Design Methodology of The Power Receiver with High Efficiency and Constant Output Voltage for Megahertz Wireless Power Transfer

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***Abstract*—Megahertz (MHz) wireless power transfer (WPT) has been widely studied due to its lighter and more compact system and higher spatial freedom. This paper proposes a design methodology of the power receiver with high efficiency and constant output voltage in MHz WPT systems. The power receiver consists of four parts, the receiving coil, the Class E rectifier, the buck converter and the DC load. Firstly, the inductance, equivalent series resistance (ESR) and the coupling coefficients are formulated based on the physical model of the coupling coils. The Class E rectifier and the buck converter are also derived and analyzed. Secondly, the receiving coil and the Class E rectifier are designed and optimized simultaneously to maximize the efficiency while the buck converter is designed to works in a self-regulation mode to provide the constant output voltage. The system parameters design is formulated as an optimization problem and solved using the Genetic Algorithm (GA). Finally, simulation tool Advanced Design System (ADS) is used to verify the proposed design methodology.**

***Index Terms*—wireless power transfer, constant output voltage, Class E rectifier, buck converter, coil optimization**

I. Introduction

Megahertz (MHz) wireless power transfer (WPT) is now being considered as a promising candidate for the mid-range transfer of a medium amount of power [1], [2]. A higher operating frequency (such as 6.78 and 13.56 MHz) is desirable for a more compact and lighter WPT system with a longer transfer distance and makes it possible to charge multiple receivers simultaneously. Lots of researches have been done on the design and optimization of MHz WPT from a system-level or component-level, and great progresses have been made recently, such as the power amplifier design [3], [4], the coupling coils optimization [5]–[9], and control strategy of the WPT system [10], [11].

The power receivers are usually mounted in the user end, i.e., portable consumer electronic devices such as cell phones or laptops, while the power transmitters are usually installed in a fixed place with less restrictions on the space, weight and power. Therefore, the design and optimization of the power receivers are more challenging than that of the power transmitters. For example, the heating problem usually occurs when the power conversion efficiency of the power receivers is low, which seriously affects the user experience. Therefore, the efficiency of the power receivers should be as high as possible to minimize the power consumption at the user end. At present, there is no work that optimizes the WPT system from a sub-system level, i.e., the power transmitters or power receivers. For a predetermined power transmitter or the magnetic field, the power receivers, consisting of the receiving coil, rectifier, DC-DC converter and a load, can be seen as an sub-system of the WPT system and can be designed and optimized as an integral part.

This paper, for the first time, proposes a design and optimization methodology of the power receiver with high efficiency and constant output voltage in the MHz WPT systems. The power receiver consists of a receiving coil, a Class E rectifier, a buck converter and a DC load. Firstly, the physical model of the coils is proposed and the inductance, equivalent series resistance (ESR) and the coupling coefficients are formulated based on this physical model. The Class E rectifier and the buck converter are also derived and formulated. In this system, the buck converter works in self-regulation mode to provide a constant output voltage. Based on above analysis and derivations, a numerical optimization problem is formulated to achieve the high efficiency and constant output voltage simultaneously. Genetic algorithm (GA) toolbox in Matlab is used to solve this optimization problem. Finally the design and optimization methodology is verified by the radio frequency simulation tool Advanced Design System (ADS).

II. Modeling and Analysis

The configuration of a MHz WPT system is presented in Fig. 1, including a power transmitter TX and a power receiver RX. The TX consists of a MHz power source and a transmitting coil. The transmitting coil is resonant with the compensation capacitor C_{tx} to achieve an unit power factor. The RX consists of a receiving coil, a current driven Class E rectifier, a buck converter and a DC load. The receiving coil is compensated by a series-connected capacitor C_{rx}. The buck converter here is used to regulate the output voltage of the rectifier to realize a constant output voltage. In the following subsections, the design parameters of the power receiver are defined and efficiencies of the receiving coil and the rectifier are formulated considering the parasitic parameters, including the ESR of coupling coils (r_{tx} and r_{rx}), on-resistance of the rectifying diode (r_{D_r}), and ESR of the filter inductor (r_{L_r}).

