A 6.78-MHz Class E^2 Converter with the Flexible DC-DC Voltage Ratio

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***Abstract*—High frequency DC-DC converters working at megahertz (MHz) are promising candidates for making compact, highly efficient converters with galvanic isolation. However, it is challenging when requiring the system design of different final loads and voltage ratios. In this paper, a design method is proposed to adjust the voltage ratio of a Class E^2 DC-DC converter by only changing the capacitors. the experimental 6.78 MHz Class E^2 converter system works with the overall efficiency of 80.1%, 83.1% and 83.8% and the DC-DC voltage ratio ranging from 2 to 5. The inductors of the system remain unchanged under different voltage ratios. In addition, a expansion approach is proposed to further increase the system voltage ratio when the input resistance of the coupling coils is already high to the PA.**

***Index Terms*—DC-DC power converters, resonant converters, voltage ratio, Class E.**

I. Introduction

Power conversion circuits with high operation frequency own the potential to be lighter and more compact [1], which are also applied to wireless power transfer system to improve transfer distance and efficiency [2], [3]. Class E power amplifier (PA) is a good candidate high frequency converter due to its circuit simplicity and high efficiency [4], [5], which is designed under zero voltage switching (ZVS) and zero voltage derivative switching (ZVDS) conditions. The traditional Class E PA is neither a current source nor a voltage source, and it can not ensure the output power when tuning the load of PA. The Class E rectifier was first proposed in 1988 [6], with various Class E topologies developed later. The Class E full-wave current-driven rectifier is a promising candidate for low-harmonic-contents and high-efficiency rectification at MHz [2]. In this paper, a Class E^2 DC-DC converter consisting of a Class E PA, coupling coils, and a Class E full-wave rectifier is used as the basic circuit topology for the following work.

Compared to conventional DC-DC converter typologies such as boost or buck converters, a Class E^2 DC-DC converter with coupling coils owns the advantage of galvanic isolation, weight reduction and efficiency improvement. Besides, the using of coupling coils without an iron core provides possibility for the converter to be flat and more compact. However, conventional converters can provide a desired DC-DC voltage ratio preciously, while for resonant converters as Class E^2 converters it is more challenging to analytical derive the DC-DC voltage ratio. The article [7] proposes an equivalence approach to transform the Class E^2 DC-DC converter to a new typology. The method is able to reduce the number of inductors and achieve the desired voltage ratio, which is practical in system design. However, the method has two main disadvantages. First, an original design with a known voltage ratio is required to perform circuit transformation. Second, when changing different voltage ratios, the inductors of both primary and secondary sides need to be replaced, which increase designing and manufacturing complexity.

In this paper, the author proposes a method to design a Class E^2 DC-DC converter with flexible voltage ratios based on the idea of impedance transformation. Different system voltage ratios are obtained through changing the compensation capacitors of the primary and second sides. In the proposed design procedure, the inductors of the system remain unchanged, which dramatically reduce the complexity to design and manufacture such a system. Besides, an expansion approach is proposed to further increase the maximum voltage ratio.

II. System Voltage Ratio Calculation

A. System Configuration

Fig. 1 shows the typical configuration of a Class E^2 DC-DC converter. A Class E full-wave rectifier is used in this paper to reduce harmonic contents and to increase the rectifier efficiency. In the figure, P_{dc} is the input power of the PA, P_{in} is the input power of coupling coils, P_{rec} is the input power of the rectifier, P_o is the power dissipated on the load. In wireless power transfer (WPT) systems, researchers tend to compensate the imaginary parts of both the primary side and the secondary side through compensation capacitors [8]. The main advantage of this strategy is to increase the reflected impedance to make the system own high efficiency, even when the coupling coefficient k of the transmission coils is low to 0.13 [9]. Under the circumstance of complete compensation, the input impedance of the coupling coils will be pure resistive. However, in DC-DC converter case, due to higher coupling coefficient and stable alignment of the coils, the system can maintain high efficiency in a detuned state, which can provide greater flexibility to system design. Based on the equivalent circuit proposed in [10], the system shown in Fig. 1 can be transformed to Fig. 2 (a). Note that the imaginary part of the

