Fractional Order Control and Its Applications in Motion Control

Chengbin Ma
(馬　澄斌)
D3(October), Hori Laboratory

The Department of Electrical Engineering

July 26th 2004

Outline

- Brief Review
- My Contributions
- Conclusions & Future Works
- Publications

Section 1

- Brief Review
  - What is FOC?
  - When did FOC begin?
  - Why need FOC?
- My Contributions
- Conclusions & future works
- Publications

What is "Fractional"

- Models and/or controllers described by fractional order differential equations.
  - Fractional order modeling:

$$0.7943y^{2.5708}(t)+5.2385y^{0.8372}(t)+1.5560y(t)=u(t)$$

  - Fractional PID controller:

$$C(s)=K_p+\frac{K_i}{s^{\alpha}}+K_d s^{\beta}$$

Mathematic definitions

- Riemann-Liouville Definition:

$$ {}_{a}D_{t}^{\alpha} f(t) = \frac{1}{\Gamma(\gamma-\alpha)} \frac{d^{\gamma}}{dt^{\gamma}} \int_{a}^{t} \frac{1}{(t-\xi)^{\alpha-\gamma+1}} f(\xi) d\xi, \qquad \gamma - 1 < \alpha < \gamma $$

- Grünwald-Letnikov Definition:

$$ {}_{a}D_{t}^{\alpha} f(t) = \lim_{\substack{h\to 0 \\ nh = t-a}} h^{-\alpha} \sum_{r=0}^{n} (-1)^{r} \binom{\alpha}{r} f(t-rh) $$

Fractional order systems have an unlimited memory (infinite dimension)

- Fractional order of s means fractional order calculus.

$$ L\left\{ {}_{0}D_{t}^{\alpha} f(t) \right\} = s^{\alpha} F(s), \quad F_{e}\left\{ {}_{0}D_{t}^{\alpha} f(t) \right\} = (j\omega)^{\alpha} F(j\omega) $$

Mathematical presentations

- Fractional differential equation:

$$a_n D_t^{\alpha_n} y(t) + \cdots + a_1 D_t^{\alpha_1} y(t) + a_0 D_t^{\alpha_0} y(t)$$
$$= b_m D_t^{\beta_m} u(t) + \cdots + b_1 D_t^{\beta_1} u(t) + b_0 D_t^{\beta_0} u(t)$$

- Fractional transfer function:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^{\beta_m} + \cdots + b_1 s^{\beta_1} + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + \cdots + a_1 s^{\alpha_1} + a_0 s^{\alpha_0}}$$

- State-space model in s-plane:

$$sX(s) = A(s)X(s) + B(s)U(s)$$
$$Y(s) = C(s)X(s)$$

Frequency response

- Sinusoidal input

$$r(t) = R\sin(\omega t)$$

For FOC system with transfer function: $M(j\omega)$

$$Y = R\left|M(j\omega)\right|, \qquad \phi = \angle M(j\omega)$$

Using an asymptotic expansion of the incomplete gamma function

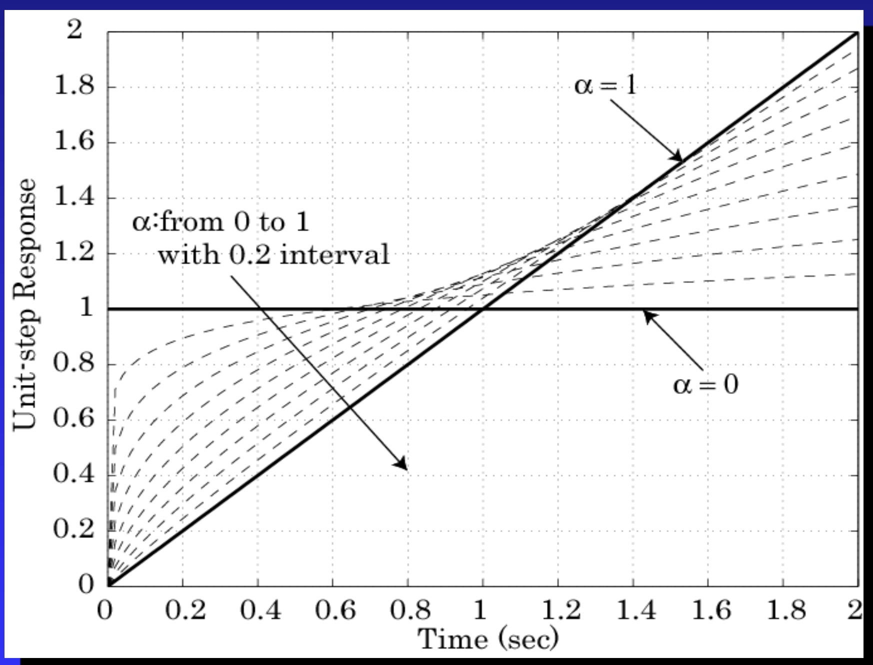
$$\left. {}_0D_t^{\alpha} \sin(\omega t)\right|_{t\to\infty} = \omega^{\alpha}\sin(\omega t + \frac{\pi}{2}\alpha) + \sum_{n=0}^{\infty} \frac{(-1)^n}{\omega^{1+2n}\Gamma(-\alpha-2n)} \cdot \frac{1}{t^{1+2n+\alpha}}$$

$$= \omega^{\alpha}\sin(\omega t + \frac{\pi}{2}\alpha)$$

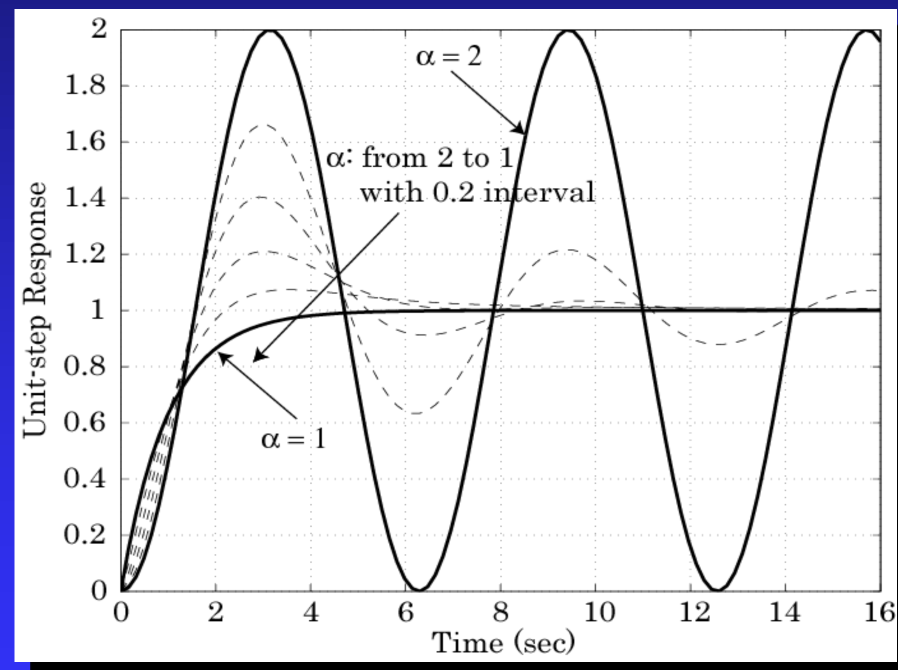
$$y(t) = Y\sin(\omega t + \phi) \qquad Y = R\omega^{\alpha}, \quad \phi = \frac{\pi}{2}\alpha$$

Example of time responses

- Basically interpolation, but still quite different.



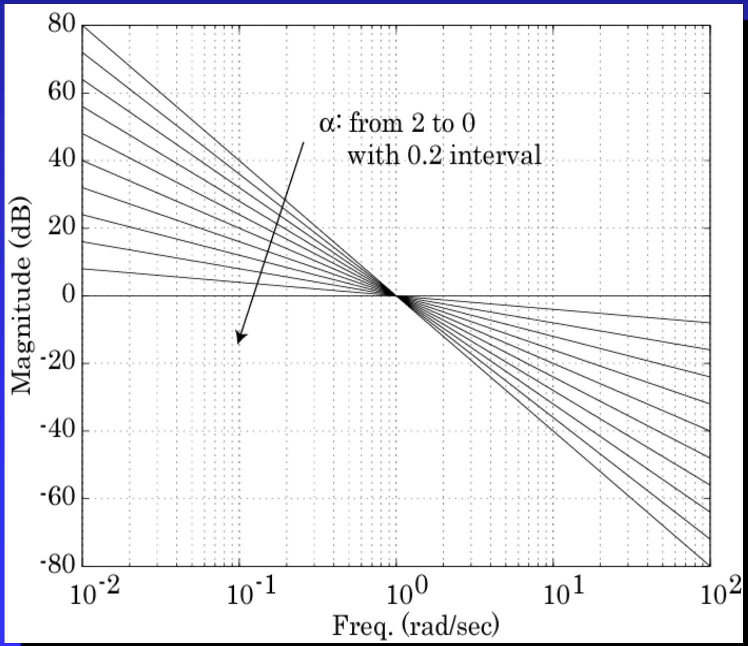
Open-loop 1/s^α system

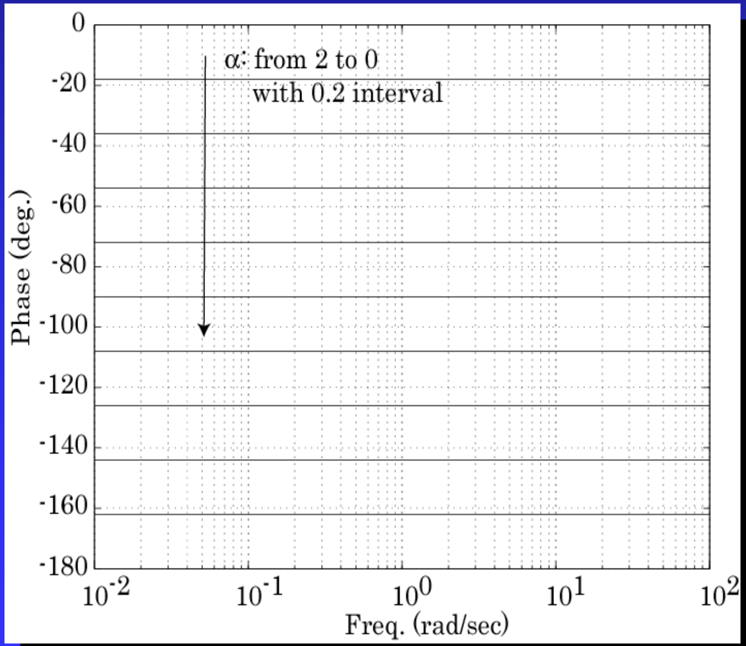


Unity feedback 1/s^α system

Example of frequency responses

- Perfect interpolation, both in magnitude and phase.





Open-loop $1/s^\alpha$ system

Section 1

- Brief Review
  - What is FOC?
  - When did FOC begin?
  - Why need FOC?
- My Contributions
- Conclusions & future works
- Publications

History of 300 years

- Leibnitz introduced the notation $d^n y/dx^n$. A letter to L'Hospital in 1695 Leibnitz raised the following question:

  Can the meaning of derivatives with integer order $d^n y(x)/dx^n$ be generalized to derivatives with non-integer orders?

  The story goes that L'Hospital was somewhat curious about that question and replied by another question to Leibnitz:

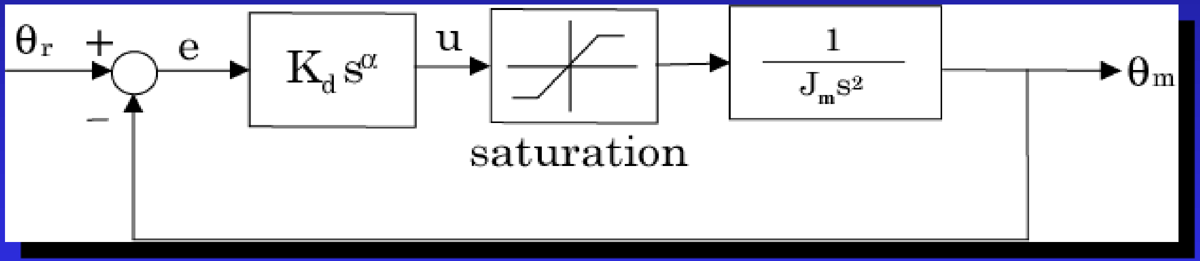
  What is n=1/2?

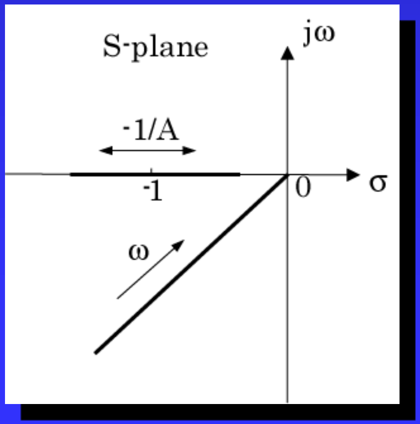
  Leibnitz in a letter dated September 30, 1695 replied:

  It will lead to a paradox, from which one day useful consequences will be drawn.

"New" research with long history

- Fractional Order Control (FOC) was introduced by Tustin for the position control of massive objects half a century ago in 1958.



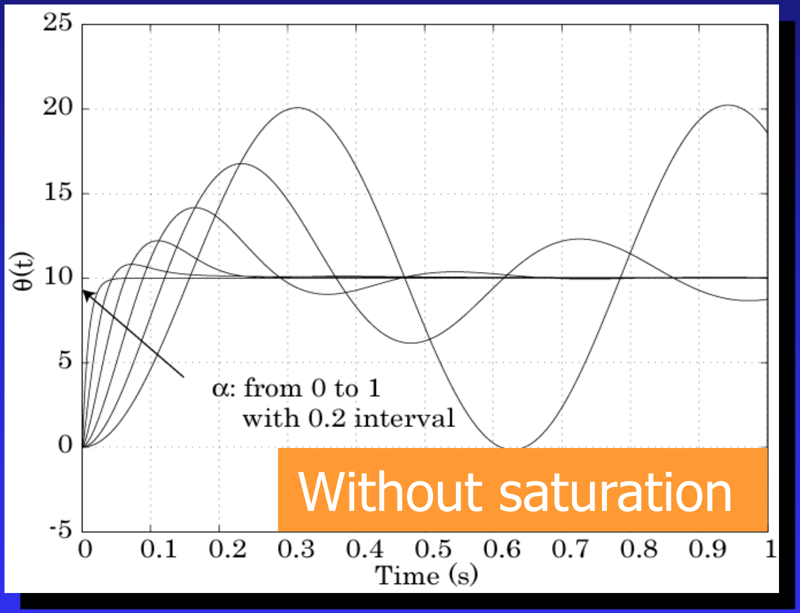


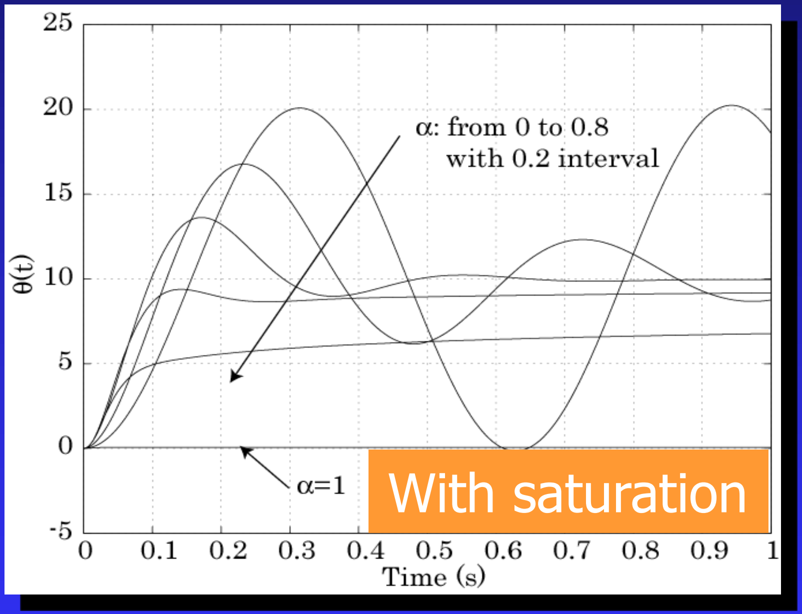
$$PM = 180 - (2 - \alpha) \times 90 (\mathrm{deg.})$$

$$\alpha = 1 \rightarrow PM = 90 (\mathrm{deg.})$$

$$\alpha = 0 \rightarrow PM = 0 (\mathrm{deg.})$$

Robust against gain variation





- α=0.6, namely 1/s^1.4 system, has the best robustness against saturation non-linearity

Main obstacles in the past

- FOC was not widely incorporated into control engineering mainly due to:
  - The unfamiliarity of taking fractional order
  - The existence of so few physical applications
  - The limited computational power available at that time.

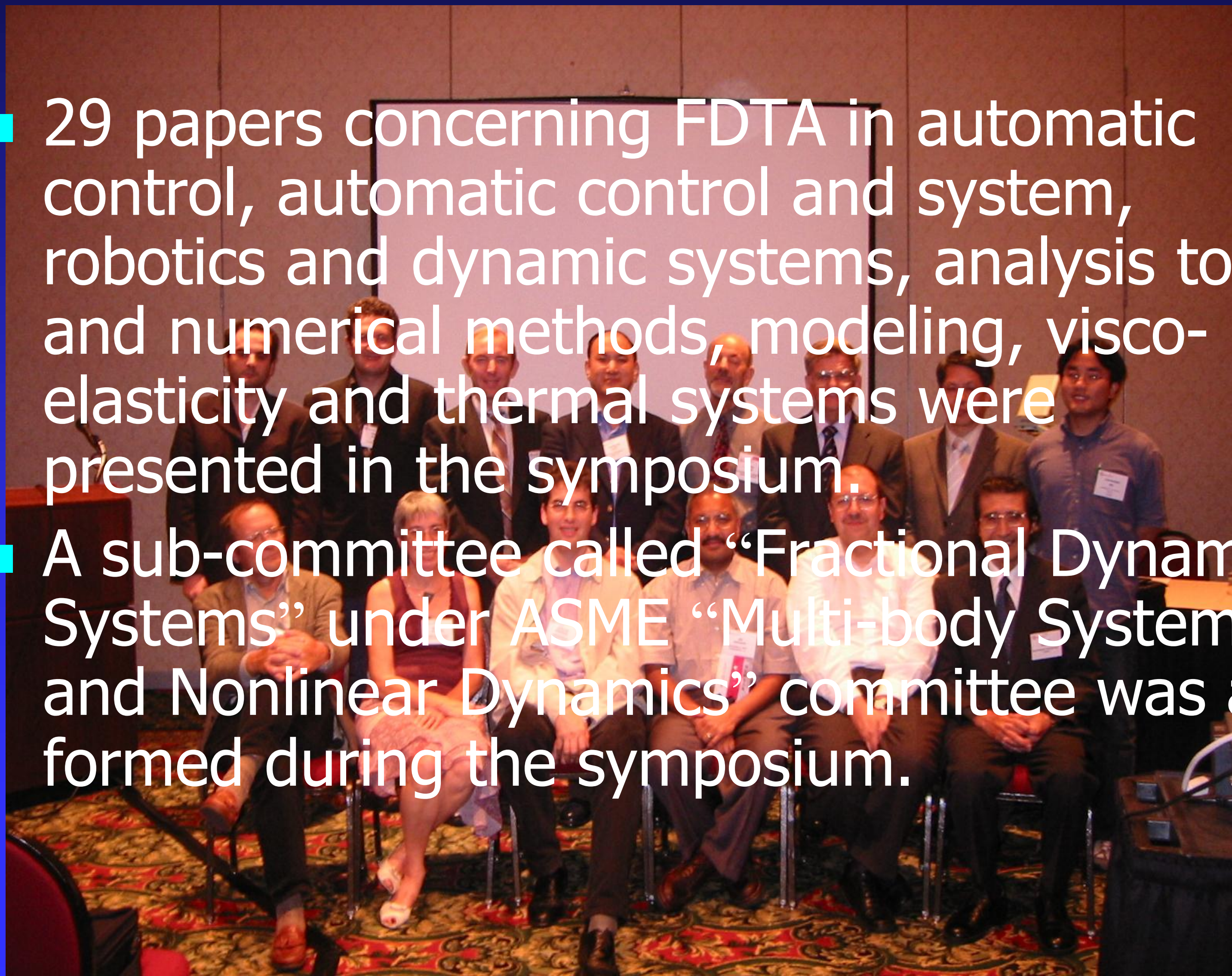
Resurging interest

- Fractional order differential equations could describe dynamic processes more adequately.
- Fractional dynamic systems need fractional order controllers for more effective control.
- Computational power's progress also makes modeling and realization of FOC systems much easier.
- FOC can achieve clear-cut design of robust control systems.