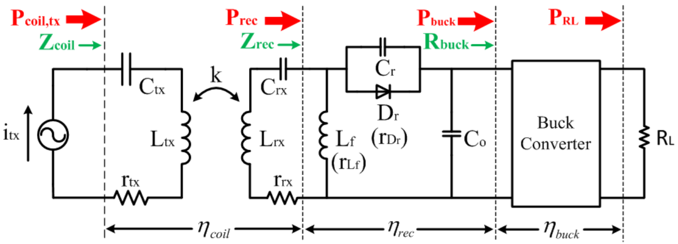

Fig. 1. Configuration of the MHz WPT system.

A. Coupling Coils

The coupling coils are modeled according to the layout of the spiral coil on printed circuit board. As shown in Fig. 2, the outer and inner diameters of the coil, d_o and d_i, the width of each trace w, and the distance between two adjacent traces s are specified as the design parameters of the coils. The thickness of the trace, denoted by t_c, is fixed as 1 oz, i.e., 35 μm. The inductance, ESR as well as the coupling coefficients of the coupling coils can be formulated and serve as the basis of the system optimization.

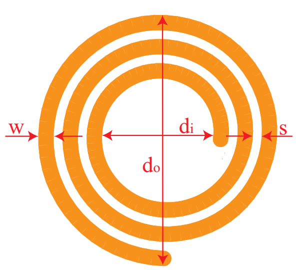

Fig. 2. Design parameters of the coils.

The self inductance of the printed spiral coil can be written as

$$L = \frac{\mu N^2 (d_o + d_i) c_1}{4} \left(\ln \left({}^{c_2}\!/_{\rho} \right) + c_3 \rho^2 \right), \tag{1}$$

where ρ can be written as

$$\rho = \frac{d_o - d_i}{d_o + d_i}. \tag{2}$$

In the above equations μ is the permeability of the copper, N is the number of turns of the coil. c_1, c_2 and c_3 are all constant parameters related to the layout of the coils and their values are listed in Table I.

TABLE I
CONSTANT PARAMETERS FOR INDUCTANCE EXPRESSION

c_1	c_2	c_3
1.00	2.46	0.20

Before determining the ESR of the spiral coil, the length of the trace of the spiral coil needed to be calculated. The planar equidistant helix model can be used to calculate the length of the trace accurately. Here a simplified equation is given to do the calculation.

$$l_c \approx \sum_{n=1}^{N} \pi \left(d_o - (w + s)(n - 1) \right). \tag{3}$$

The the DC ESR of the printed spiral coil can be written as follows.

$$r_{dc} = l_c \frac{\rho_c}{w t_c}. \tag{4}$$

where ρ_c is the electrical resistivity of copper. Since the coupling coils work in MHz frequency, the skin effect is quite obvious, thus the AC ESR of the print spiral coil needed to be determined, as given as follows.

$$r_{ac} = r_{dc} \frac{t_c}{\delta \left(1 - e^{-t_c/\delta} \right)}. \tag{5}$$

In this equation δ is the skin depth, which can be represented by the resistivity ρ_c, the permeability μ, and the frequency f, as shown below.

$$\delta = \sqrt{\frac{\rho_c}{\pi \mu f}}. \tag{6}$$

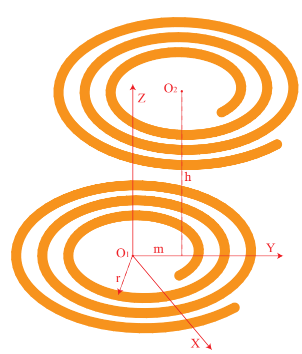

Fig. 3. Relative position of two coils.

As shown in Fig 3, m and h are used to denote the horizontal misalignment and vertical distance between two coils and r is used to denote the approximate radius of a certain turn of trace. When calculating the mutual inductance of the coupling coils, each coil can be seen as a set of concentric single turn coils with shrinking radius, connected in series. Thus, once the mutual inductance between a pair of single-turn coils is determined, the overall mutual inductance can be calculated by summing the partial mutual inductance values of every turn of one coil and all the turns of the other coil.