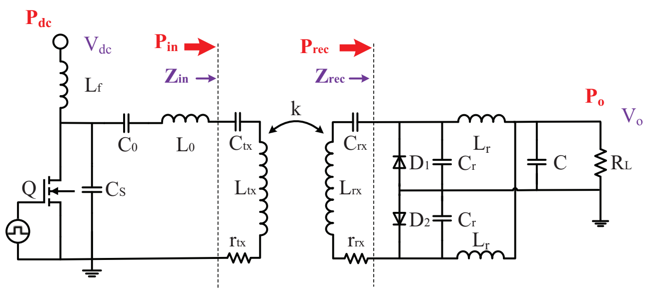

Fig. 1. System configuration of the Class E^2 DC-DC converter.

input impedance of the rectifier is also compensated by C_{rx}. The reflected impedance R_{in} is formulated as follows, which is pure resistive.

$$R_{in} = r_{tx} + R_{in} = r_{tx} + \frac{\omega^2 M^2}{R_{rec} + r_{rx}}. \tag{1}$$

Where M is the mutual inductance calculated by $M = k\sqrt{L_{tx}L_{rx}}$. Assuming that the system deviates from the resonant state on both sides, i.e., the capacitors C_{tx} and C_{rx} are adjusted so as to generate reactances ΔX_{tx} and ΔX_{rx} on each sides of the converter (Fig. 2 (b)). This operation will provide two new variables to effectively adjust the input impedance Z_{in} seen by the PA, accordingly, changing the DC-DC voltage ratio of the converter, which will be explained in the following sections.

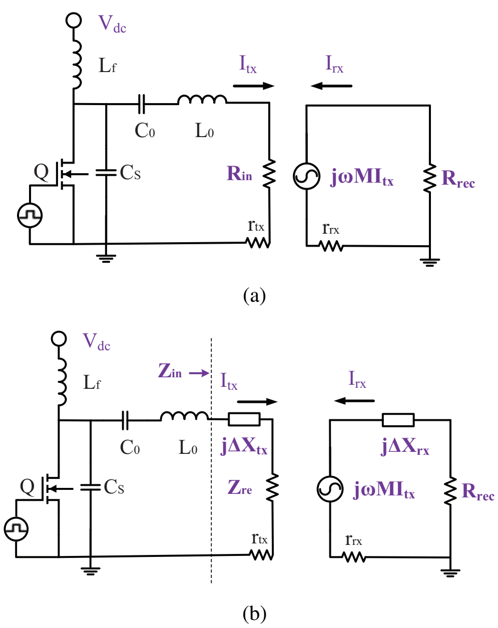

Fig. 2. System equivalence of the Class E^2 DC-DC converter. (a) Resonant condition. (b) Detuned condition.

B. Representation of the system DC-DC voltage ratio

The DC-DC voltage ratio of such a nonlinear Class E^2 system is related with several factors, such as the coil inductance, the final load and the parameters of the PA and rectifier, etc. However, since any converters can be regarded as impedance transformers, it will be more convenient to analyze and represent the DC-DC voltage ratio through the way of impedance transformation. The optimum parameter design procedure as well as the corresponding R_{rec} and R_{in} of the system are given in [2]. This paper will discuss the influence of ΔX_{tx} and ΔX_{rx} caused by the variation of C_{tx} and C_{rx}. Note that the initial capacitances of C_{tx} and C_{rx} are calculated based on the optimum design procedure.