International community

- The researches on FOC are centered in European universities.
- France *CRONE*（Controle Robuste d'Ordre Non-Entier）team, *Denis Matignon*, *Ivo Petras*, *Igor Podlubny*, *J. A. Tenreiro Machado*, *Yangquan Chen*, etc.
- 1st ASME Symposium on Fractional Derivatives and Their Applications (FDTA) was held in last year's ASME Chicago conference.
- 1st IFAC Workshop on Fractional Differentiation and its Applications (FDA'04) was held last week in Bordeaux, France.

ASME Chicago symposium

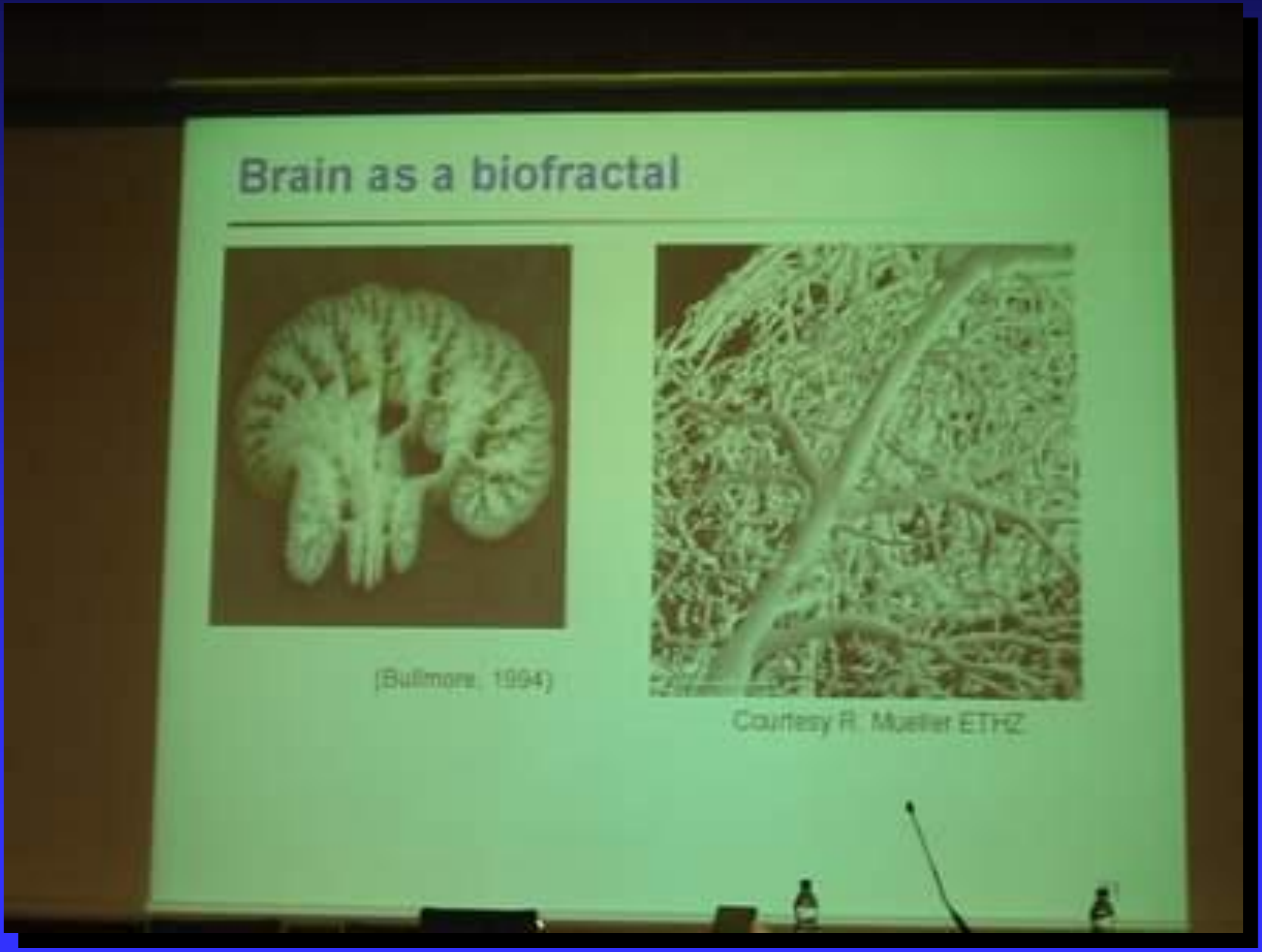


- 29 papers concerning FDTA in automatic control, automatic control and system, robotics and dynamic systems, analysis tools and numerical methods, modeling, visco-elasticity and thermal systems were presented in the symposium.
- A sub-committee called “Fractional Dynamic Systems” under ASME “Multi-body Systems and Nonlinear Dynamics” committee was also formed during the symposium.

IFAC Bordeaux workshop

- 16 sections:
  - Material Modeling
  - Physical and Biological Systems Modeling
  - Material Tools, Control
  - System Identification
  - Software for Fractional Systems
  - Applications in Econophysics
  - Technological Transfers
  - Implementation/Discretisation of Fractional Operators
  - Thermal and Fluid Systems Modeling
  - Applications in Electrical Engineering
  - Anomalous transport/Random walks
  - Applications in Control, Robotics and Mechatronics
  - Fractional Diffusion Equations and Their Applications
  - Fractional Systems Analysis
  - Numerical methods for Fractional Systems

Impressive presentations in IFAC Bordeaux workshop

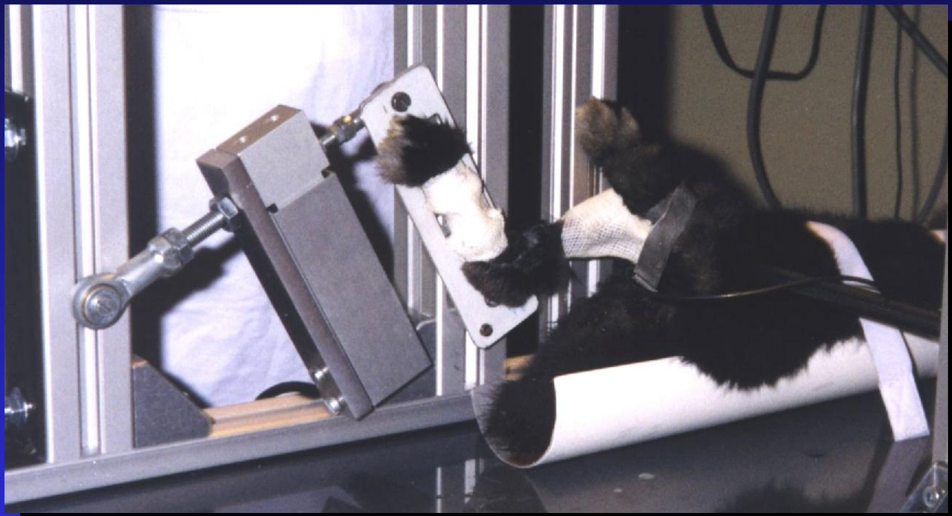


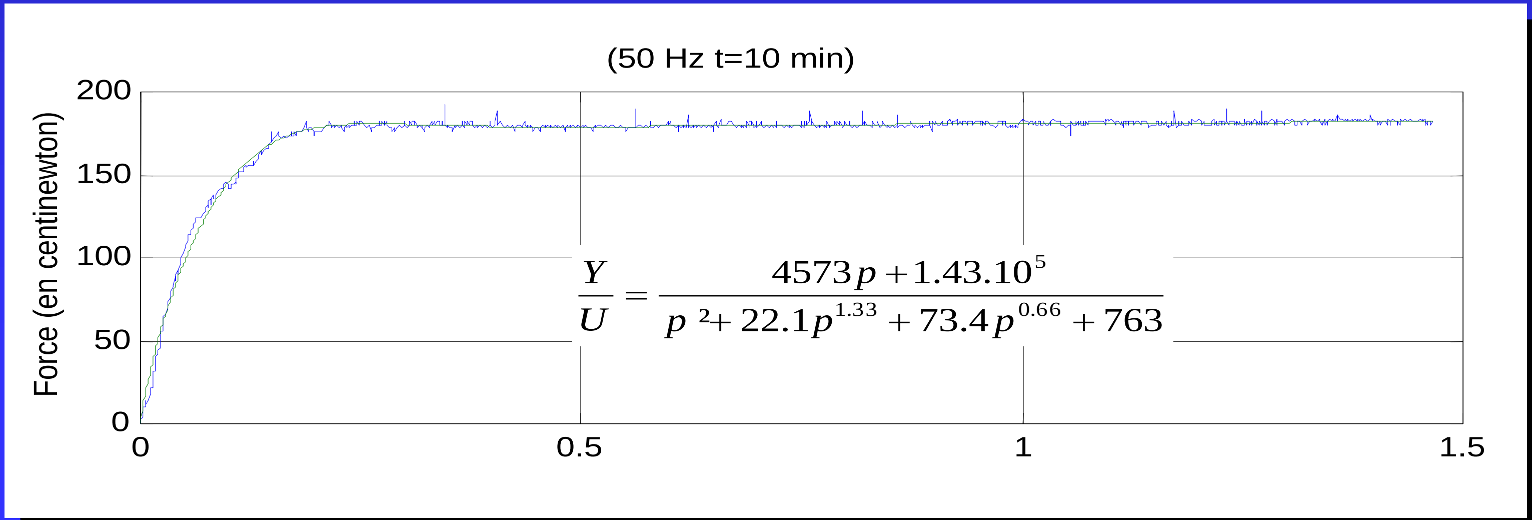
Impressive presentations in IFAC Bordeaux workshop

Impressive presentations in IFAC Bordeaux workshop

Identification of the rabbit muscle behavior

Bio-Control with FOC?





Section 1

- Brief Review
  - What is FOC?
  - When did FOC begin?
  - Why need FOC?
- My Contributions
- Conclusions & future works
- Publications

Three main advantages

- Natural modeling of control plant's dynamic features
- Clear-cut and effective robust control design
- Reasonable realization by proper approximation

Section 2

- Brief Review
- My Contributions
  - Basic attitude
  - Fractional order modeling
  - Stability determination
  - Effective gain-phase tradeoff
  - Sampling time scaling property
  - Applications
- Conclusions & future works
- Publications

Basic attitude

- **Responsibility of pioneer**: FOC is still in a primitive stage. AS a FOC pioneer (maybe), I want to establish a good basis for future FOC researches.
- **FOC should be an engineering research**: The mathematics of FOC can be extremely difficult. However, as an engineering student, I should not be restricted by difficult mathematics, but try to apply FOC from engineering view.
- **FOC should be useful**: Like all the other researches, the FOC research is also inevitably a team work. Superior application results will absorb excellent researchers into FOC field.

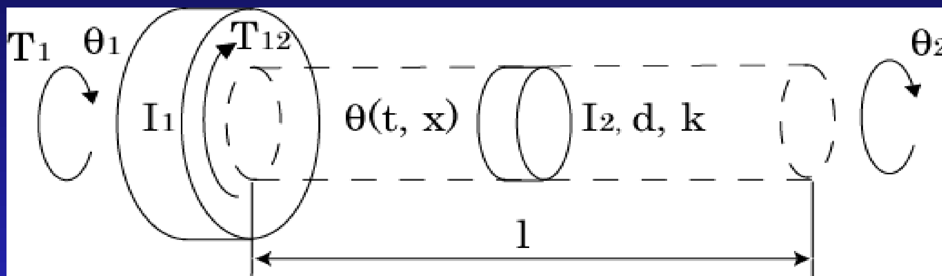
Section 2

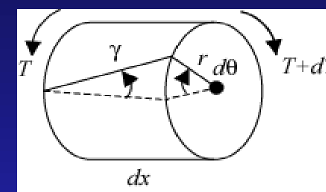
- Brief Review
- My Contributions
  - Basic attitude
  - Fractional order modeling
  - Stability determination
  - Effective gain-phase tradeoff
  - Sampling time scaling property
  - Applications
- Conclusions & future works
- Publications

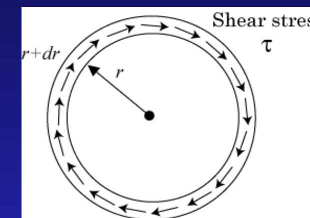
More natural modeling

- The fractional order model can provide a new possibility to acquire more adequate modeling of dynamic processes.
- Fractional order models have been applied to describe re-heating furnace, visco-elasticity, chemical processes and Chaos system, etc.
- Using fractional order model for describing distributed-parameter systems is quite natural since the Laplace transform of partial differential equations will inevitably introduce fractional order s operator.

An example of fractional order modeling



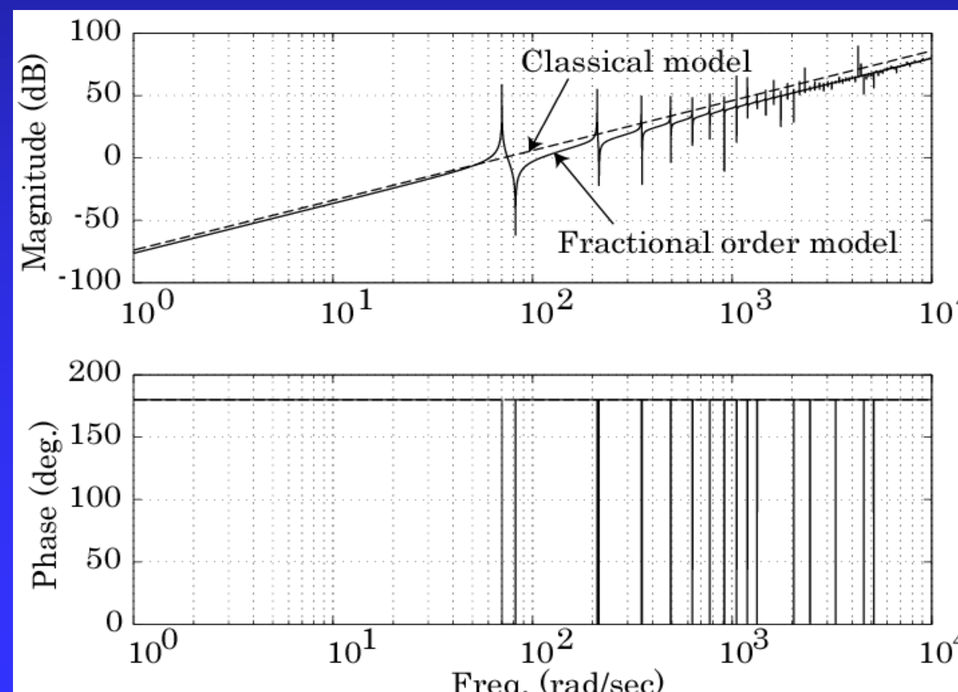




$$\frac{I_2}{l}\frac{\partial^2\theta(t,x)}{\partial t^2}-kl\frac{\partial^2\theta(t,x)}{\partial x^2}=0$$

$$\frac{I_2}{l}s^2\theta(x)-kl\frac{\partial^2\theta(x)}{\partial x^2}=0$$

$$\frac{T_1}{\theta_1}=I_1 s+\sqrt{klI_2}\tanh\left(\sqrt{\frac{lI_2}{k}}s\right)s$$



Section 2

Stable pole for N(s^α) system

- Characteristic equation:

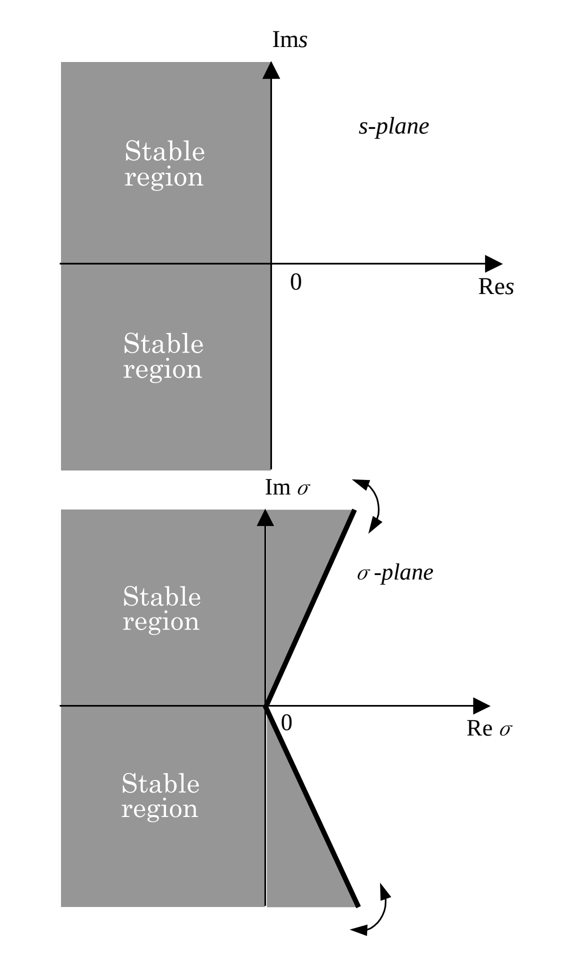
$$(s^{\alpha})^{n} + a_{n-1}(s^{\alpha})^{n-1} + \cdots + a_1 s^{\alpha} + a_0 = 0$$

$$\sigma \Leftarrow s^{\alpha}$$

$$\sigma^{n} + a_{n-1}\sigma^{n-1} + \cdots + a_1 \sigma^{\alpha} + a_0 = 0$$

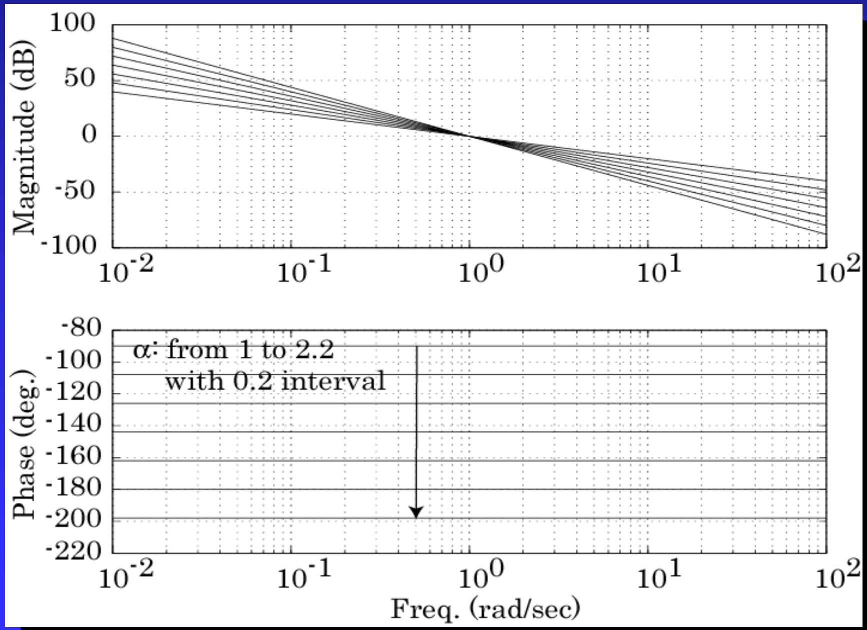
$$s^{n} + a_{n-1}s^{n-1} + \cdots + a_1 s^{\alpha} + a_0 = 0$$

$$outside : \left|\arg(\sigma)\right| \le \alpha \frac{\pi}{2}$$

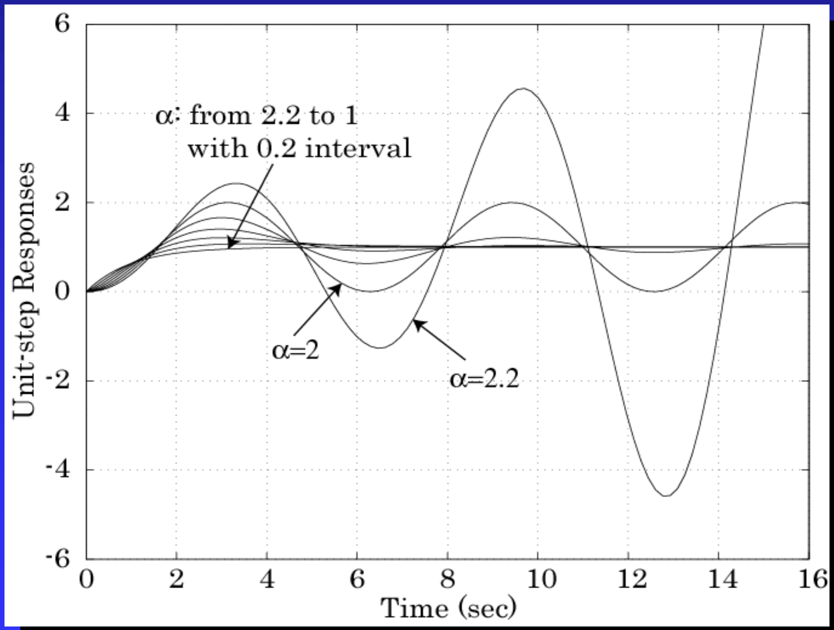


Nyquist stability criterion

- Take unity-feedback 1/s^α as an example:



Frequency responses



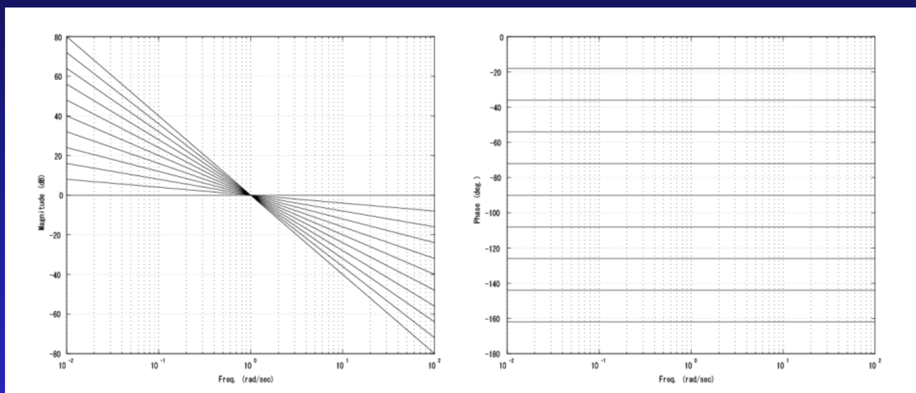
Time responses

Section 2

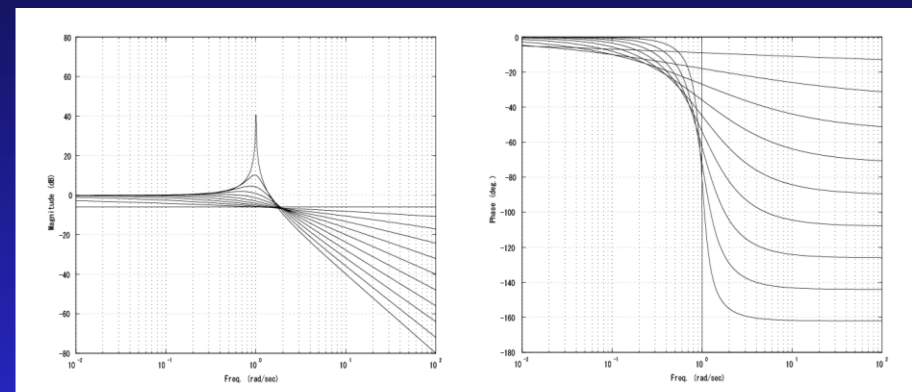
- Brief Review
- My Contributions
  - Basic attitude
  - Fractional order modeling
  - Stability determination
  - Effective gain-phase tradeoff
  - Sampling time scaling property
  - Applications
- Conclusions & future works
- Publications

Impact of s^{α}

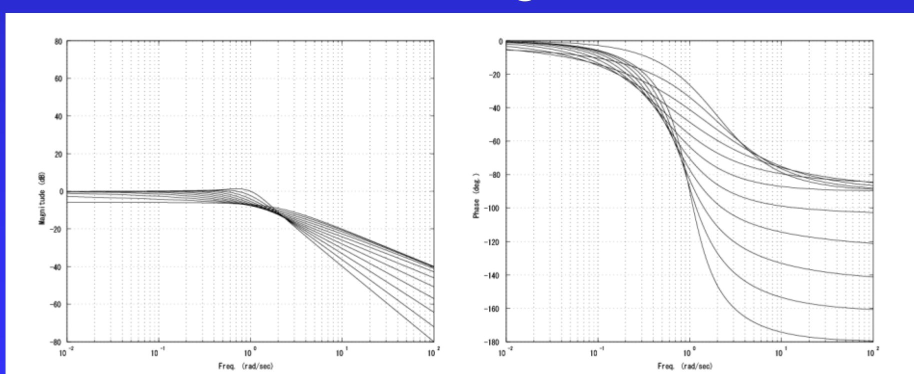
α: 0.0 → 2.0 with 0.2 interval



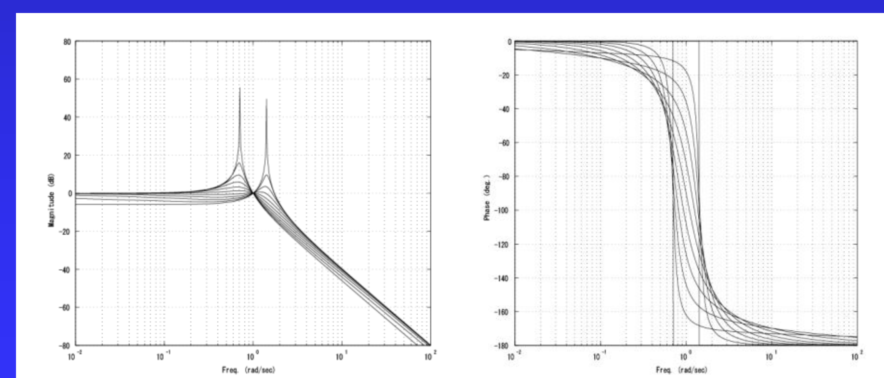
$$G(s) = \frac{1}{s^{\alpha}}$$



$$G(s) = \frac{1}{s^{\alpha} + 1}$$

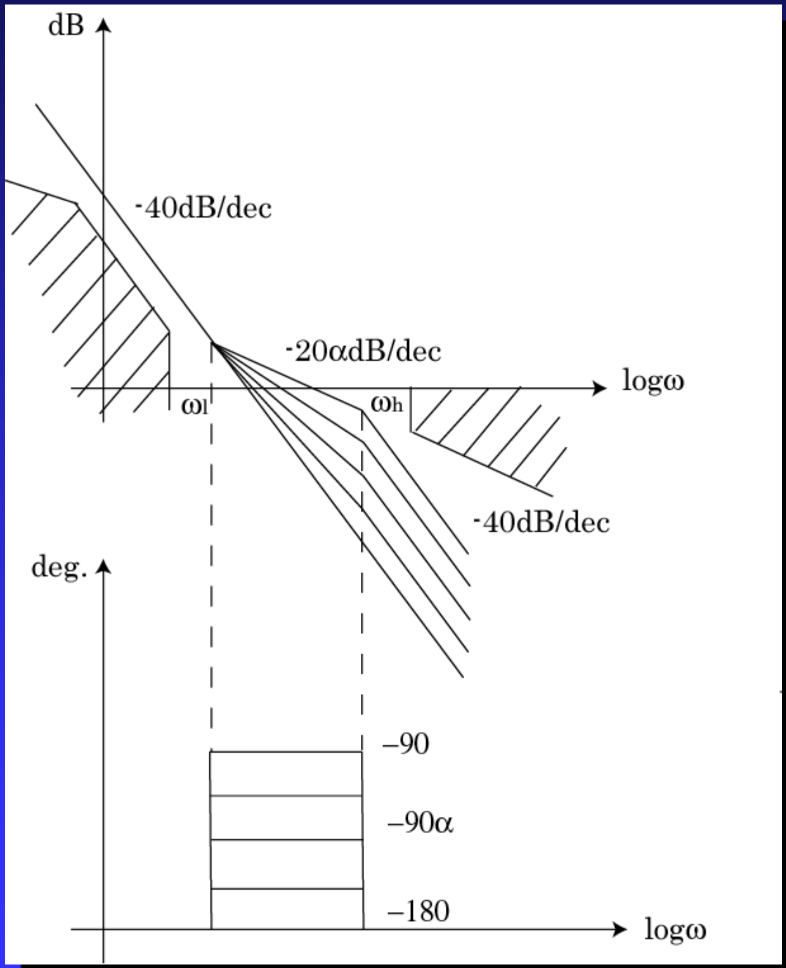


$$G(s) = \frac{1}{s + s^{\alpha} + 1}$$



$$G(s) = \frac{1}{s^{2} + s^{\alpha} + 1}$$

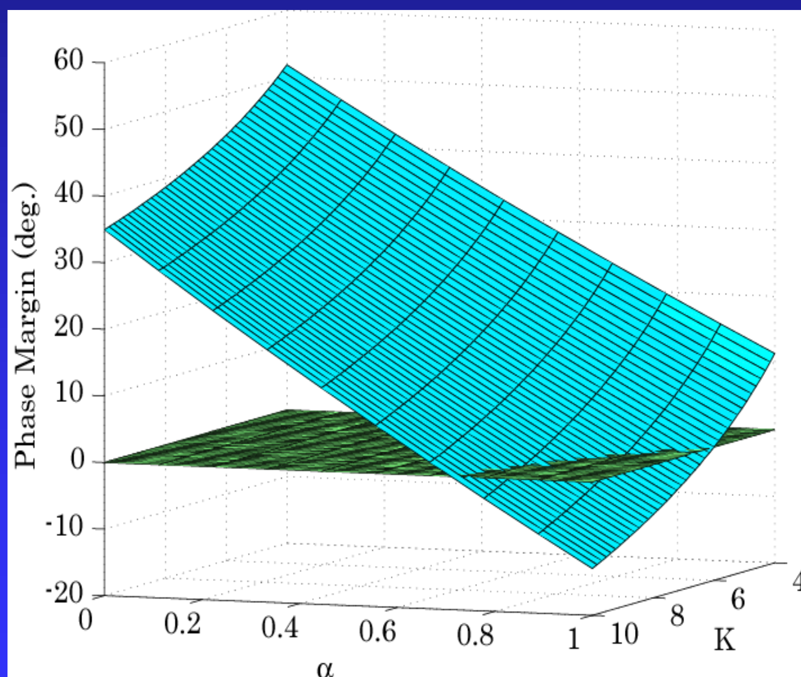
Smooth transition

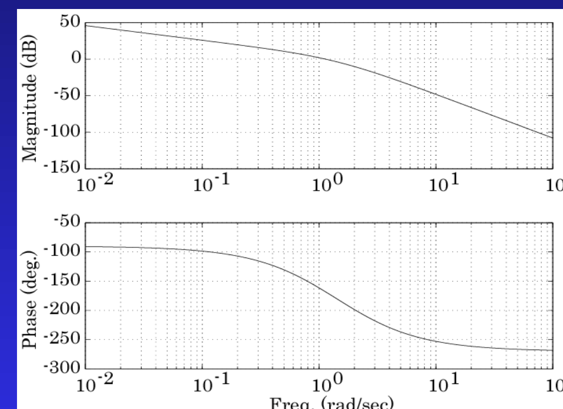


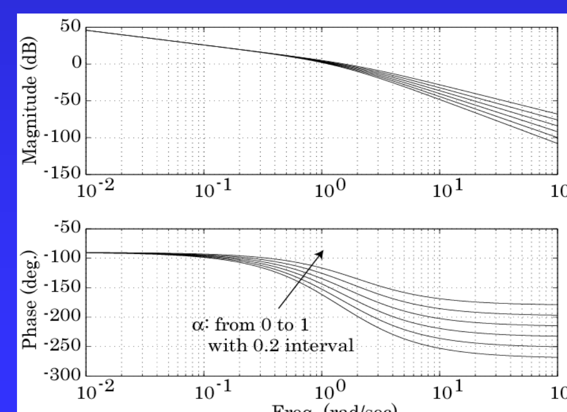
- Good control performance: large loop gain in low-frequency range
- Good robustness: small loop gain in high-frequency
- A smooth transition is important to design control system, which is neither too conservative (too large stability margin) nor too aggressive (too small stability margin).

Desirable stability margin

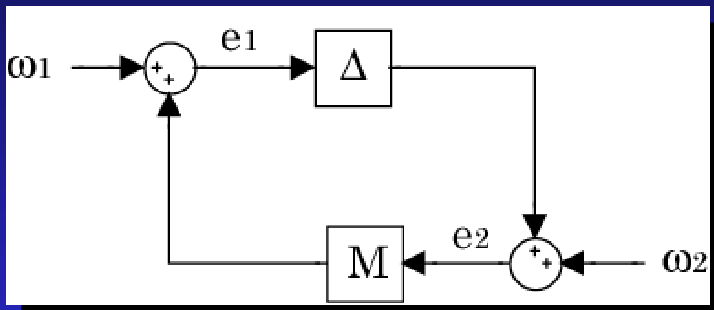
- Unity-feedback system $G(s)=\dfrac{K}{s(s+1)^{\alpha}(s+2)}, K=4$

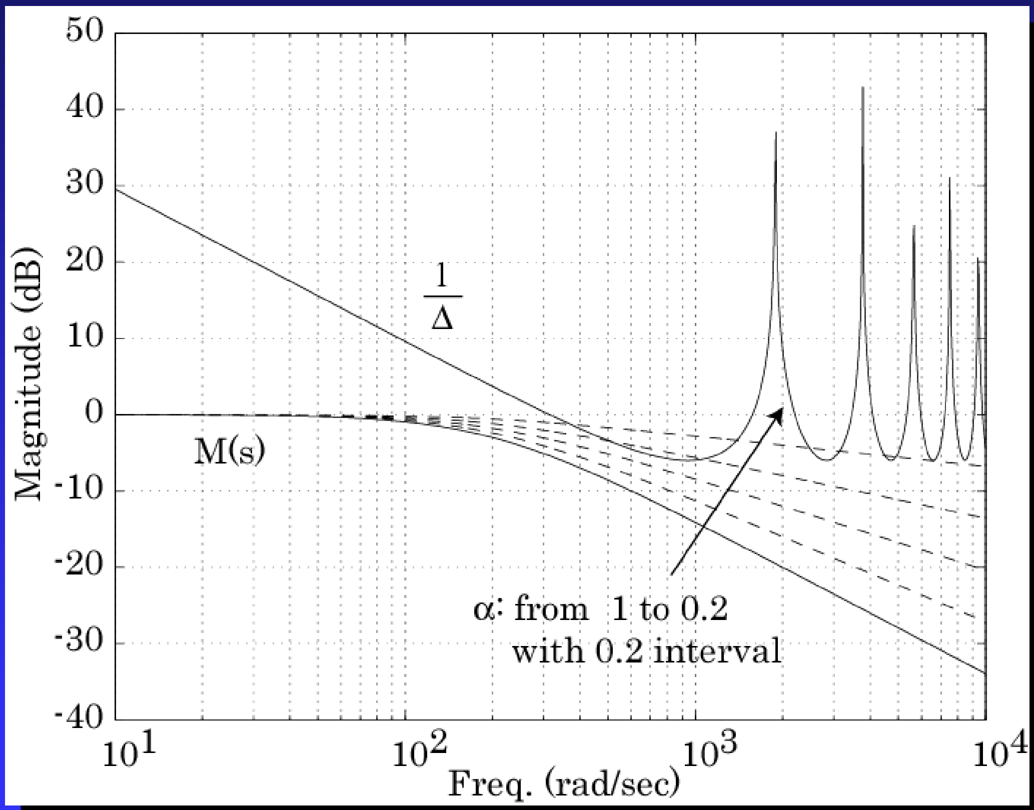






Robust stability





$$M(s) = \frac{1}{(\tau s + 1)^{\alpha}}$$

$$\Delta(s) = e^{-sT_d} - 1$$

Section 2

Riemman-Liouville definition

$$ {}_0I_t^\alpha f(t) = \int_0^t f(\tau)dg_t(\tau), \quad 0<\alpha<1 $$

$$ g_t(\tau) = \frac{1}{\Gamma(1+\alpha)}\left[t^\alpha - (t-\tau)^\alpha\right] $$

$t = nt_s$

$$ g_{nt_s}(kt_s) = \frac{\left[n^\alpha - (n-k)^\alpha\right]}{\Gamma(1+\alpha)} t_s^{\ \alpha}, \quad k = 1,\dots,n $$

Let

$$ \Delta g_{nt_s}(kt_s) = g_{nt_s}(kt_s) - g_{nt_s}[(k-1)t_s] $$

$$ \Delta g_{nt_s}(kt_s) = \frac{(n-k+1)^\alpha - (n-k)^\alpha}{\Gamma(1+\alpha)} t_s^{\ \alpha} $$