According to the Maxwell equations, the mutual inductance between a pair of single turn coils, M_{ij}, as well as the overall mutual inductance of two spiral printed coils, M, can be represented as follows.

$$\begin{aligned} M_{ij} &= \mu\pi\sqrt{r_i r_j}\int_0^{\infty} J_1\left(x\sqrt{\frac{R_i}{R_j}}\right)J_1\left(x\sqrt{\frac{R_j}{R_i}}\right) \\ &\times J_0\left(x\frac{m}{\sqrt{r_i r_j}}\right)\exp\left(-x\frac{h}{\sqrt{r_i r_j}}\right)dx, \end{aligned} \tag{7}$$

$$M = \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} M_{ij}. \tag{8}$$

In the above equations, r_i and r_j are used to denote the approximate radius of each turn of the two printed spiral coils. J_0 and J_1 are the Bessel functions of the zeroth and first order, respectively. The coupling coefficient of the coupling coils can be represented by the self-inductance of the coils and the mutual inductance between them.

$$K = \frac{M}{\sqrt{L_{tx}L_{rx}}}. \tag{9}$$

B. Class E rectifier

The Class E rectifier working in zero-voltage-switching (ZVS) and zero-voltage-derivative-switching (ZVDS) mode is suitable for high frequency rectification. The circuit of the current-driven Class E rectifier in Fig. 1 consists of a radio frequency choke L_f, a rectifying diode D_r, a parallel capacitor C_r, and a filter capacitor C_o. C_r is the only design parameter in the Class E rectifier. The choke L_f and filter capacitor C_o should be large enough to facilitate a DC output current and low ripple output voltage. Based on the equivalent circuit analysis [12], the efficiency and input impedance of the rectifier, η_{rec} and Z_{rec} (=$R_{rec} + jX_{rec}$) can be obtained.

$$\eta_{rec} = \frac{P_{buck}}{P_{rec}} = \frac{R_{buck}}{R_{buck} + r_{L_r} + \frac{cr_{D_r}}{\sin^2\phi_{rec}}}, \tag{10}$$

$$\begin{aligned} c &= \frac{D}{2} + D\sin^2\phi_{rec} - \frac{1}{\pi}\sin\phi_{rec}\cos(\phi_{rec} - 2\pi D) \\ &+ \frac{1}{8\pi}sin(2\phi_{rec} - 4\pi D) + \frac{3}{8\pi}sin2\phi_{rec}, \end{aligned} \tag{11}$$

$$\phi_{rec} = \arctan\left[\frac{1-\cos 2\pi D}{\sin(2\pi D) + 2\pi(1-D)}\right], \tag{12}$$

$$R_{rec} = 2(R_{buck} + r_{L_r})\sin^2\phi_{rec} + 2cr_{D_r}, \tag{13}$$

$$X_{rec} = -\frac{1}{\pi}\left[\frac{e}{\omega C_r} + r_{D_r}f\right], \tag{14}$$

$$\begin{aligned} e &= \pi(1-D)[1 + 2\sin\phi_{rec}\sin(\phi_{rec} - 2\pi D)] \\ &+ \frac{1}{4}[\sin(2\phi_{rec} - 4\pi D) - \sin(2\phi_{rec})] \\ &+ \sin(2\pi D), \end{aligned} \tag{15}$$

$$\begin{aligned} f &= \frac{1}{2} - \frac{\cos(2\phi_{rec})}{4} - \frac{\cos(2\phi_{rec} - 4\pi D)}{4} \\ &- \sin\phi_{rec}\sin(\phi_{rec} - 2\pi D), \end{aligned} \tag{16}$$