Under the condition of Fig. 2 (b), the impedance Z_{in} can be calculated,

$$\begin{aligned} &Z_{in} = Z_{re} + j\Delta X_{tx} = \tfrac{\omega^2 M^2}{j\Delta X_{rx} + r_{rx} + R_{rec}} + r_{tx} + j\Delta X_{tx} \\ &= R_{in} + jX_{in} = \omega^2 M^2 \tfrac{R_{rec}+r_{rx}}{(R_{rec}+r_{rx})^2+\Delta X_{rx}^2} + rtx \\ &+ j\left(\Delta X_{tx} - \tfrac{\omega^2 M^2 \Delta X_{rx}}{(r_{rx}+R_{rec})^2+\Delta X_{rx}^2}\right). \end{aligned} \tag{2}$$

From (2), it is obvious that ΔX_{tx} only influences the imaginary part X_{in} while ΔX_{rx} can affect both R_{in} and X_{in}. In another word, by adjusting C_{rx}, R_{in} can be changed. Meanwhile, the input impedance Z_{in} may maintain pure resistive by tuning C_{tx}. The new capacitances of C_{rx} and C_{tx} can be derived as follows,

$$C'_{rx} = \frac{C_{rx}}{1 - \omega C_{rx}\Delta X_{rx}}, \tag{3}$$

$$\Delta X_{tx} = \frac{\omega^2 M^2 \Delta X_{rx}}{(r_{rx}+R_{rec})^2 + \Delta X_{rx}^2}, \tag{4}$$

$$C'_{tx} = \frac{C_{tx}}{1 - \omega C_{tx}\Delta X_{tx}}. \tag{5}$$

The only independent variable here is ΔX_{rx} used to adjust R_{in}. After ΔX_{rx} is chosen, ΔX_{tx}, C_{rx} and C_{tx} will be determined.

Although the parameter design of the rectifier and compensation circuit is already finished, the parameters of the PA should be calculated after adjusting C_{rx} and C_{tx} since Z_{in} seen by the PA has just been determined. In order to achieve zero-voltage switching (ZVS) and maximize the PA efficiency, (6) and (7) can be used to determine C_S and C_0 [11],

$$C_S = \frac{0.1836}{\omega R_{in}}, \tag{6}$$

$$C_0 = \frac{1}{\omega^2 L_0 - 1.1525\omega R_{in}}. \tag{7}$$

Afterwards, the DC input impedance of the system (i.e., R_{dc}) can be calculated as follows,

$$R_{dc} = \frac{\pi^2 - g\left(2\pi\cos\phi + 4\sin\phi\right)}{4\pi\omega C_S}, \tag{8}$$

where

$$g = \frac{2\pi\sin(\phi+\psi) + 4\cos(\phi+\psi)}{4\cos\phi\sin(\phi+\psi) + \pi\cos\psi} \tag{9}$$

$$\psi = \arctan\frac{\omega L_0 - \frac{1}{\omega C_0}}{R_{in}}, \tag{10}$$

$$\phi = \arctan \frac{\frac{\pi^2}{2} - 4 - \pi\omega C_S \left[2R_{in} + \pi\left(\omega L_0 - \frac{1}{\omega C_0}\right)\right]}{\pi + \pi^2\omega C_S R_{in} - 2\pi\omega C_S \left(\omega L_0 - \frac{1}{\omega C_0}\right)} \tag{11}$$

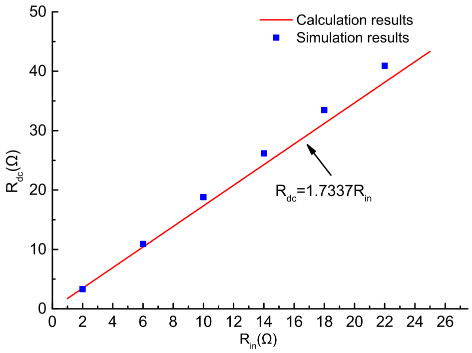

Fig. 3. The relationship between R_{dc} and R_{in}.