Sampling time between $f(kt_s)$

$$ T_n(n) = \frac{1^\alpha - 0^\alpha}{\Gamma(1+\alpha)} t_s^{\ \alpha} $$

$$ T_n(n-1) = \frac{2^\alpha - 1^\alpha}{\Gamma(1+\alpha)} t_s^{\ \alpha} $$

...

$$ T_n(1) = \frac{n^\alpha - (n-1)^\alpha}{\Gamma(1+\alpha)} t_s^{\ \alpha} $$

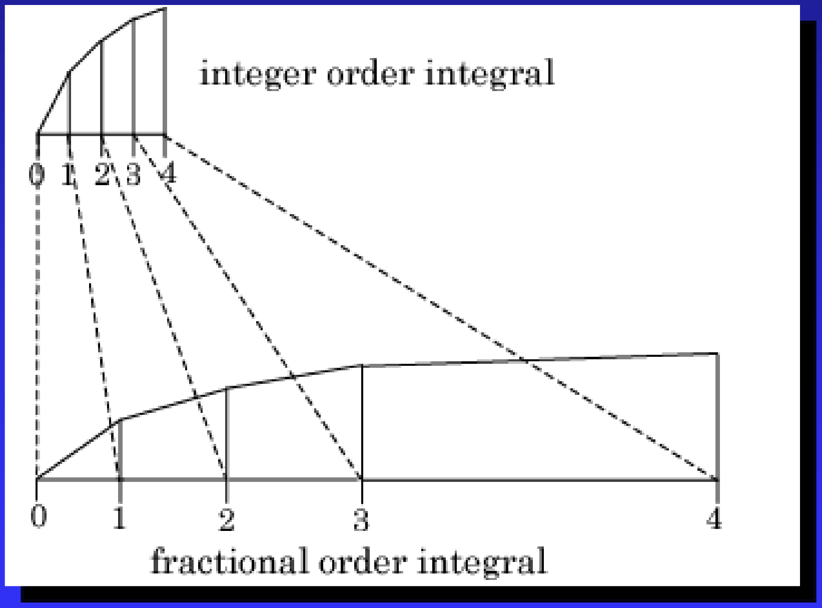
Scaled integral

- Based on the trapezoidal integration rule:

$$ {}_0 I_{nt_s}^{\alpha} \approx \sum_{k=1}^{n} \frac{f(kt_s) + f[(k-1)t_s]}{2} T_n(k) $$

and

$$ {}_0 I_{nt_s}^{\alpha} = \lim_{\substack{t_s \to 0 \\ t = nt_s}} \sum_{k=1}^{n} \frac{f(kt_s) + f[(k-1)t_s]}{2} T_n(k) $$



Derivative of scaled integral

- Riemman-Liouville definition:

$$
{}_0D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha}} d\tau, \quad 0 < \alpha < 1
$$

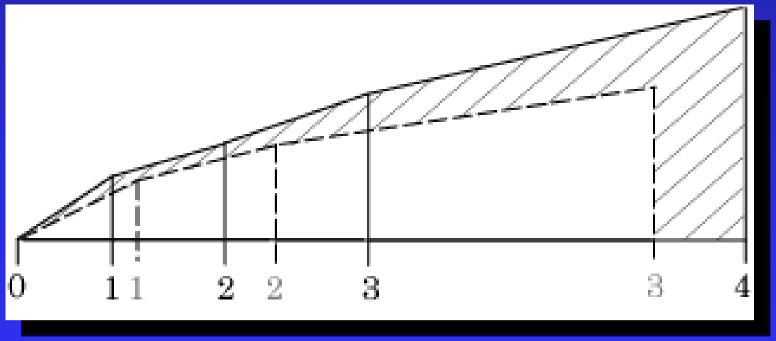
$$
= \frac{d\left[\int_0^t f(\tau) dg_t^{'}(\tau)\right]}{dt}
$$

$$
T_n^{'}(n) = \frac{1^{1-\alpha} - 0^{1-\alpha}}{\Gamma(2-\alpha)} t_s^{1-\alpha}
$$

$$
T_n^{'}(n-1) = \frac{2^{1-\alpha} - 1^{1-\alpha}}{\Gamma(2-\alpha)} t_s^{1-\alpha}
$$

$$
\cdots
$$

$$
T_n^{'}(1) = \frac{n^{1-\alpha} - (n-1)^{1-\alpha}}{\Gamma(2-\alpha)} t_s^{1-\alpha}
$$

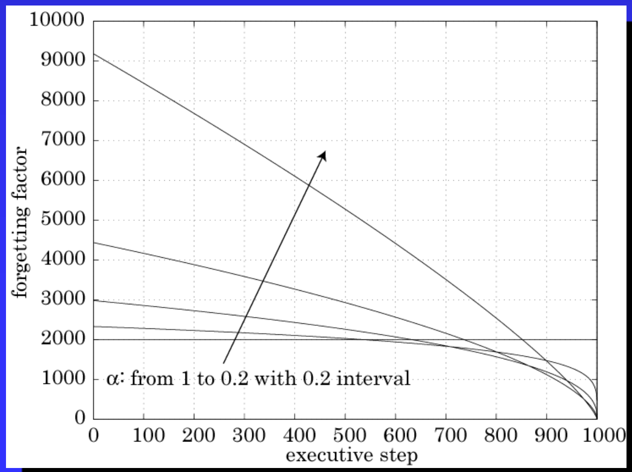


Control strategy

- Apply strong control action to latest sampled inputs:

$$u(n) = \sum_{k=1}^{n} \frac{1}{\lambda_n(k)}[e(k)+e(k-1)], \qquad \lambda_n(k) := \frac{1}{T_n(k)}$$

- The rapidly fading influences of the old values and dominance of the latest ones make FOC " <u>passively adaptive</u>" to the present changes of dynamic processes.



The forgetting factor for discrete I^α controller (sampling time = 0.001sec)

Realization by scaled sampling time

- Memorize the behavior of $x(t)$ only in "recent past":

$$ {}_{t_0}D_t^{\alpha}x(t) \approx {}_{t-L}D_t^{\alpha}x(t), \qquad t > t_0 + L $$

- Therefore:

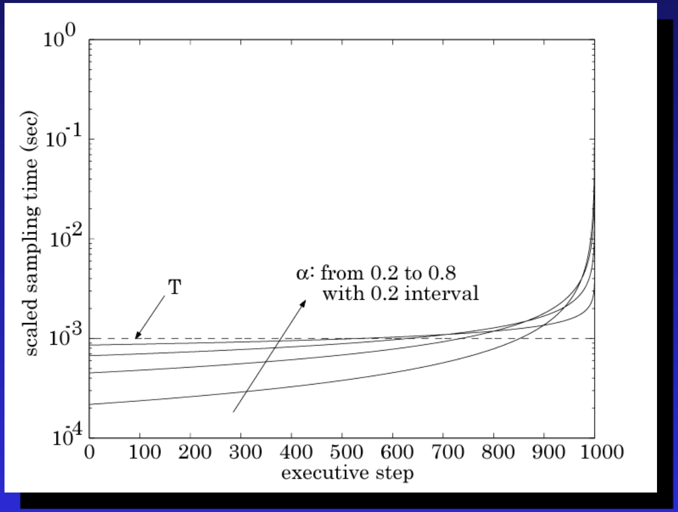
$$ Z(D^{\alpha}[x(t)]) \approx \frac{1}{T^{\alpha}} \sum_{j=0}^{[L/T]} c_j z^{-j} $$

  - integral controllers $D^{-\alpha}$:

$$ c_0 = \frac{1}{2\Gamma(1+|\alpha|)} $$

$$ c_j = \frac{(j+1)^{|\alpha|} - (j-1)^{|\alpha|}}{2\Gamma(1+|\alpha|)}, \qquad j \geq 1 $$

  - derivative controllers $D^{-\alpha}$:

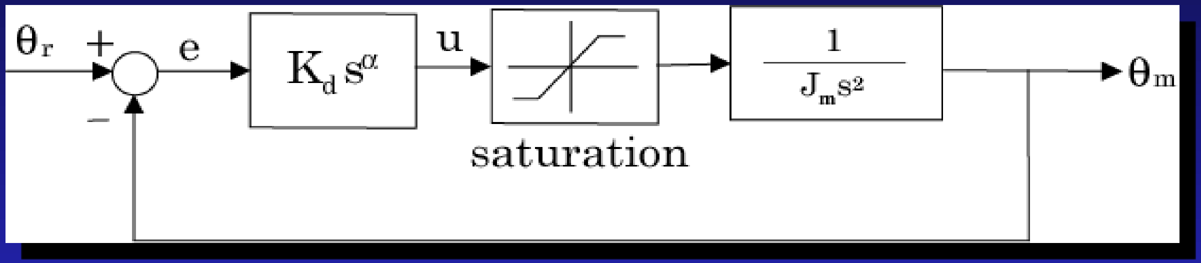


$$ c_0 = \frac{1}{2\Gamma(2-\alpha)} $$

$$ c_1 = \frac{2^{1-\alpha} - 1}{2\Gamma(2-\alpha)} $$

$$ c_j = \frac{1}{2\Gamma(2-\alpha)}[(j+1)^{1-\alpha} - j^{1-\alpha} - (j-1)^{1-\alpha} - (j-2)^{1-\alpha}], \qquad j \geq 2 $$

An example



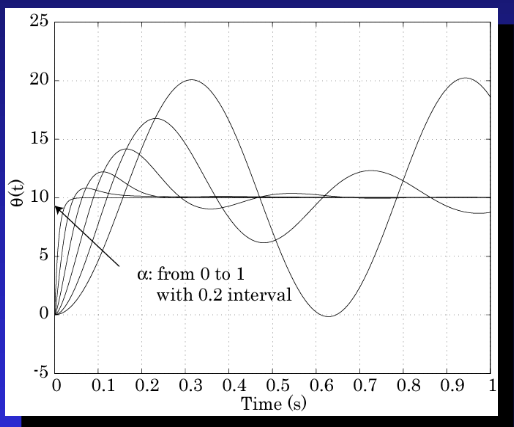
One-mass position control loop

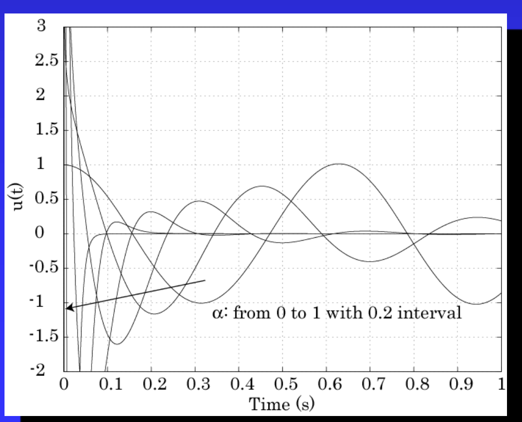
- Fractional order D^a controllers are realized by sampling time scaling method, in which whole past values will be remembered.
- The integer order D controller is discretized by the backward-difference rule:

$$Z\left\{ \frac{df(t)}{dt}\Big|_{t=kT} \right\} = Z\left\{ \frac{1}{T}\left(f(kT) - f[(k-1)T] \right) \right\} = \frac{z-1}{Tz} F(z)$$

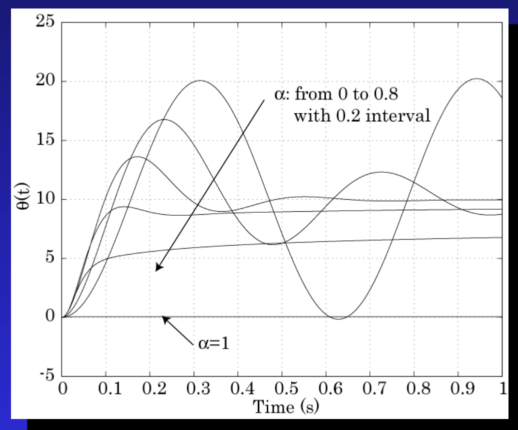
Time responses

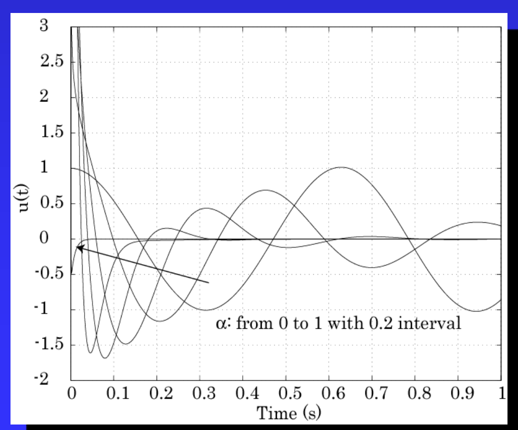
Without saturation





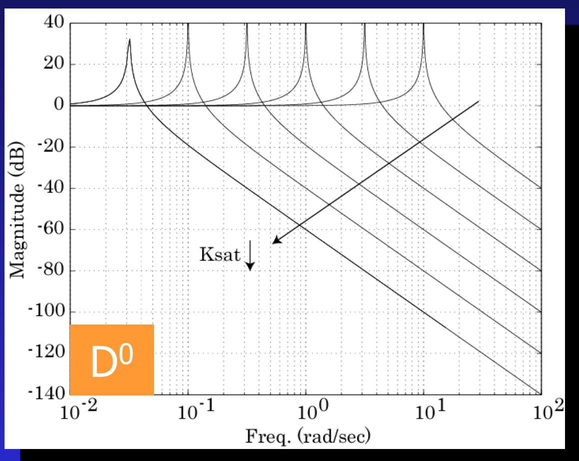
With saturation

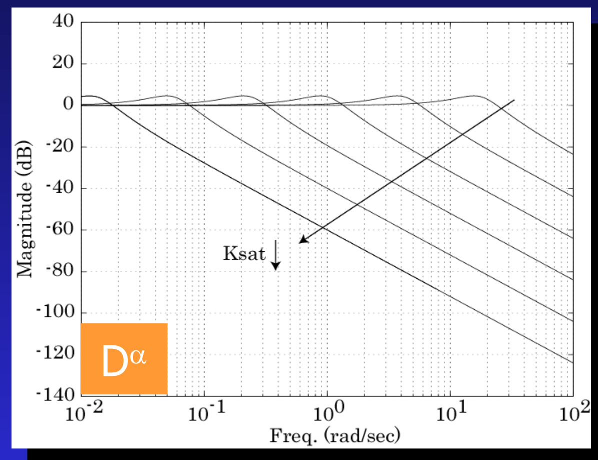


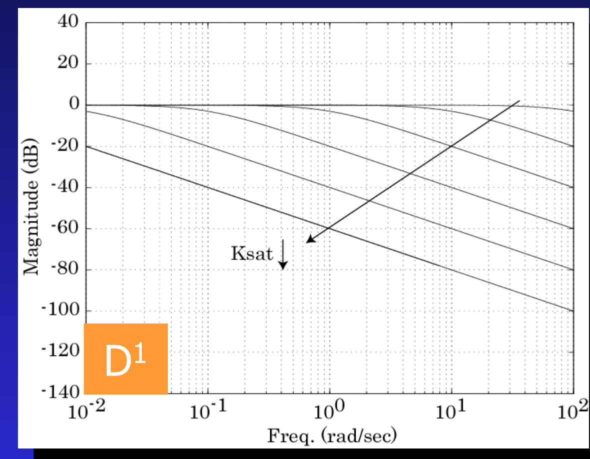


	D^0	D^α	D^1
Inputs memorized	latest one	whole	latest two
Forgetting factor	1	scaled	T
Control action	weak	middle & scaled	strong
Robustness	-	good	poor

Close-loop frequency responses







	D^0	D^α	D^1
Resonant peak	large	intermediate	small
Bandwidth BW	small	intermediate	large
Change of BW	small	intermediate	large
Cut-off rate	large	intermediate	small

Section 2

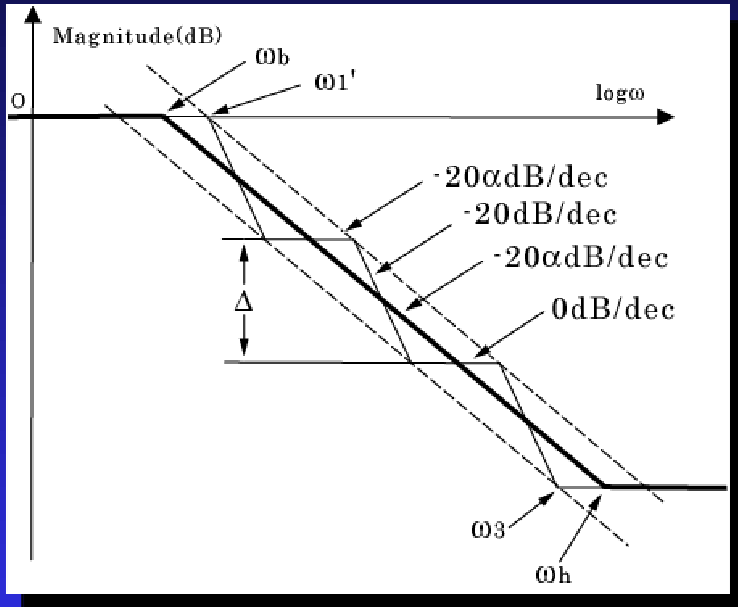
- Brief Review
- My Contributions
  - Basic attitude
  - Fractional order modeling
  - Stability determination
  - Effective gain-phase tradeoff
  - Sampling time scaling property
  - Realization issues
  - Applications
- Conclusions & future works
- Publications

Various methods

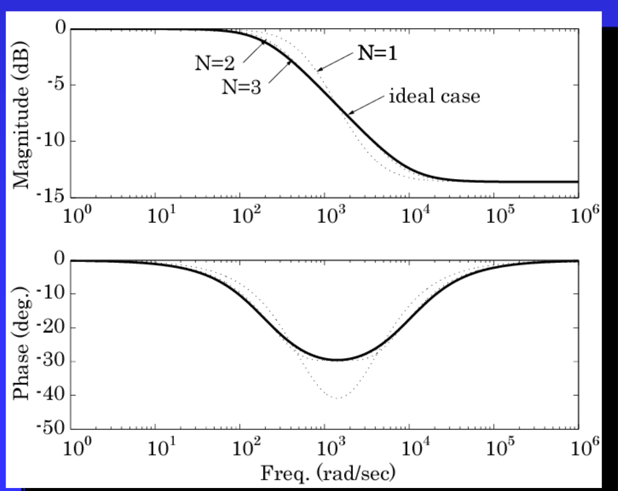
- Since the fractional order systems have infinite dimension, proper approximation by finite differential or difference equation must be introduced.
- For broken-line approximation, further discretization is needed.
- Three direct discretization approaches: Short Memory Principle, Tustin Taylor Expansion, Lagrange Function Interpolation.