In the above equations ϕ_{rec} is the initial phase of the input current to the rectifier and c, e, f are all intermediate variables. The input resistance of the buck converter, R_{buck}, can be seen as the load of the rectifier. The duty cycle of the diode, D, can be implicitly expressed as

$$C_r = \frac{1 + \frac{[\sin(2\pi D) + 2\pi(1-D)]^2}{1-\cos(2\pi D)} - 2\pi^2(1-D)^2 - \cos(2\pi D)}{2\pi\omega(R_{buck} + r_{L_r} + r_{D_r})}, \tag{17}$$

where r_{L_r} and r_{D_r} are the ESR and on-resistance of the filter inductor and the diode. From (10)–(16), it can be seen that the η_{rec} and Z_{rec} are uniquely determined by the load R_{buck} and duty cycle of the diode D. Since the duty cycle D can be determined by the shunt capacitor C_r and the load R_{buck}, C_r is chosen as the only design parameter in the Class E rectifier. The efficiency of the receiving coil can be represented by the R_{rec} and the ESR of the receiving coil r_{rx}.

$$\eta_{coil,rx} = \frac{R_{rec}}{R_{rec} + r_{rx}}, \tag{18}$$

C. Buck Converter

The circuit of the buck converter is omitted in the Fig. 1 for simplicity. Since the coupling coefficients may vary according to the relative positions of the coupling coils, the buck converter is assumed to work in the self-regulation mode to facilitate a constant output voltage. This function can be fulfilled by many commercialized integrated DC-DC buck converter. In this paper, the efficiency derivation of the buck converter is omitted, and the duty cycle is derived and serves as a constraint during the design optimization. Since the duty cycle and the efficiency of the buck converter is inverse correlated, the efficiency of the buck converter is considered indirectly. The input resistance of the buck converter, R_{buck}, can be represented as

$$R_{buck} = \frac{R_L}{{D_{buck}}^2}. \tag{19}$$

Here D_{buck} is the duty cycle of the buck converter. Assuming the output voltage of the buck converter is constant, the duty cycle D_{buck} can be represented by the parameters of the coupling coils and the rectifier. The derivation results are given as follows.

$$D_{buck} = \frac{V_{R_L}}{V_{buck}} = \frac{I_{buck}R_L}{V_{R_L}}, \tag{20}$$

$$I_{buck} = \frac{\sin\phi_{rec}\omega M I_{tx}}{\sqrt{(R_{rec} + r_{rx})^2 + \left(X_{rec} + \frac{1}{j\omega C_{rx}} + j\omega L_{rx}\right)^2}}, \tag{21}$$

where V_{buck} and V_{R_L} are the input and constant output voltage of the buck converter and I_{buck} is the input current of the buck converter. I_{tx} is the amplitude of i_{tx}, the current driving the transmitting coil. From the above equations it can be seen that D_{buck} is proportional to the mutual inductance of the coupling coils. As M changes with the coupling coil misalignments, the variation of D_{buck} can be determined accordingly.

III. Optimization problem

Based on the aforementioned analysis and derivations, a systematic parameter design approach is developed that maximizes the efficiency of the power receiver for a predetermined power transmitter. This parameter design first needed to be formulated as an optimization problem by defining the design parameters, constant parameters, uncertain parameters, objective function and the constraints. Then several optimization algorithms such as GA or particle swarm optimization (PSO) can be applied to find the optimum or near-to-optimum solution of the optimization problem.

A. Parameters Definition

The constant parameters $\bar{\boldsymbol{P}}$, uncertain parameters $\tilde{\boldsymbol{P}}$ and the design parameters $\boldsymbol{X}$ are redefined in vectors as

$$\begin{aligned}\bar{P} \;=\; &(I_{tx}, t_c, w_{tx}, s_{tx}, d_{o,tx}, N_{tx},\\ &d_{o,rx}, f, h, r_{L_f}, r_{D_r}, R_L, V_{R_L}),\end{aligned} \tag{22}$$

$$\tilde{\boldsymbol{P}} = (m)\,, \tag{23}$$

$$\boldsymbol{X} = (w_{rx}, s_{rx}, N_{rx}, C_{rx}, C_r)\,. \tag{24}$$

Here the subscript tx and rx of w_{tx}, s_{tx}, $d_{o,tx}$, N_{tx} and w_{rx}, s_{rx}, $d_{o,rx}$, N_{rx} are used to differentiate the parameters of the transmitting coil and the receiving coil. The feasible range of the design variable $\boldsymbol{X}$ can be defined as