Based on (6)-(11), the red line in Fig. 3 is formed, which indicates the linear relationship between R_{dc} and R_{in}. Also, Advanced Design System (ADS) is used to simulate R_{dc} versus different R_{in}, showing good matching results. The expression of the relationship can be formulated as

$$R_{dc} = 1.7337R_{in}. \tag{12}$$

The system input power P_{dc} is

$$P_{dc} = \frac{V_{dc}^2}{R_{dc}}. \tag{13}$$

Since all system parameters are under optimum design condition, it is safe to assume the overall system efficiency is high and remains unchanged (say $\eta_{sys} = 80\%$). The power transfered to the final load is

$$P_o = P_{dc}\eta_{sys}. \tag{14}$$

Therefore, the DC output voltage and the system voltage ratio can be formulated as (15) and (16),

$$V_o = \sqrt{P_o R_L}, \tag{15}$$

$$K_{sys} = \frac{V_{dc}}{V_o} = \sqrt{\frac{R_{dc}}{\eta_{sys} R_L}}. \tag{16}$$

By observing equations above, it is not difficult to find that R_{in} is determined by R_L (R_{rec} is determined by R_L [2]), M and C_{rx} under optimum design. Meanwhile, R_{in} is the only independent variable which influence R_{dc}. From (16), it can be seen that the mutual inductance M and the compensation capacitor C_{rx} are the only two independent variables that determine the DC-DC voltage ratio under the condition of given R_L.

III. System parameter design

In this section, the author will explain the parameter design procedure of a Class E^2 DC-DC converter with required voltage ratios. The operation frequency of the system is 6.78 MHz. It can be seen from (2) that Z_{in} reaches its maximum when ΔX_{rx} is 0, i.e., the system is completely resonant. Under that condition, the voltage ratio K_{sys} also reaches its maximum. Thus, in order to design a converter with the highest voltage ratio of $K_{sys,max}$, the mutual inductance must be large enough. Substituting (2), (4) and (12) into (16) yields the minimum value of M,

$$M_{\min} = \frac{1}{\omega}\sqrt{(R_{rec} + r_{rx})\left(\frac{K_{sys,\max}^2 \eta_{sys} R_L}{1.7337} - r_{tx}\right)}. \tag{17}$$

When the converter is tuned to be resonant, it will provide the maximum voltage ratio $K_{sys,\max}$. ΔX_{rx} and ΔX_{tx} are introduced into the system to adjust the voltage ratio at the range of $K_{sys} \leq K_{sys,max}$, calculated as (18) and (4),

$$\Delta X_{rx} = -\sqrt{\left(\frac{\omega^2 M^2}{0.5768 K_{sys}^2 \eta_{sys} R_L - r_{tx}} - R_{rec} - r_{rx}\right)(R_{rec} + r_{rx})}. \tag{18}$$

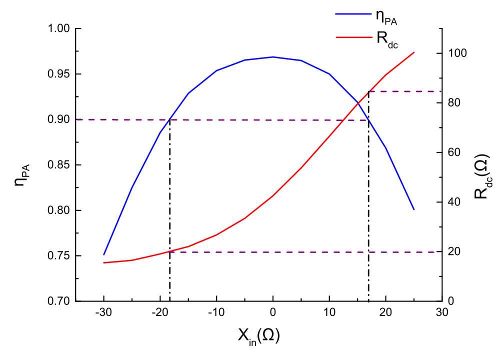

Fig. 4. The PA efficiency and the DC input impedance versus X_{in} when $R_{in} = 20\Omega$.

What is noteworthy is that although the receiving circuit is tuned to own a imaginary part $j\Delta X_{rx}$, the input impedance Z_{in} of the coils is still pure resistive due to $j\Delta X_{tx}$. $j\Delta X_{tx}$ is designed to compensate the imaginary part of the reflected impedance Z_{re} (Fig. 2 (b)), which makes the PA work with high efficiency and reduces the design complexity. However, a Class E PA also maintain high efficiency when the reactance seen by it is limited in an optimum range. Here ADS is used to simulate the PA performance under varying X_{in}. Fig. 4 gives the simulation result, where the efficiency of the PA can stay above the limit (say 0.9) with the input reactance X_{in} varying in the range of $-18\sim17\Omega$. Within the range, the DC input impedance's variation is relatively large ($-20\sim82\Omega$). As shown in (16), the variation of R_{dc} is going to bring a change in the voltage ratio K_{sys}. It is known that the efficiency of a

Class E PA with optimum design will decrease significantly if R_{in} is too high. Therefore, it is quiet helpful to adjust ΔX_{rx}, in another word, to adjust C_{tx} to further increase the system voltage ratio when the real part R_{in} is already high to the PA.