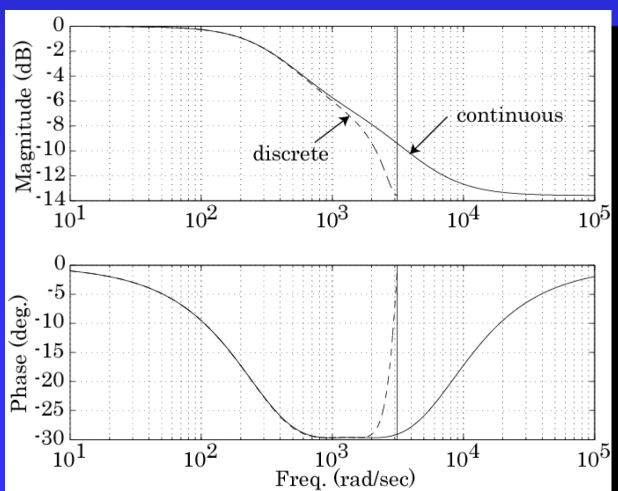
Frequency-domain approach

$$\left(\frac{\dfrac{s}{\omega_h}+1}{\dfrac{s}{\omega_b}+1}\right)^{\alpha} \approx \prod_{i=0}^{N-1} \frac{\dfrac{s}{\omega_i'}+1}{\dfrac{s}{\omega_i}+1} \Rightarrow \begin{array}{l} \omega_i = \left(\dfrac{\omega_h}{\omega_b}\right)^{\frac{i+1/2-\alpha/2}{N}} \omega_b \\ \omega_i' = \left(\dfrac{\omega_h}{\omega_b}\right)^{\frac{i+1/2+\alpha/2}{N}} \omega_b \end{array}$$



Nice approximation





$D^{-0.4}$ in [200, 10000]

$$\frac{0.2091(s+786.4471)(s+5561.0205)}{(s+359.6462)(s+2543.0828)}$$

$$\Downarrow$$

$$\frac{0.4110z^2+0.0146z-0.0843}{z^2-0.5756z-0.0831}$$

Short memory principle

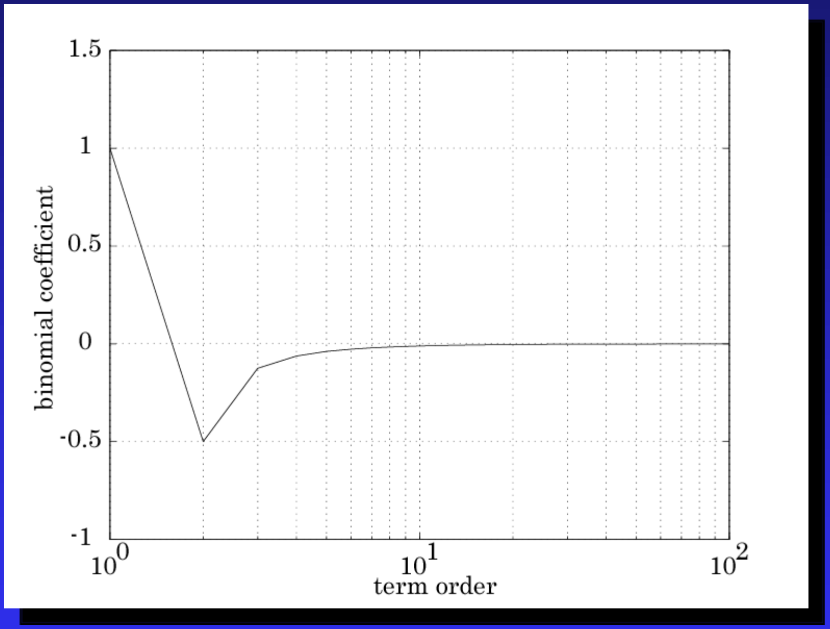
Grünwald-Letnikov Definition

$$ {}_aD_t^k = \lim_{\substack{h\to 0 \\ nh=t-a}} h^{-k} \sum_{r=0}^{n} (-1)^r \binom{k}{r} f(t-rh), $$

$$ Z\left(s^k\right) \approx T^{-k} \sum_{j=0}^{\infty} c_j^k z^{-j} $$

Binomial coefficient

$$ c_j^k = (-1)^k \binom{k}{j}, \quad c_0^k = 1 $$



Only recent past remembered

$$ {}_{t_0}D_t^k[f(t)] \approx {}_{t-L}D_t^k[f(t)] $$

Tustin Taylor expansion

- Taylor expansion of fractional *k* order well-known Tustin operator:

$$Z(s^k) \approx \left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right)^k \longrightarrow \left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right)^k = \frac{1}{T^k}\sum_{j=0}^{\infty} c_j z^{-j}$$

$$where \quad c_j = \frac{2^k}{j!}\left[\left(\frac{1-x}{1+x}\right)^k\right]^{(j)}\Big|_{x=0}$$

- For implementation, truncation is needed:

$$\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right)^k = \frac{1}{T^k}\sum_{j=0}^{m} c_j z^{-j}$$

Lagrange function interpolation

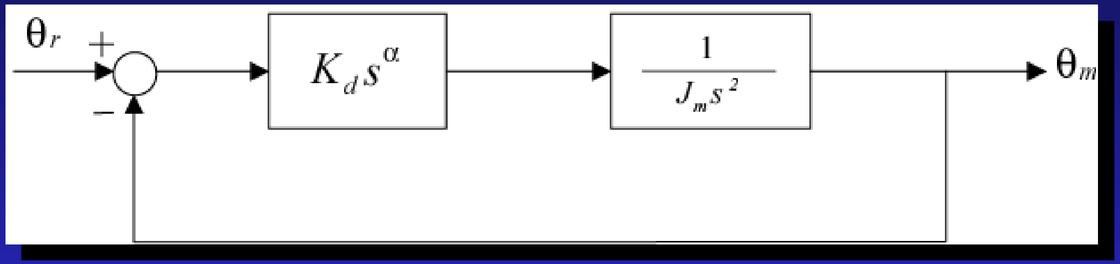
- Quadratic Lagrange interpolation between $x(k\text{-}2)$, $x(k\text{-}1)$ and $x(k)$:

$$x(t)=\frac{x(k)-2x(k-1)+x(k-2)}{2}\left(\frac{t}{T}\right)^2-\frac{x(k)-4x(k-1)+3x(k-2)}{2}\frac{t}{T}$$
$$+\quad x(k-2)$$

- k order derivative of t^n: ${}_0D_t^k(t^n)=\dfrac{n!t^{n-k}}{\Gamma(n-k+1)}$
- For t=2T, the z-transformation is:

$$Z(s^k)=\frac{1}{T^k}\frac{1}{2^k\Gamma(3-k)}\left[(2+k)-4kz^{-1}+k^2z^{-2}\right]$$

SMP best



$$J_m = 0.001$$
$$K_d = 0.01$$
$$T_s = 0.001\text{sec}$$

- Tustin Taylor expansion and Lagrange function interpolation are not reliable.
- Sampling time scaling and short memory principle have similar performances.
- Short memory principle method is most practically superior due to its simple algorithm. Take memory length 100 should give good approximation. In real application, 10 is also acceptable.

Examples for SMP and STS

$$Z_{SMP}\{s^{0.4}\} = 15.8489 - 6.3396z^{-1} - 1.9019z^{-2} - 1.0143z^{-3} - 0.6593z^{-4} - 0.4747z^{-5} - 0.3639z^{-6} - 0.2912z^{-7} - 0.2402z^{-8} - 0.2028z^{-9}$$

$$Z_{STS}\{s^{0.4}\} = 8.8689 + 4.5738z^{-1} - 5.1664z^{-2} - 1.3436z^{-3} - 0.7834z^{-4} - 0.5373z^{-5} - 0.4008z^{-6} - 0.3150z^{-7} - 0.2567z^{-8} - 0.2149z^{-9}$$

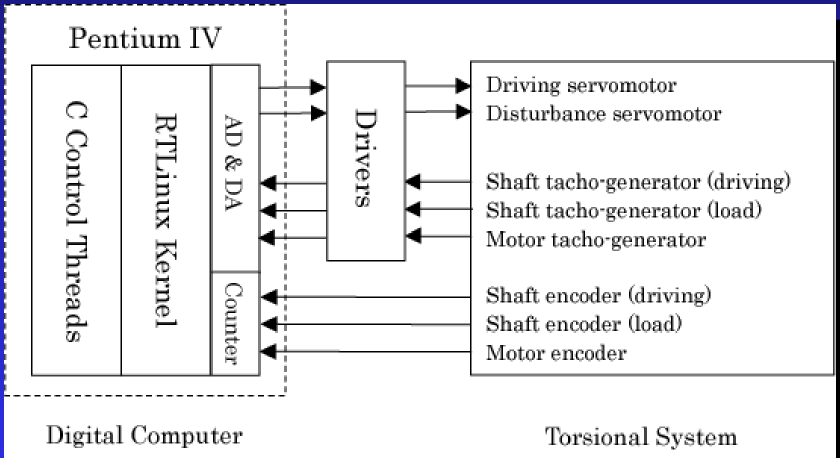
- The 10 latest sampled inputs are memorized. The approximated fractional order controllers can be easily realized by computer program.
- In sampling time scaling method, gamma function $\Gamma(x)$ needs to be calculated. The short memory principle's algorithm is much easier. Only the four basic operations of arithmetic are needed.

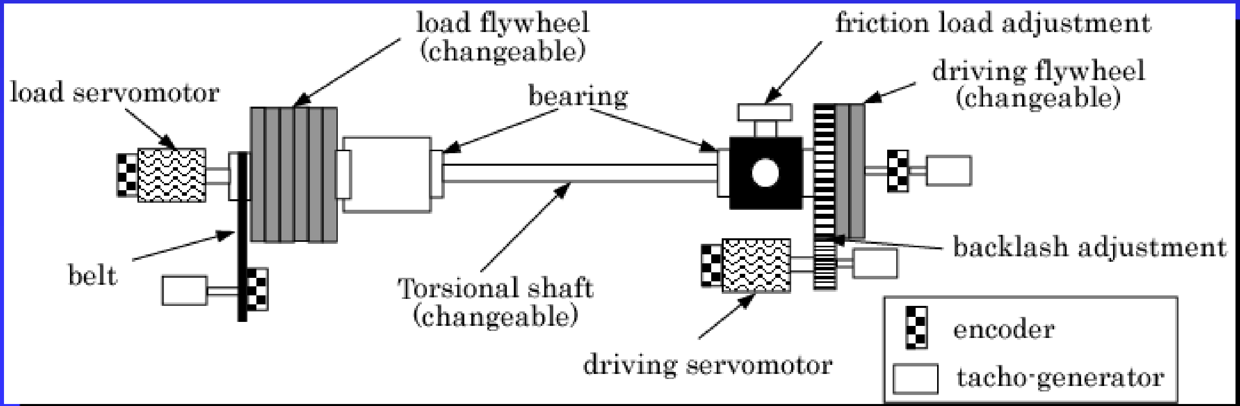
Section 2

- Brief Review
- My Contributions
  - Basic attitude
  - Fractional order modeling
  - Stability determination
  - Effective gain-phase tradeoff
  - Sampling time scaling property
  - Realization issues
  - Applications
- Conclusions & future works
- Publications

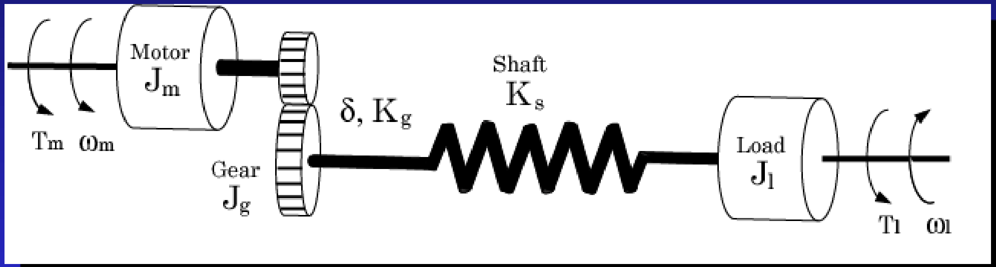
The experimental setup

redhat

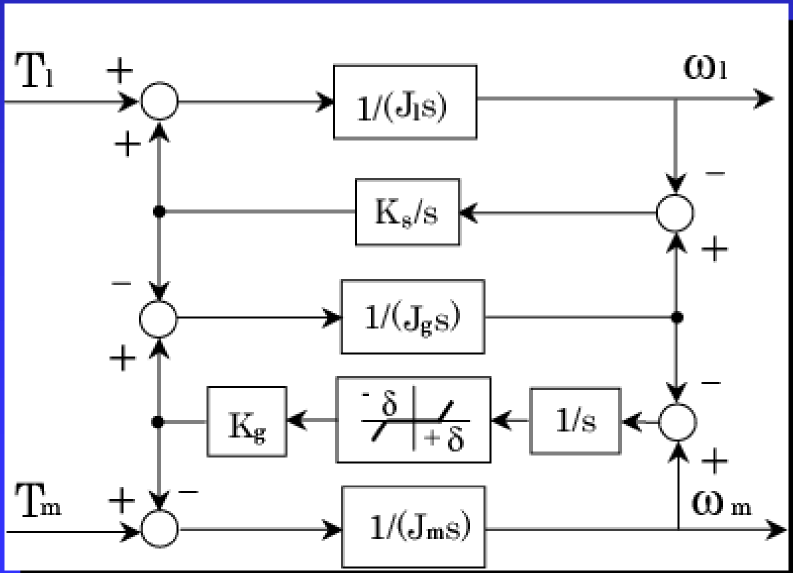


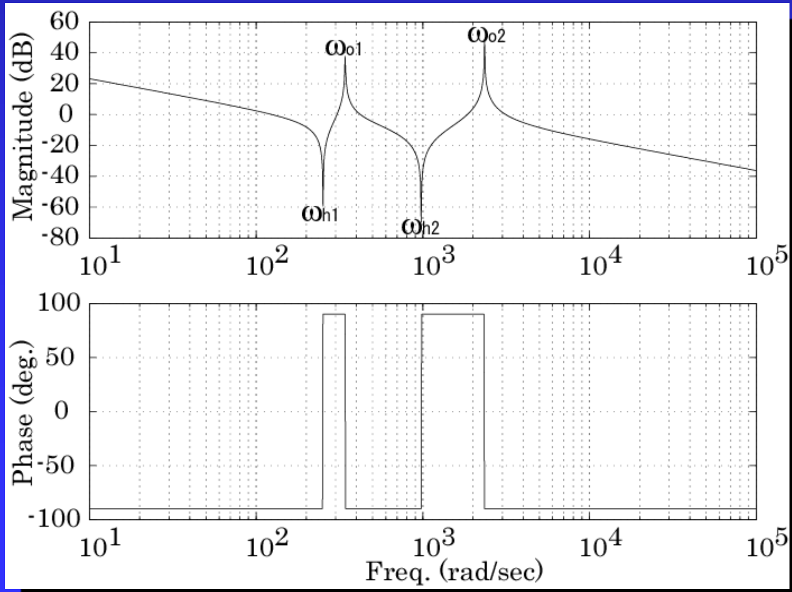


Three-mass Model



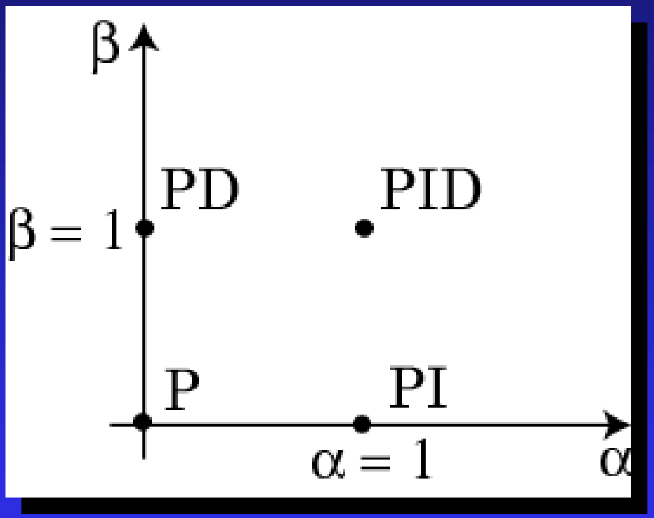
$$G(s) = \frac{(s^2+\omega_{h1}^2)(s^2+\omega_{h2}^2)}{J_m s(s^2+\omega_{o1}^2)(s^2+\omega_{o2}^2)}$$

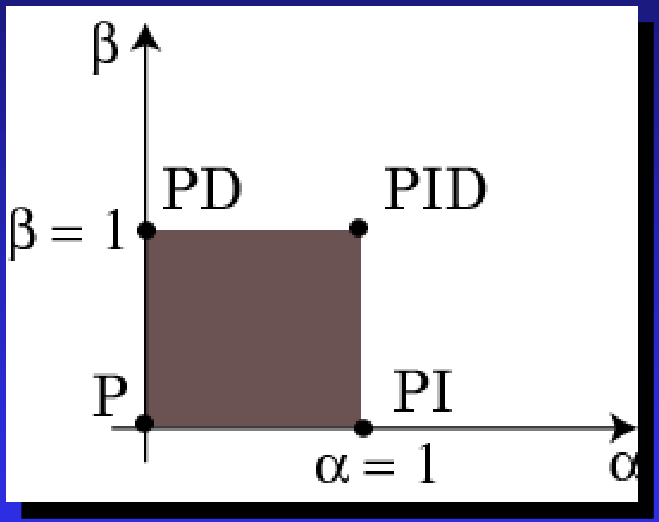




Fractional order PI$^{\alpha}$D$^{\beta}$ controller

Point to plane

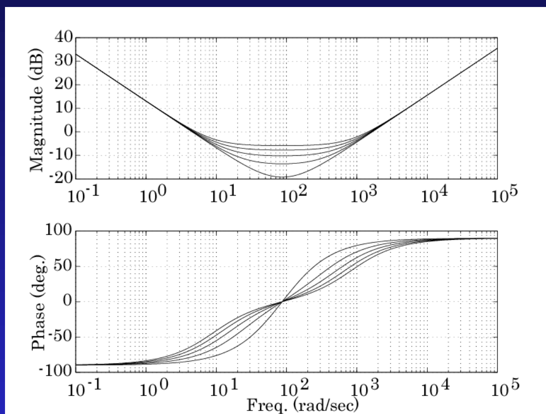




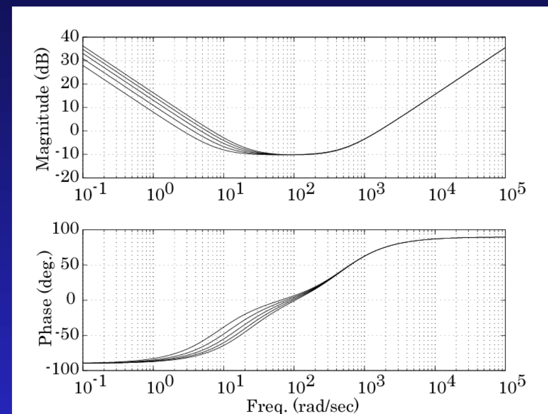
$$\mathrm{C(s)=K_p} + \frac{K_i}{s} + K_d s$$