$$\boldsymbol{X} \in (\boldsymbol{X}^{lower}, \boldsymbol{X}^{upper}), \tag{25}$$

where $\boldsymbol{X}^{lower}$ and $\boldsymbol{X}^{upper}$ are the lower and upper bounds of the design parameters $\boldsymbol{X}$, respectively. It can be seen that all the parameters of the transmitting coil, I_{tx}, w_{tx}, s_{tx}, $d_{o,tx}$, t_c, N_{tx}, are all predetermined as constant parameters. Note that $d_{i,tx}$ can be calculated according to w_{tx}, s_{tx}, $d_{o,tx}$ and N_{tx}. These transmitting coil parameters can be determined according to the specific application such as the charging area or the charging power. The parameters of the receiving coil are partially predetermined, including $d_{o,rx}$ and t_c. Note that the thickness of the trace of transmitting coil and receiving coil are the same and denoted by t_c. The vertical distance between the two coil is fixed as h while the horizontal misalignment, m, serves as the uncertain parameter within a specific range. Therefore, this MHz WPT system charges the power receiver at a specific distance and can be tolerant to some misalignments, which is similar to real applications. The outer diameter of the receiving coil is fixed as $d_{o,rx}$ while the number of turns N_{rx}, the width of each trace w_{rx}, and the distance between two adjacent traces s_{rx}, serve as the design parameters of the receiving coil.

B. Objective Function and Constraints Formulation

The objective function is defined as the power transfer efficiency from the receiving coil to the Class E rectifier when the coupling coils misalignment is zero, as shown in the following.

$$\begin{aligned}&f(X,\bar{P},\tilde{P}) = \eta_{rx}\eta_{rec}|_{m=0}\\ &= \frac{R_{rec}R_{buck}}{(R_{rec}+r_{rx})\left(R_{buck}+r_{L_r}+\frac{cr_{D_r}}{\sin^2\phi_{rec}}\right)}|_m = 0.\end{aligned} \tag{26}$$

Here the duty cycle of the buck converter serves as the constraint in the optimization problem to guarantee that the buck converter works efficiently.

$$D_{buck}|_{M_{\max}} \geq 80\%. \tag{27}$$

This constraint means that when the mutual inductance of the coupling coils achieve the maximum value, the duty cycle of the buck converter reaches a minimum value, and the minimum value of D_{buck} is greater than or equal to 80%. After defining the parameters, objective function and the constraints, Matlab GA toolbox can be used to solve the problem automatically. The algorithm flow chart of the optimization problem is shown in Fig. 4.

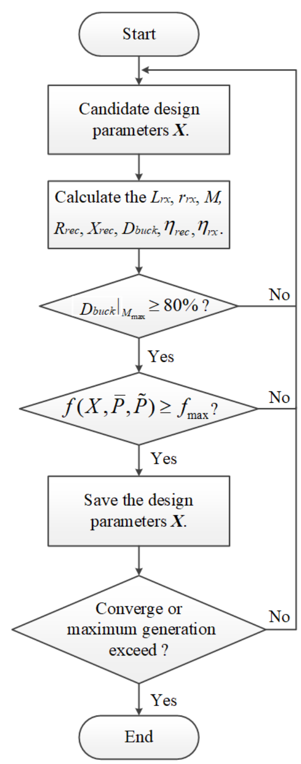

Fig. 4. Algorithm flow chart of the optimization problem.

IV. Design Case

The parameters of the MHz WPT system have been defined and the parameters design has been formulated into an optimization problem aims to achieve optimum or near-to-optimum efficiency of the power receivers. The buck converter is used to realize constant output voltage and the system is tolerant to some misalignments of the coupling coils. In this section, a real optimization design problem is formulated and solved in Matlab and the results is validated by the ADS.