Fig. 5 illustrates the design process of such a Class E^2 converter with required voltage ratio, of which the author describes the details in the following.

First, select DC load R_L to the system, obtain the optimum design of C_r and calculate the input resistance R_{rec} and reactance X_{rec} of the rectifier. Note that X_{rec} will be compensated by C_{rx} in the next step together with the reactance of the receiving coil.

Second, determine the maximum DC-DC voltage ratio $K_{sys,max}$ of such a Class E^2 converter, i.e., the converter can not be tuned to work with a voltage ratio higher than $K_{sys,max}$ unless increasing the inductance of the coupling coils or using the voltage ratio expansion method by adjusting C_{tx}.

Third, determine the actual voltage ratio K_{sys} and calculate the input impedance R_{in} of the coils. If R_{in} is too high compared to the reactance ωL_0 of the filter inductance, then reduce K_{sys} appropriately and recalculate R_{in}. Meanwhile, update C_{rx} and C_{tx} with C'_{rx} and C'_{tx} through (18) and (3)-(5) in order.

Finally, design C_s and C_0 of the PA by using (6) and (7). If K_{sys} is reduced in the third step, increase the voltage ratio by tuning C_{tx}. Fine adjust the system, with taking the effects of parasitic parameters into consideration.

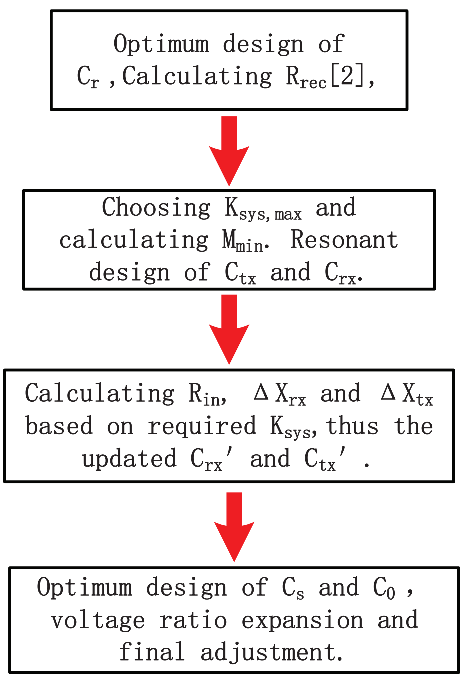

Fig. 5. The design procedure of the detuned Class E^2 converters.

IV. Experimental Verification

An example 6.78-MHz WPT system is built up to achieve different voltage ratios by only changing the capacitors. It is worth noting that in all three prototypes illustrated in the following, the inductors including the coupling coils remain unchanged, which shows major practical value. Fig. 6 gives the experimental setup used to demonstrate the design approach given in section III.

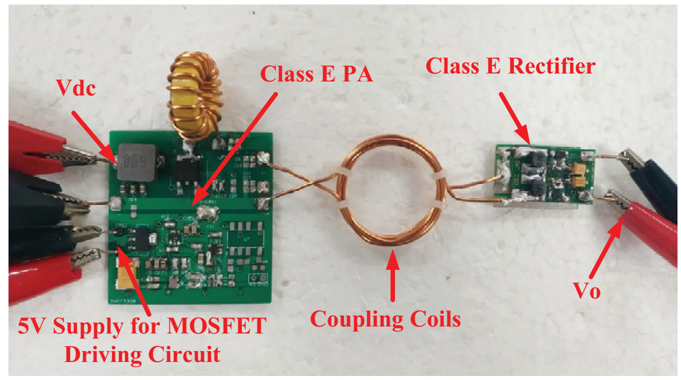

Fig. 6. The experimental 6.78-MHz DC-DC converter.

Table I provides three sets of system parameters, all of which are calculated by a MATLAB script based on the design procedure given in Fig. 5. The designed voltage ratios are 5 and 2 in the first and second setups. Note that the third setup is constructed with the voltage ratio expansion approach illustrated in section III, i.e., the third setup is built up based on the second setup by only adjusting the capacitance of C'_{tx}.