$$\mathrm{C(s)=K_p} + \frac{K_i}{s^{\alpha}} + K_d s^{\beta}$$

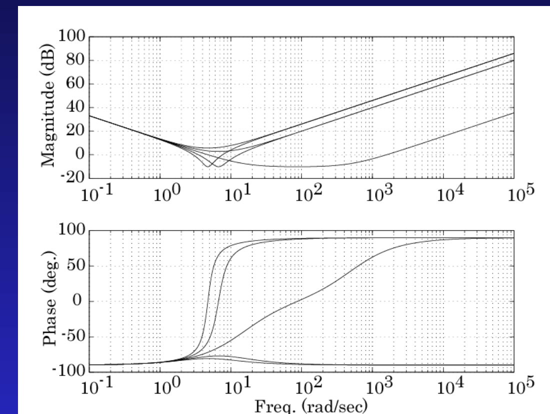
More effective and predictable



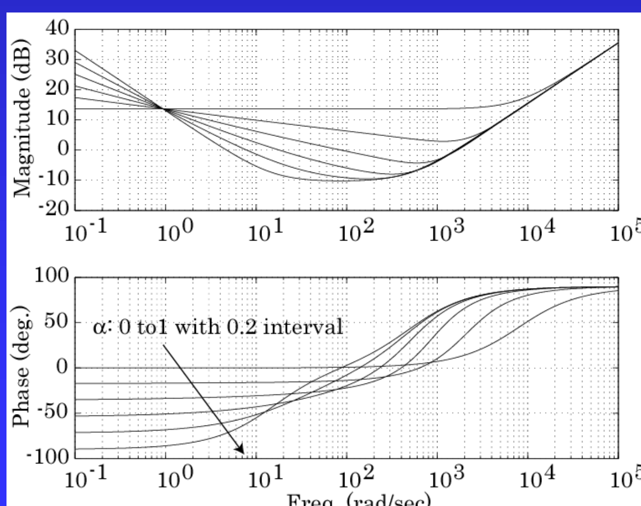
adjust K_p



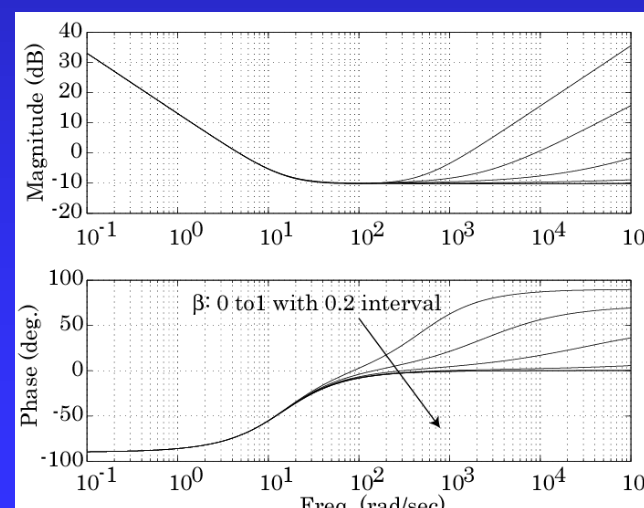
adjust K_i



adjust K_d



adjust α



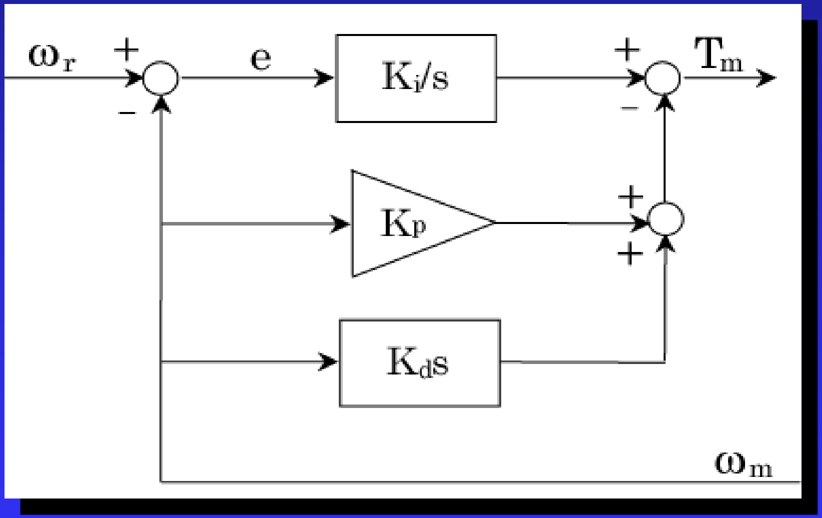
adjust β

My practices on $PI^{\alpha}D^{\beta}$ control

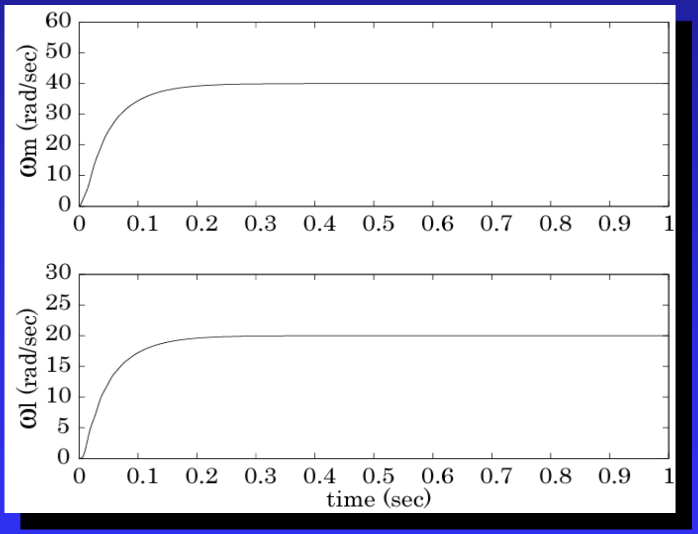
- I^{α} controller for one-mass robust speed control (ASME paper)
- $PI^{\alpha}D$ controller for two-mass robust speed control (IPEMC paper)
- PID^{β} controller for the vibration suppression control of torsional system (IEEJ and ACC papers)

Integer order PID controller

- Design PID controller with simplified two-mass model which neglects the backlash between gears.



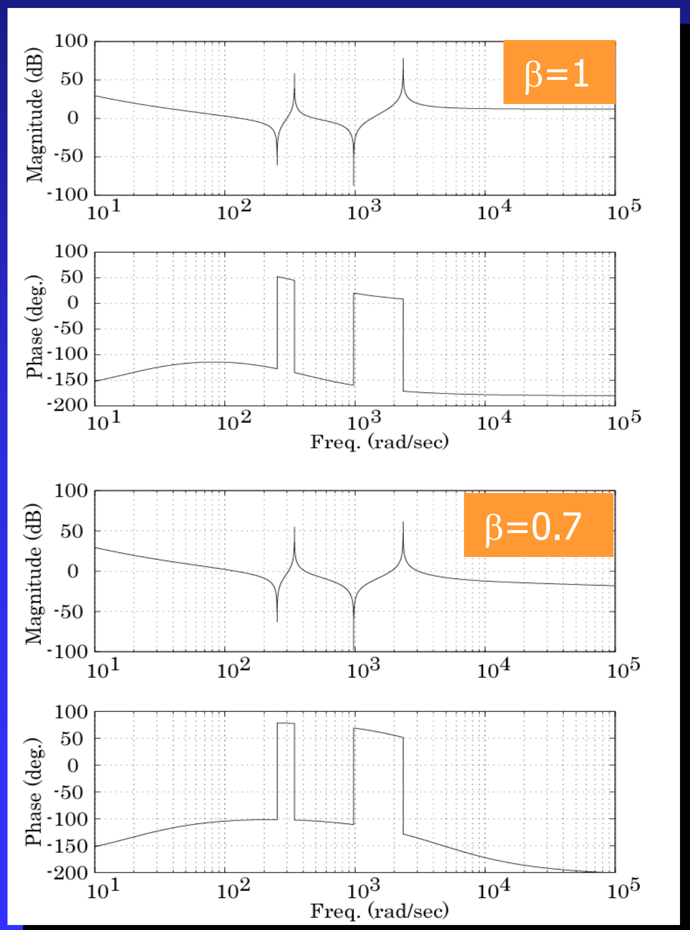
Setpoint-I PID controller

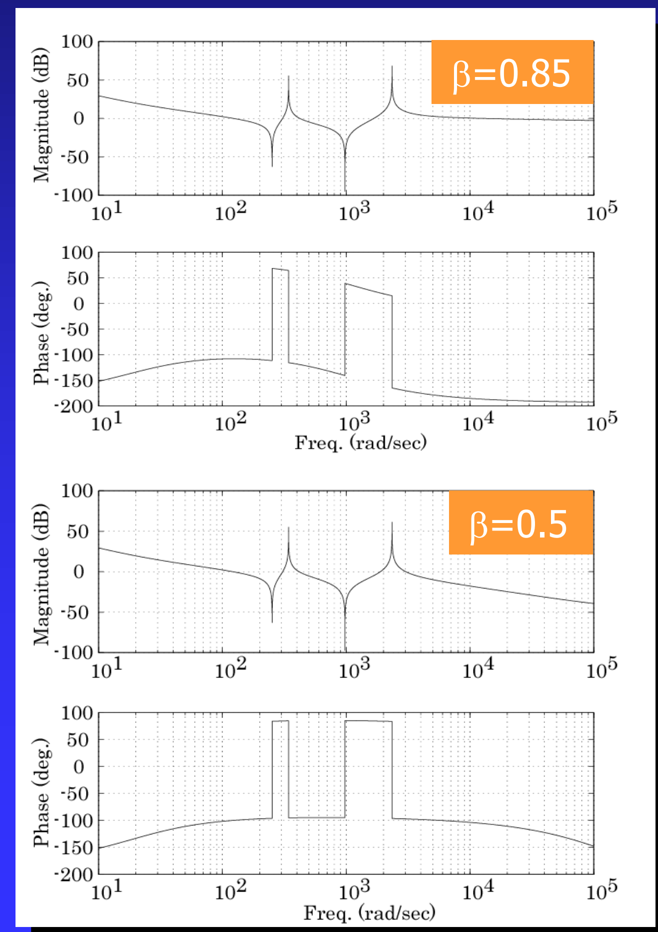


Simulation results

Introduce fractional D^{β}

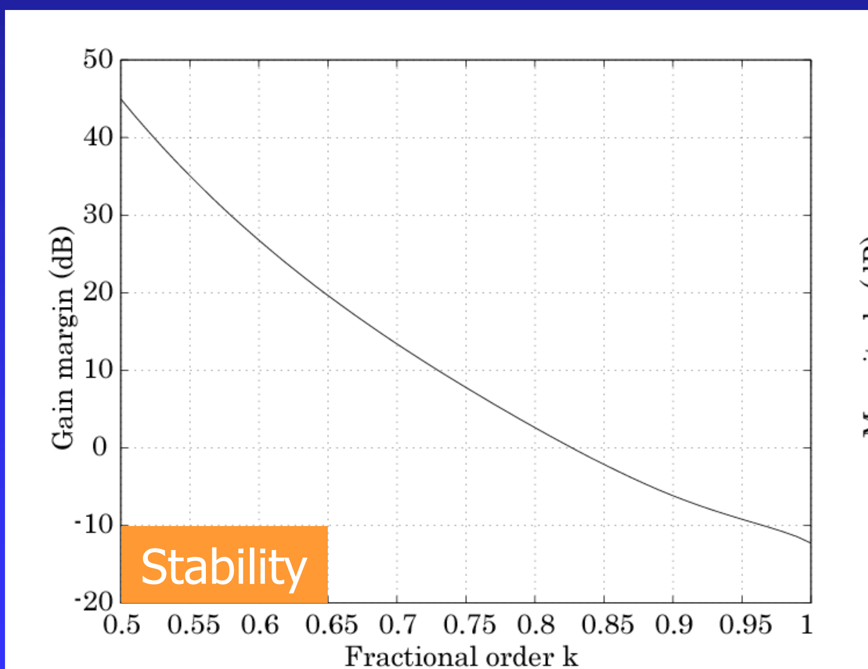
- Open-loop Bode plot with three-mass model



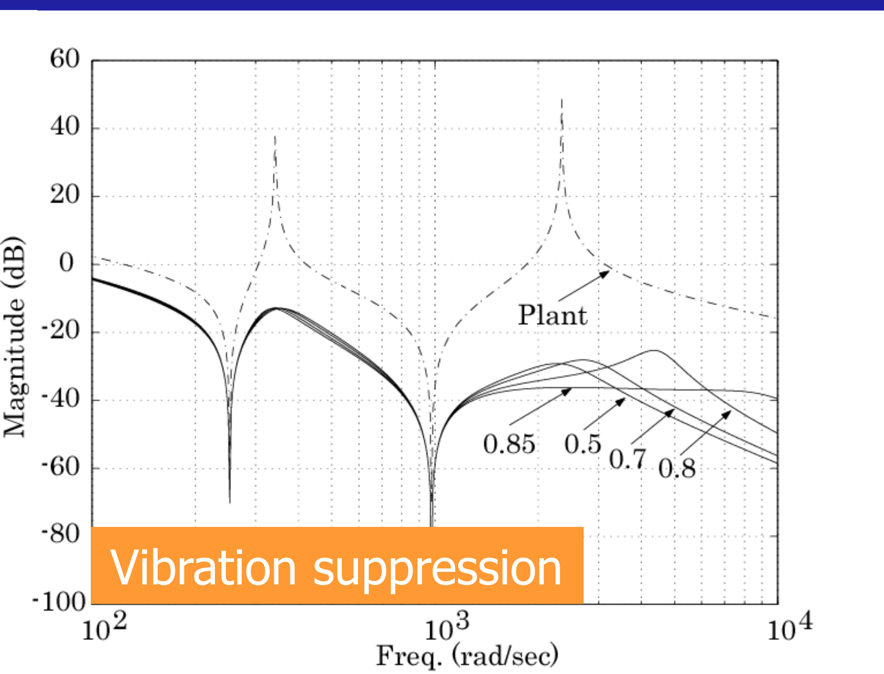


Tradeoff exists

- Tradeoff between gain margin loss and backlash vibration suppression strength exists.



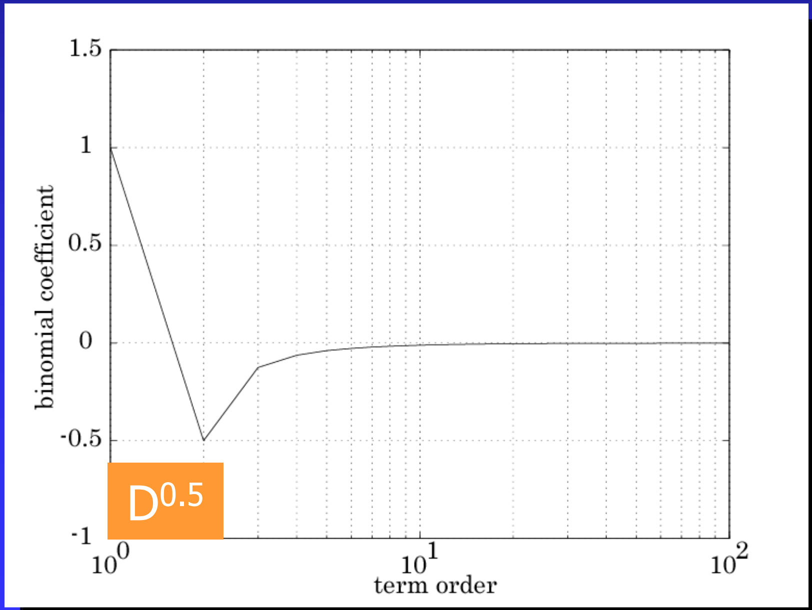
Gain margin vs β



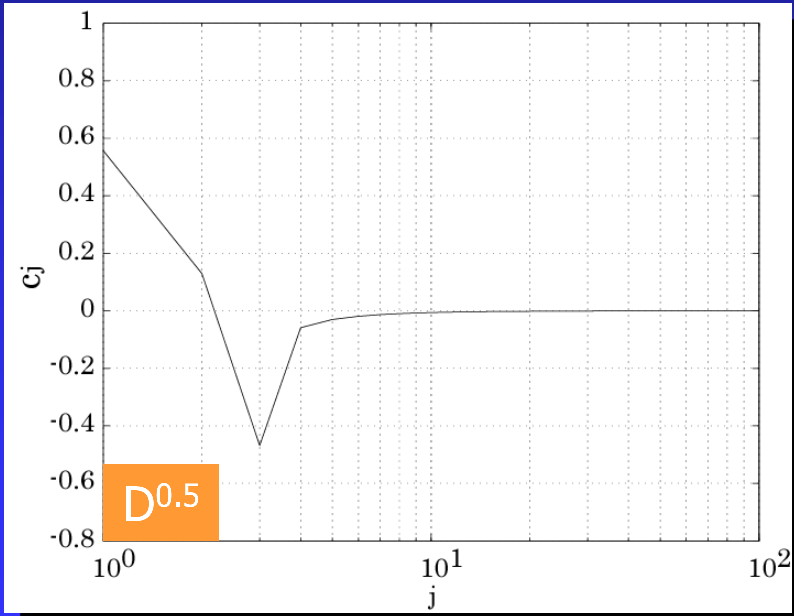
Open-loop Bode plots

Discrete realization

- For comparison, SMP and STS are both introduced. Memorizing 10 past values should be reasonable.