A. Solving the Optimization Problem in Matlab

The values of the constant parameters are listed in Table II. The range of the uncertain parameter m, is specified from 0 cm to 7 cm, as shown below.

$$m \in (0\ cm, 7\ cm)\,. \tag{28}$$

TABLE II
CONSTANT PARAMETER

I_{tx}	$d_{o,tx}$	w_{tx}	s_{tx}	t_c	N_{tx}	$d_{o,rx}$
0.93 A	200 mm	3 mm	1 mm	1 oz	3	72 mm
h	f	r_{L_f}	r_{D_r}	R_L	V_{R_L}	–
10 mm	6.78 MHz	0.2 Ω	0.2 Ω	5 Ω	5 V	–

The lower and upper bounds of the design parameters are specified as follows.

$$\boldsymbol{X}^{lower} = [0.25mm,\ 0.25mm,\ 2,\ 100pF,\ 100pF],$$

$$\boldsymbol{X}^{upper} = [4mm,\ 4mm,\ 5,\ 2000pF,\ 2000pF]. \tag{29}$$

After defining the parameters and formulating the objective function and constraints in Matlab, GA toolbox is used to solve the problem. The calculation results of design parameters and the parameters of the coupling coils are given in Table III and Table IV.

TABLE III
DESIGN PARAMETERS

w_{rx}	s_{rx}	N_{rx}	C_{rx}	C_r
2.1 mm	0.4 mm	4	328 pF	54 pF

TABLE IV
PARAMETERS OF COUPLING COILS

L_{tx}	L_{rx}	r_{tx}	r_{rx}
4.23 μH	1.77 μH	0.95 Ω	0.38 Ω

The calculation results of coupling coefficients versus the misalignments is given in Fig. 5. The coupling coefficient K achieves the minimum value at 0cm misalignment and maximum value at 7cm misalignment, which is 0.092 and 0.108 respectively. Since the transmitting coil is much larger than the receiving coil, the coupling coefficient usually achieves the maximum value when the two coils are exterior contact. Since the variation range of the duty cycle of the buck converter serves as the constraint, the optimization problem converge to the coil parameters with relatively small variation of coupling coefficients versus misalignments.

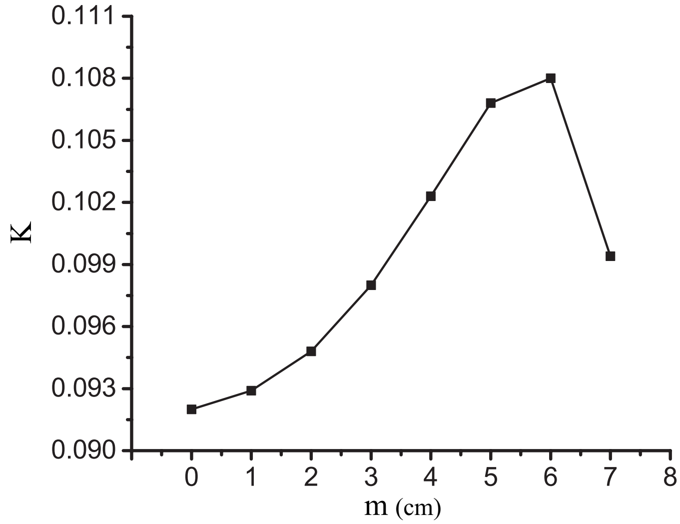

Fig. 5. Coupling coefficients versus coupling coil misalignments.

B. Verifying the Parameters design in ADS

The system parameters including the constant parameters and the calculation results, are substituted into the ADS model for simulation, and the simulation results are demonstrated as follows.

Fig. 6 shows the input voltage and duty cycle of the buck converter versus coupling coil misalignments. As m changes from 0 cm to 7 cm, D_{buck} changes within 0.81 and 0.97, which well verifies the constraint on D_{buck} during the optimization design (refer to (27)). V_{buck} increases as m increases from 0 cm to 6 cm, and begins to decrease as m increases from 6 cm to 7 cm. This tendency is the same as the change of coupling coefficient K versus m (as shown in Fig. 5). The multiplication of V_{buck} and D_{buck} is approximately equal to 5 V, which verifies the design of the buck converter. Since in ADS model the buck converter can not realize constant voltage output through self-regulation, the duty cycle of the buck converter is adjusted manually.

