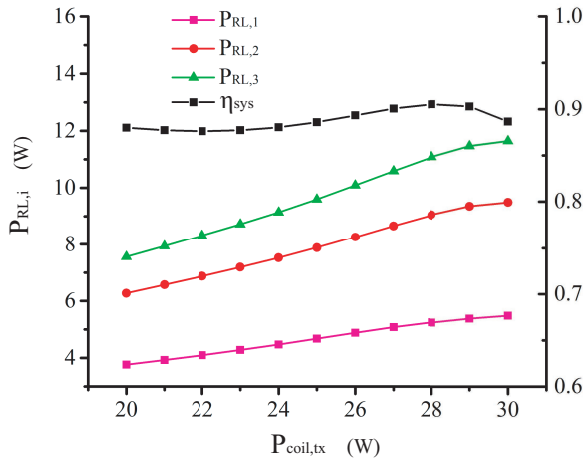


Fig. 4. Load power and overall system efficiency of the WPT system

V. CONCLUSIONS

In this paper, system efficiency and power distribution among loads are analytical derived and serve as the basis of the system design. Then an optimization problem is formulated to achieve the maximum system efficiency with constraints on power distribution of the system. Finally, an example 6.78-MHz three-receiver WPT system is built to verify the proposed analytical derivation and design methodology. The experimental results show that the proposed design can meet the power requirement of loads within 5% error and achieve system efficiency over 89.5% at a power level of 25W.

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