TABLE I
Parameters of Experimental System ($L_{tx} = L_{rx} = 430nH$, $L_0 = 1.433\mu H$, $k = 0.8$)

$NO.$	R_L (Ω)	K_{sys}	C_s (pF)	C_o (pF)	C'_{tx} (pF)	C'_{rx} (pF)	C_r (pF)
I	2.5	5	149	844	666	1556	1649
II	5	2	467	466	836	1377	824
III	5	-	-	-	1200	-	-

TABLE II
Experimental Results

$NO.$	V_{dc} (V)	V_o (V)	K_r	δ_K	P_o (W)	η_{sys}
I	20	4.011	4.99	0.27%	6.44	80.1%
II	16	8.05	1.99	0.62%	13.09	83.1%
III	26	8.43	3.08	2.67%	14.21	83.8%

The experimental results are listed in Table II, where K_r is the measured voltage ratio and $\delta_K = |K_r - K_{sys}|/K_{sys}$ is the design error of the voltage ratio. It can be seen from the results that the measured and designed voltage ratio are quite close to each other, which shows that the performance of the real system is in good agreement with the calculation. The experimental results of setup III also demonstrates the voltage ratio expansion approach by adjusting C_{tx}, with the measured voltage ratio increasing from 1.99 to 3.08.

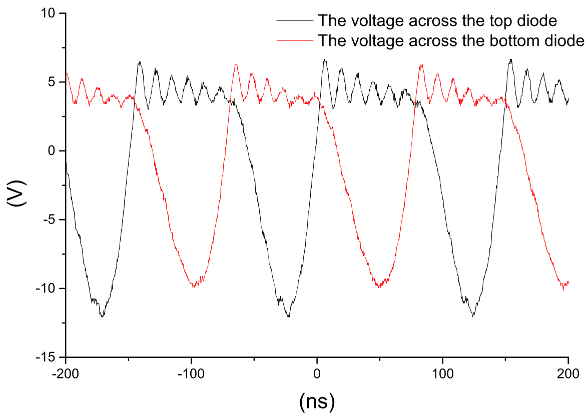

Fig. 7. Voltage waveforms of the rectifier (setup I).

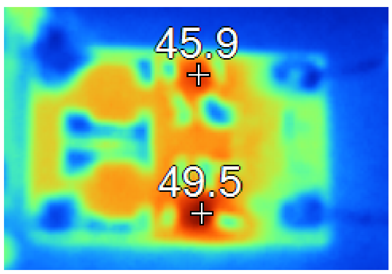

Fig. 8. Thermal image of the rectifier (setup I, unit: °C).

Fig. 7 shows the voltage waveforms of the rectifiers. The peak voltages of two diodes are not equal mainly due to the harmonic contents transfered from the primary side. By measuring the input resistance R_{in} of the coils, it is found that $R_{in} = 28.8\Omega$ is about one third of the reactance ωL_0 of PA filter inductance, which results in an insufficient filtering ability. It can be seen from Fig. 8 that there is a clear difference between the temperatures of the two diodes. This issue somehow has an adverse effect on the performance and life of the rectifier. Therefore, when the required voltage ratio is high to result in a relatively large coil input resistance, it is suggested to first decrease the designed K_{sys} and finish parameter design, then use the voltage ratio expansion method proposed in section III to increase the voltage ratio to the desired value. Besides, the design method in this paper can be extended to the systems with higher frequency or other typologies.

V. Conclusions

This paper proposes a new design method for a Class E^2 DC-DC converter possessing a flexible voltage ratio, which can be tuned by only adjusting the capacitances of the capacitors. Detailed analytical derivation and calculation procedure are presented to guide the design for such a converter. MATLAB is used to formulate the design procedure. Both ADS simulation and experiments are used to validate the proposed design approach. In addition, an effective voltage ratio increasing method is proposed, which shows good performance during experimental process.

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