Fig. 7 shows the efficiency of the transmitting coil, receiving coil and rectifier versus coupling coil misalignments m. As m increases from 0 cm to 6 cm, the input voltage of the buck converter increases (refer to Fig. 6), so D_{buck} decreases accordingly to sustain constant output voltage. Then the input resistance of the buck converter, i.e., the load of the rectifier increases(refer to (19)), which means a higher efficiency of the rectifier and the receiving coil (refer to (10) (13) and (18)), as shown in Fig. 7. Therefore, this simulation results well verify the modeling. The transmitting coil can sustain a high efficiency (almost 94%) as K changes and the variation of the η_{tx} is not obvious due to the simulation resolution.

V. CONCLUSIONS

This paper proposes a parameter design methodology of the power receiver for a predetermined power transmitter in the MHz WPT system. Firstly, the printed spiral coil is modeled and the inductance, ESR, and the coupling coefficient are

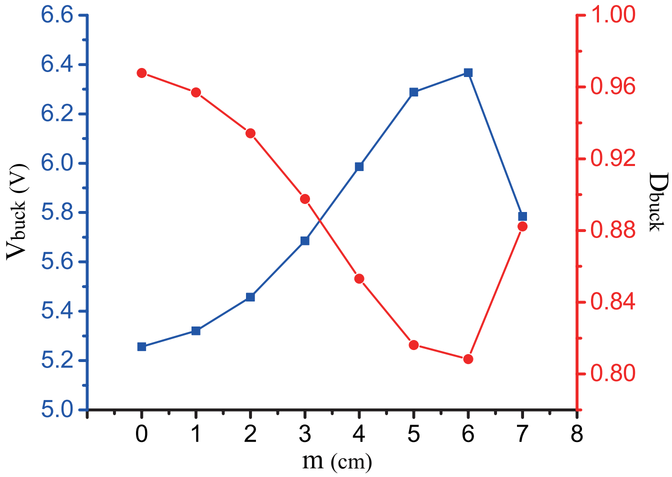

Fig. 6. Input voltage and duty cycle of buck converter versus coupling coil misalignments.

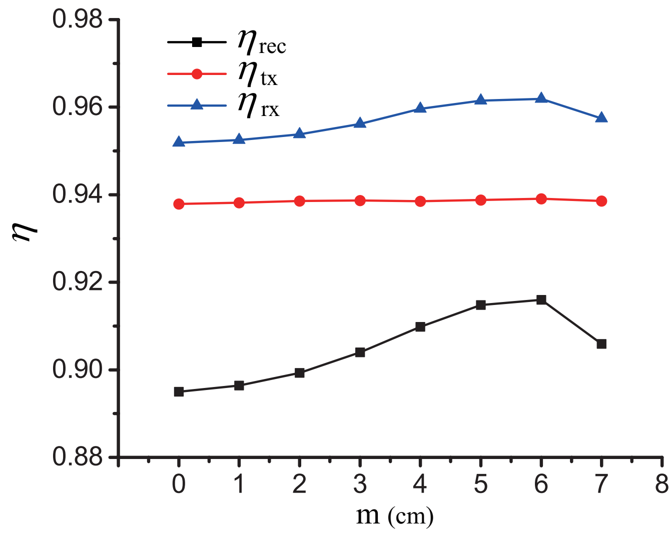

Fig. 7. Efficiency of the coupling coils and rectifier versus coupling coil misalignments.

formulated. Class E rectifier and the buck converter are also derived and serve as the basis of system design. Then the parameters design is formulated as an optimization problem to realize high efficiency and constant output voltage against the variation of coupling coil misalignments. The optimization problem is formulated in Matlab and solved using the GA toolbox. Finally, radio frequency simulation tool ADS is used to verify the proposed parameters design methodology of the power receivers.

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