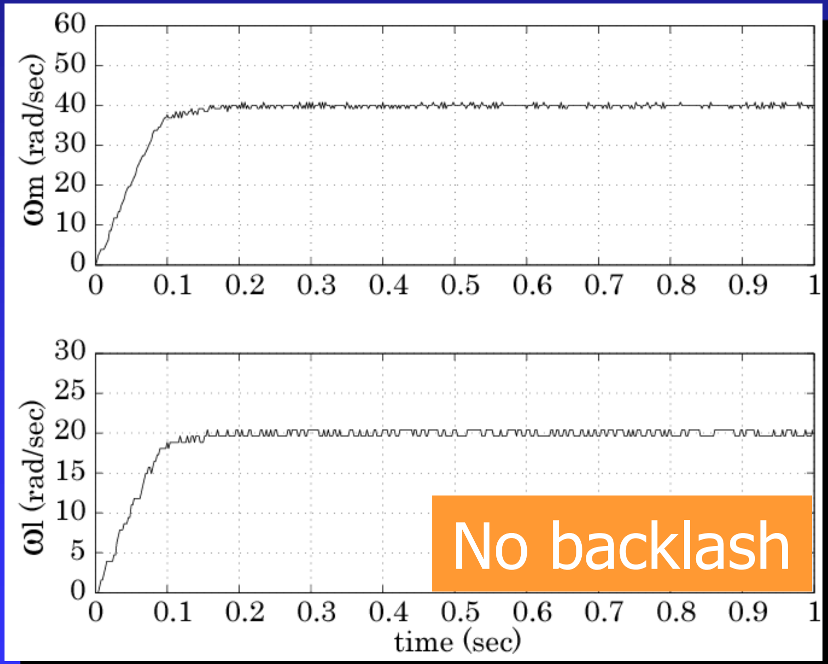
Short memory principle

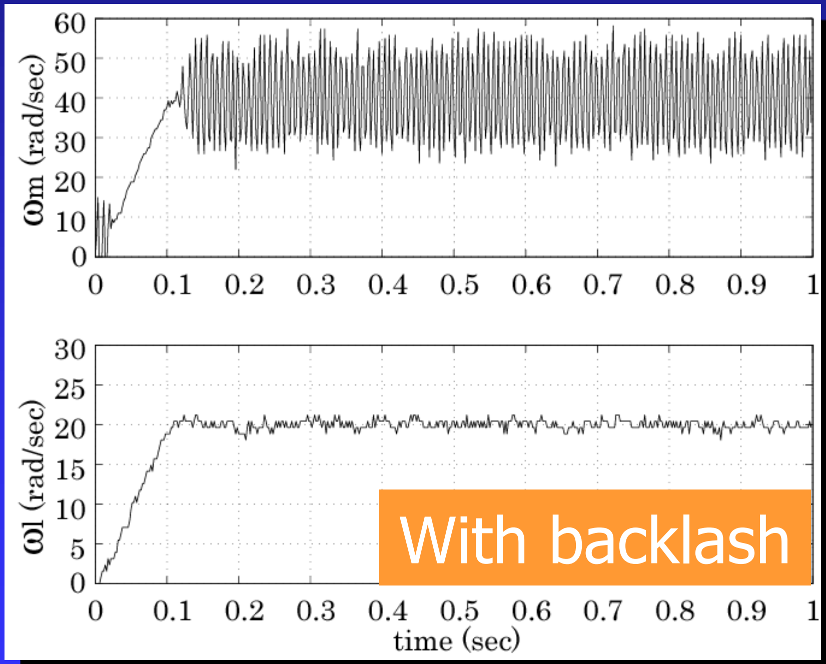


Sampling time scaling

Experiment: PID control

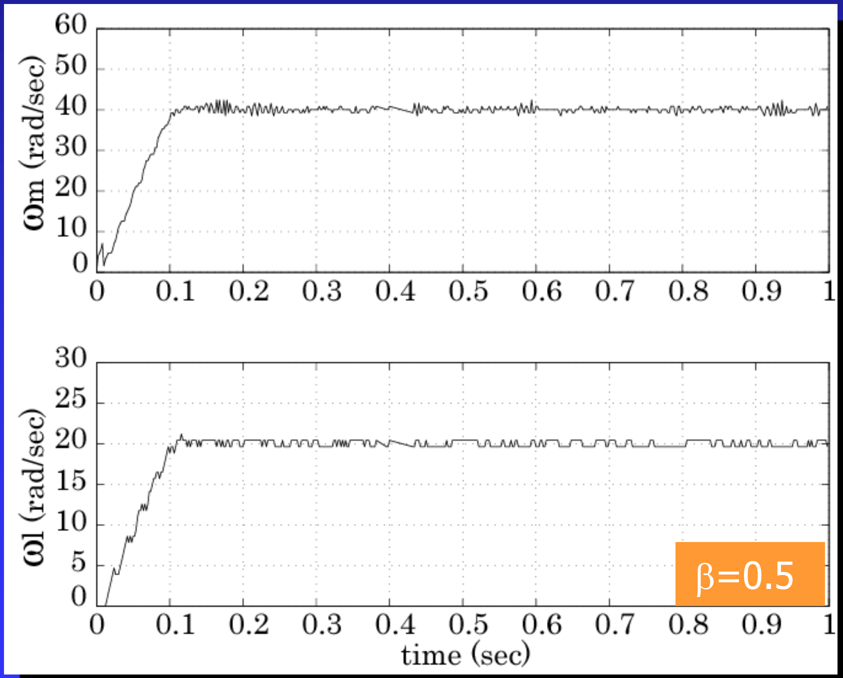
- Severe backlash vibration occurs, consistent with the analysis. PID control system is unstable.

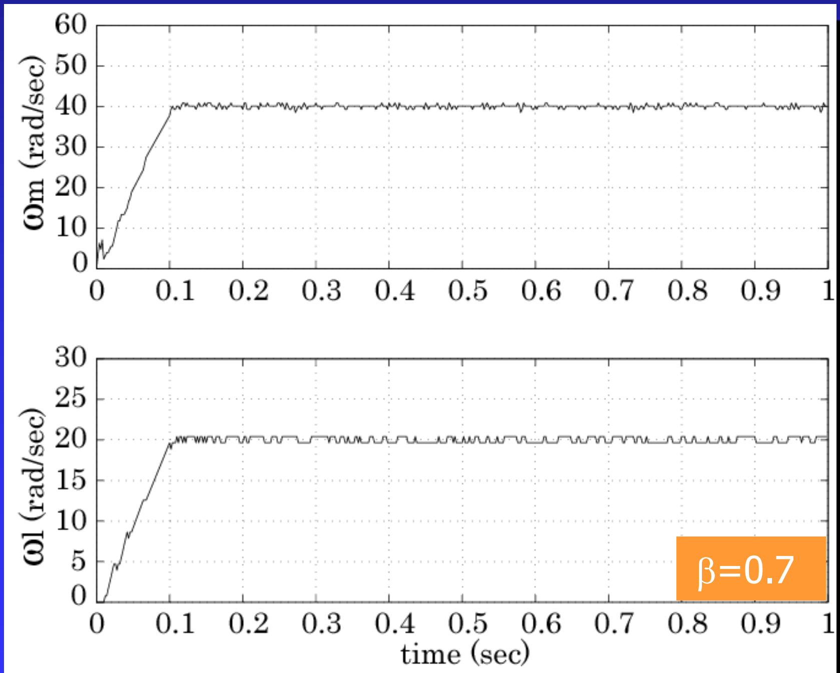




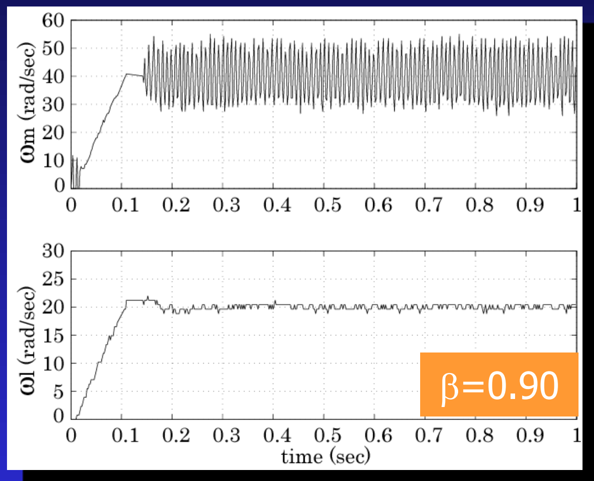
Experiment: PID$^{\beta}$ control

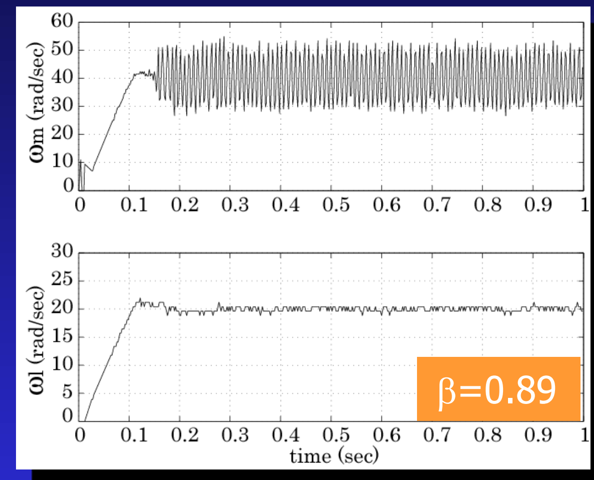
- Short memory principle: memory length 0.01sec (10 past values)

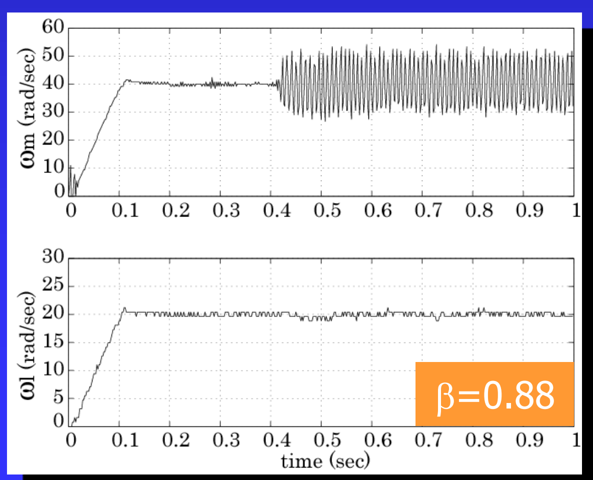


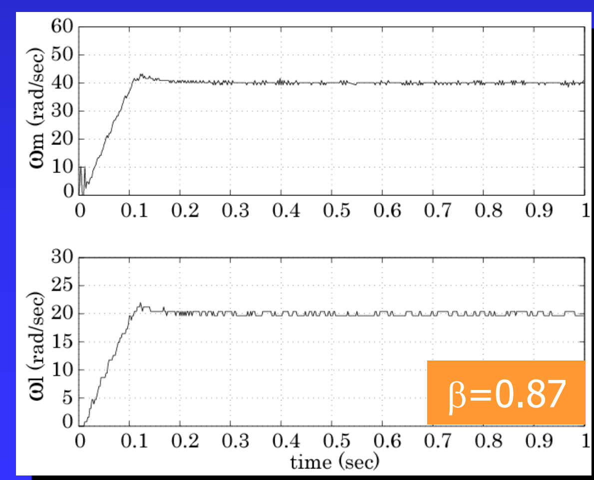


Experiment: interesting continuity

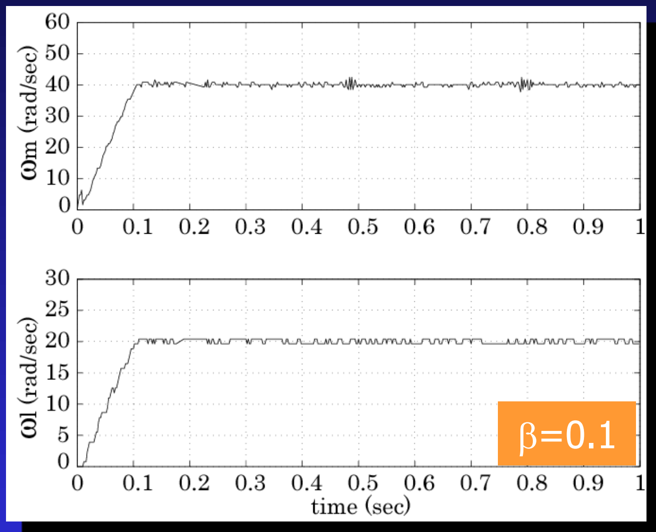


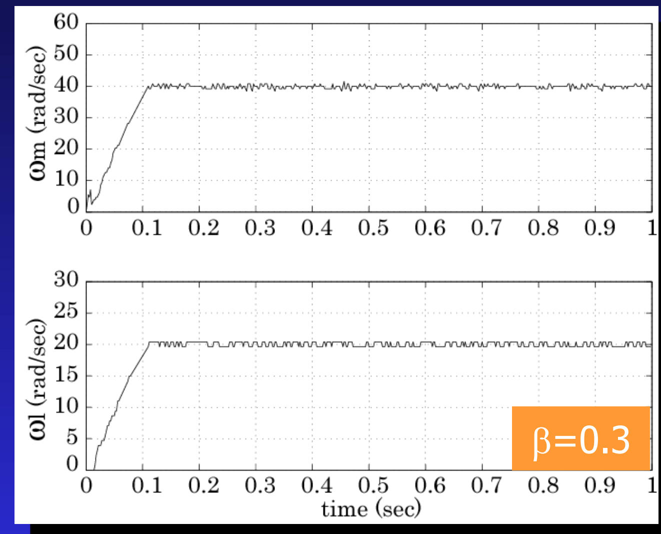


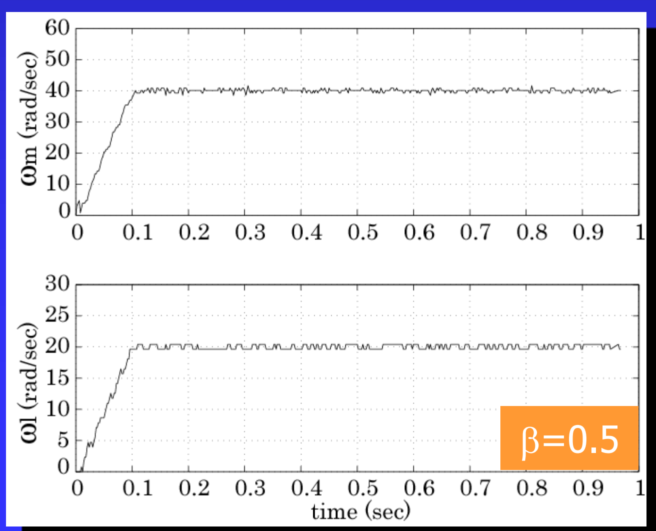


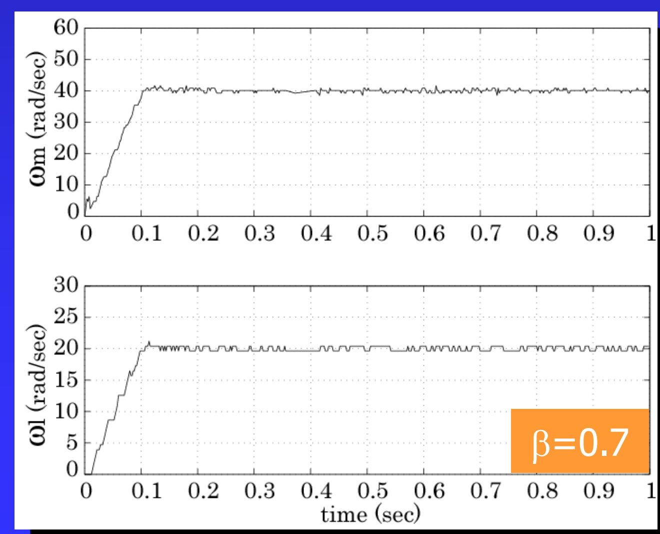


Experiment: STS method (m=10)







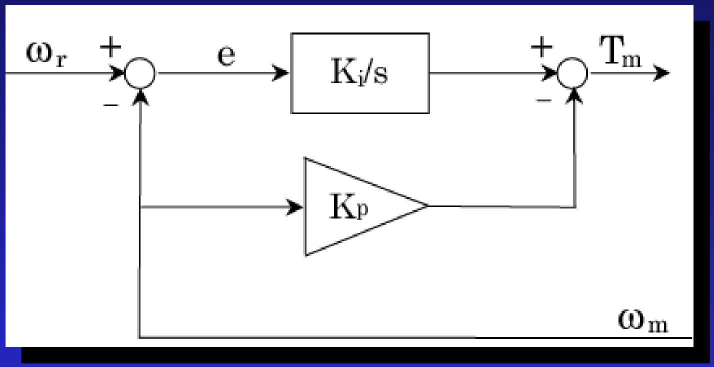


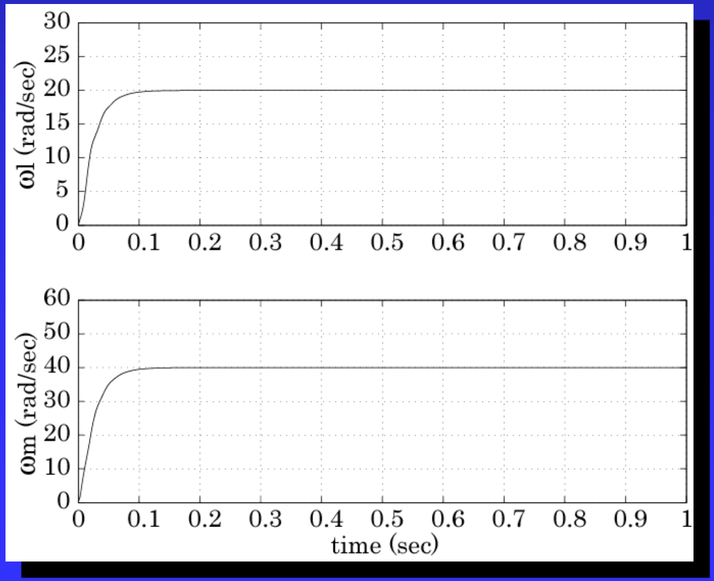
Fractional order filter

A general solution

- Tradeoff between stability margin loss and strength of vibration suppression is a common and natural problem in oscillatory system control.
- By introducing fractional order low-pass filter 1/(Ts+1)^a, this tradeoff can be adjusted directly and continuously.
- PI control with 1/(Ts+1)^a filter is proposed as a general solution with an experimental verification.

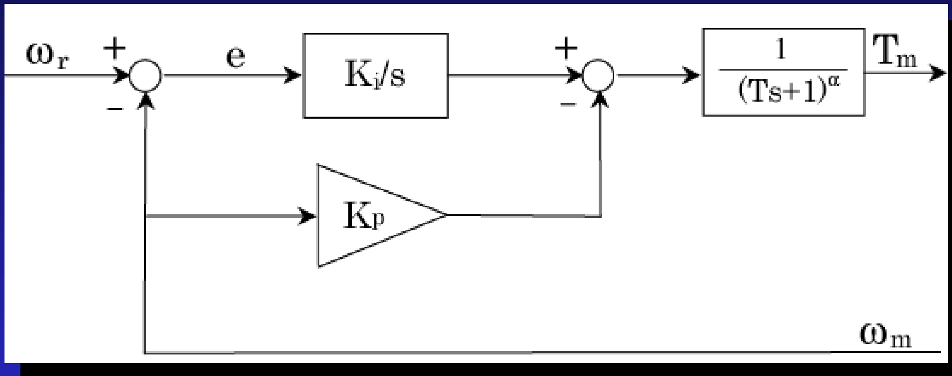
PI Control

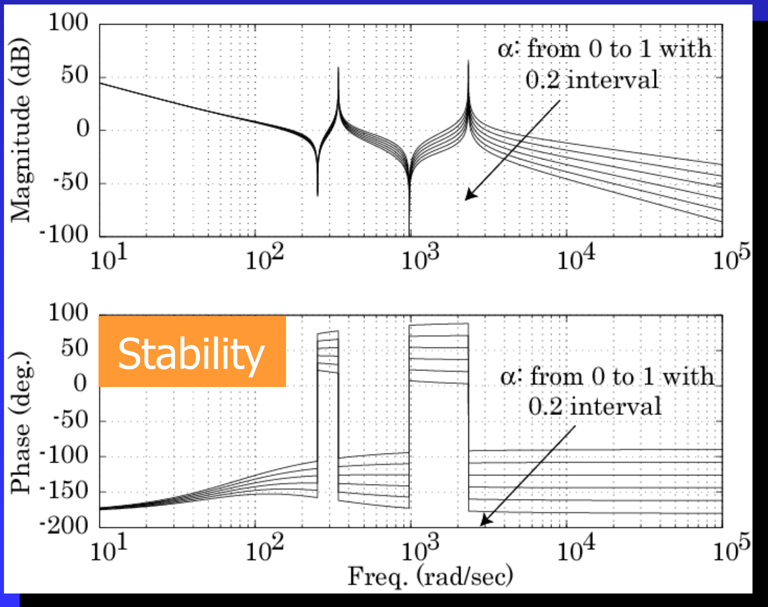


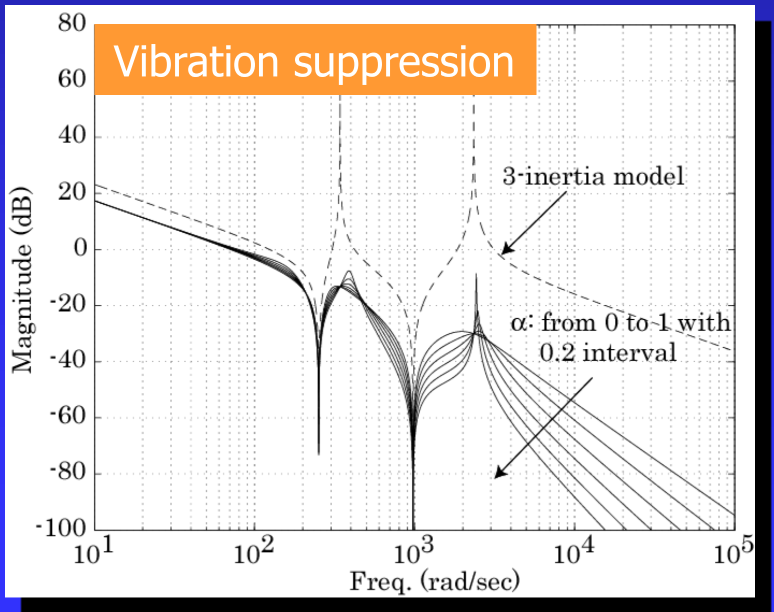


- The PI-only control has a satisfactory performance in simulation with nominal plant model.
- In torsional system's control, suppressing vibration, especially caused by gear backlash must be considered.
- In order to have a good vibration suppression, additional factors with negative slope and phase-lag are needed.

Direct tradeoff adjustment

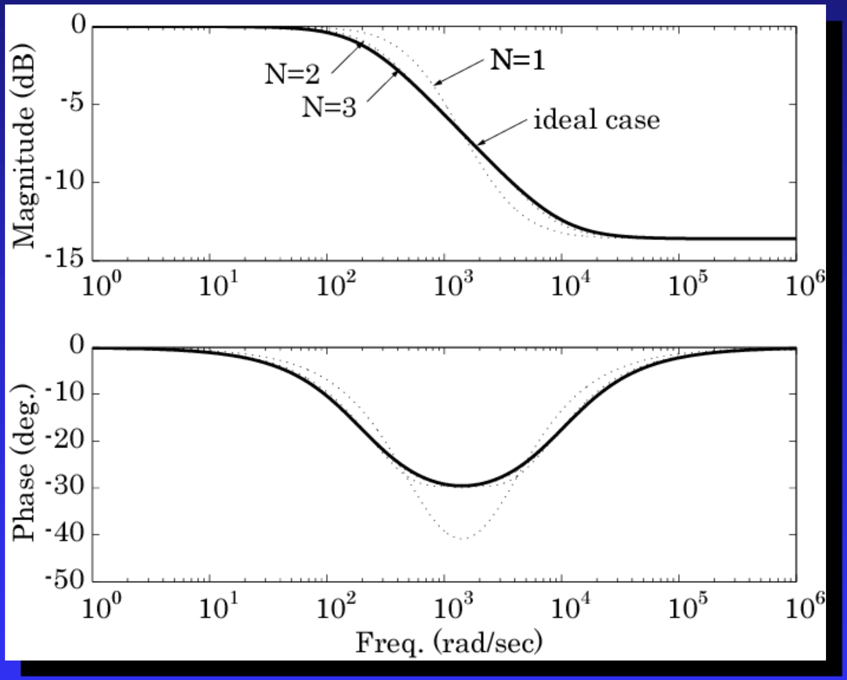






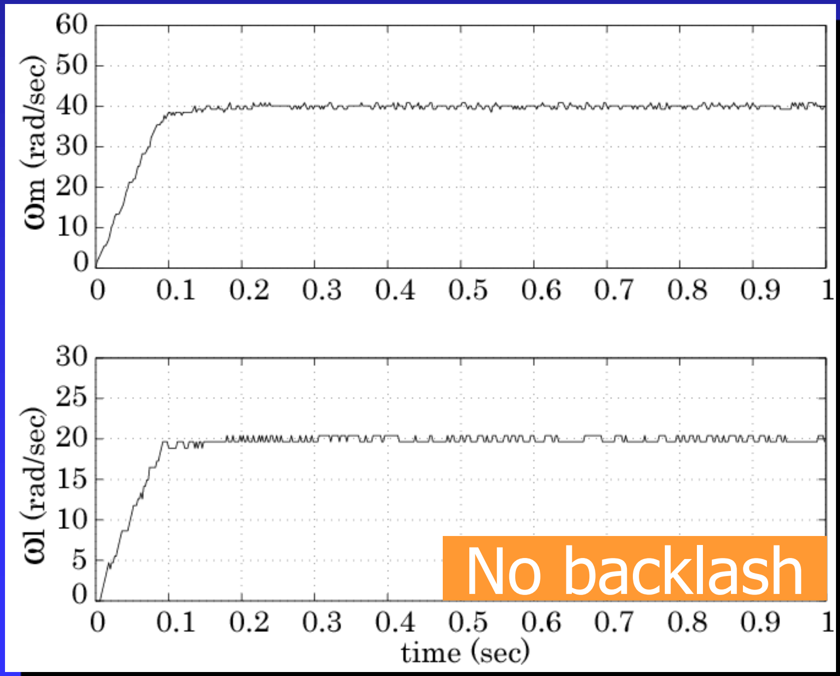
Broken-line realization

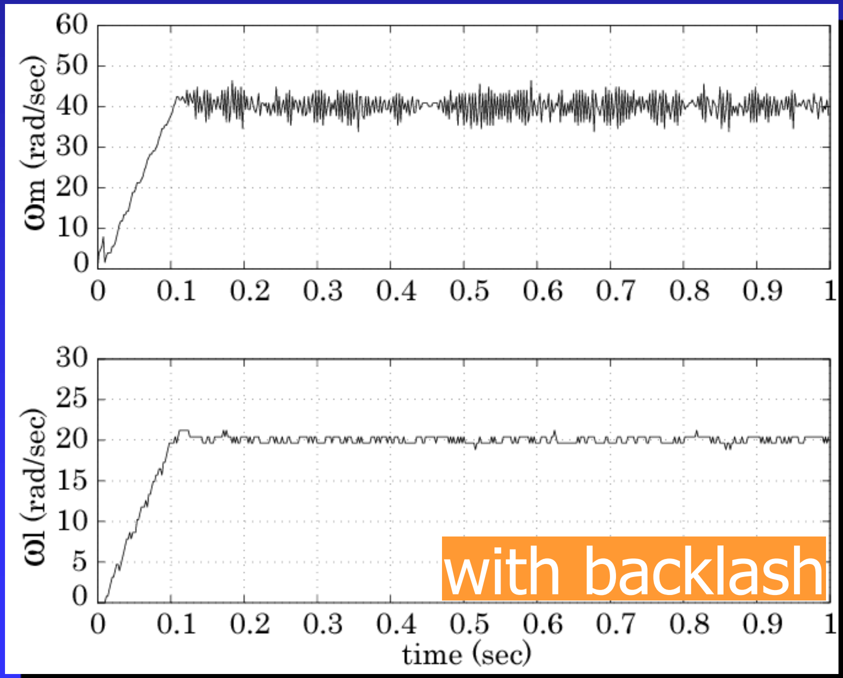
- To give control system enough band width for a fast time response, $T=1/w_b=1/200$.
- Approximate $1/(Ts+1)$^a in frequency range $[w_b, w_h]$. (w_h=10,000)
- Even taking 2nd approximation can give a good frequency response.



Experiment: PI-only

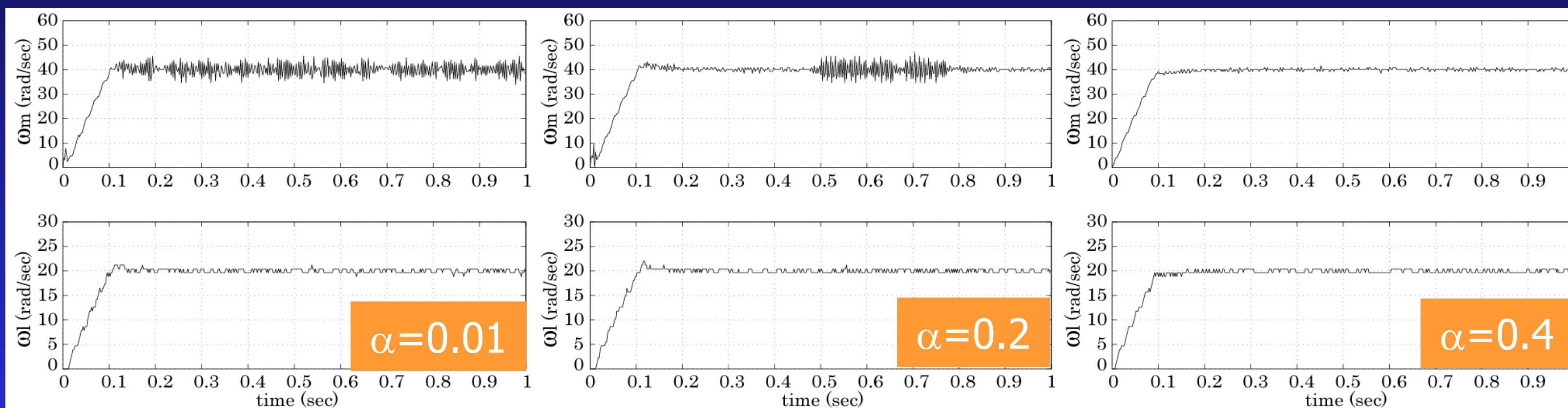
- Poor vibration suppression performance while system is still stable.



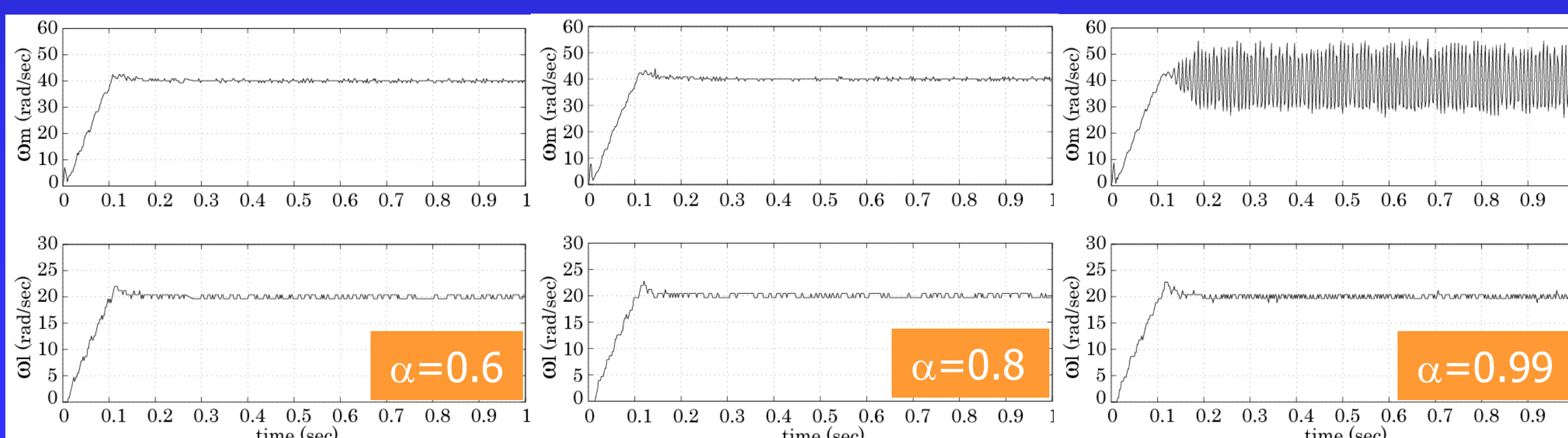


Experiment: continuous tradeoff adjustment

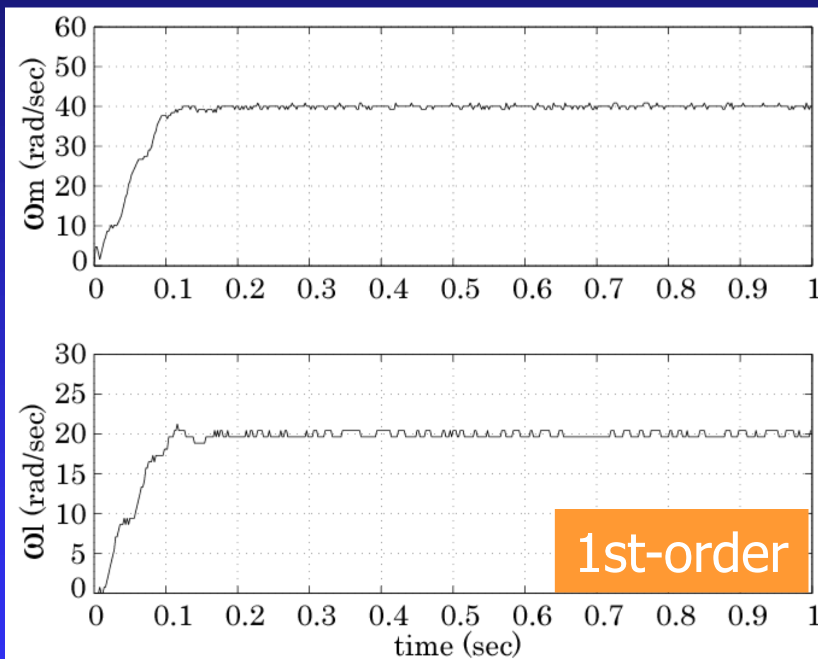
← Poor vibration suppression

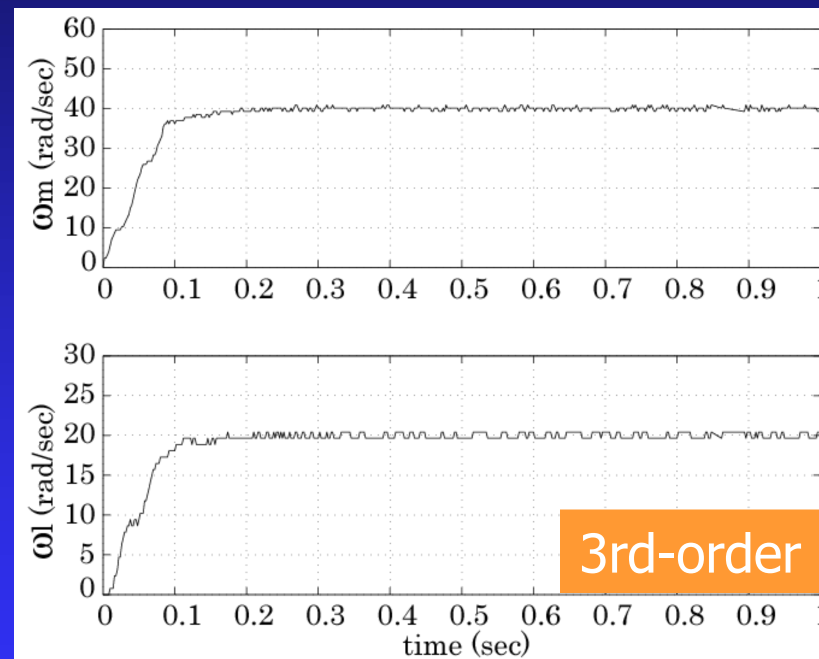


Poor stability →



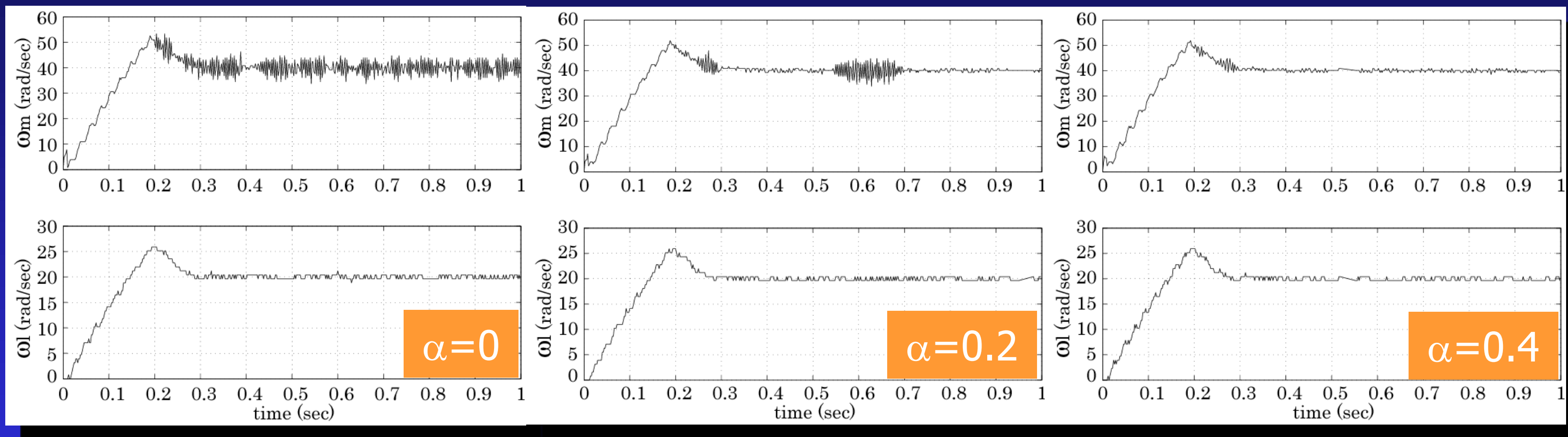
Experiment: 1st approximation worked!

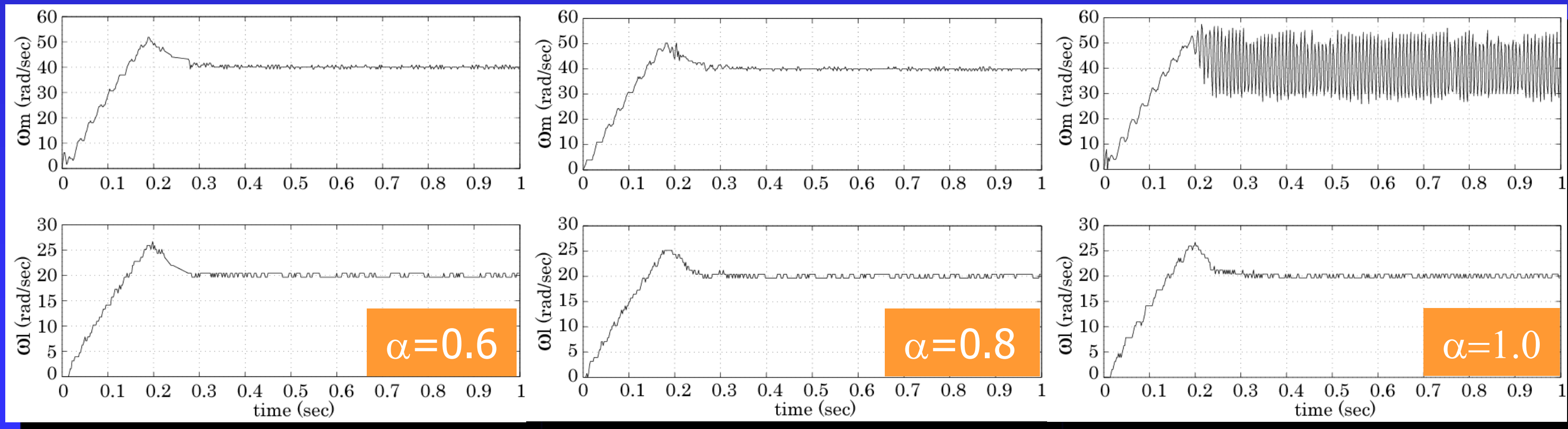




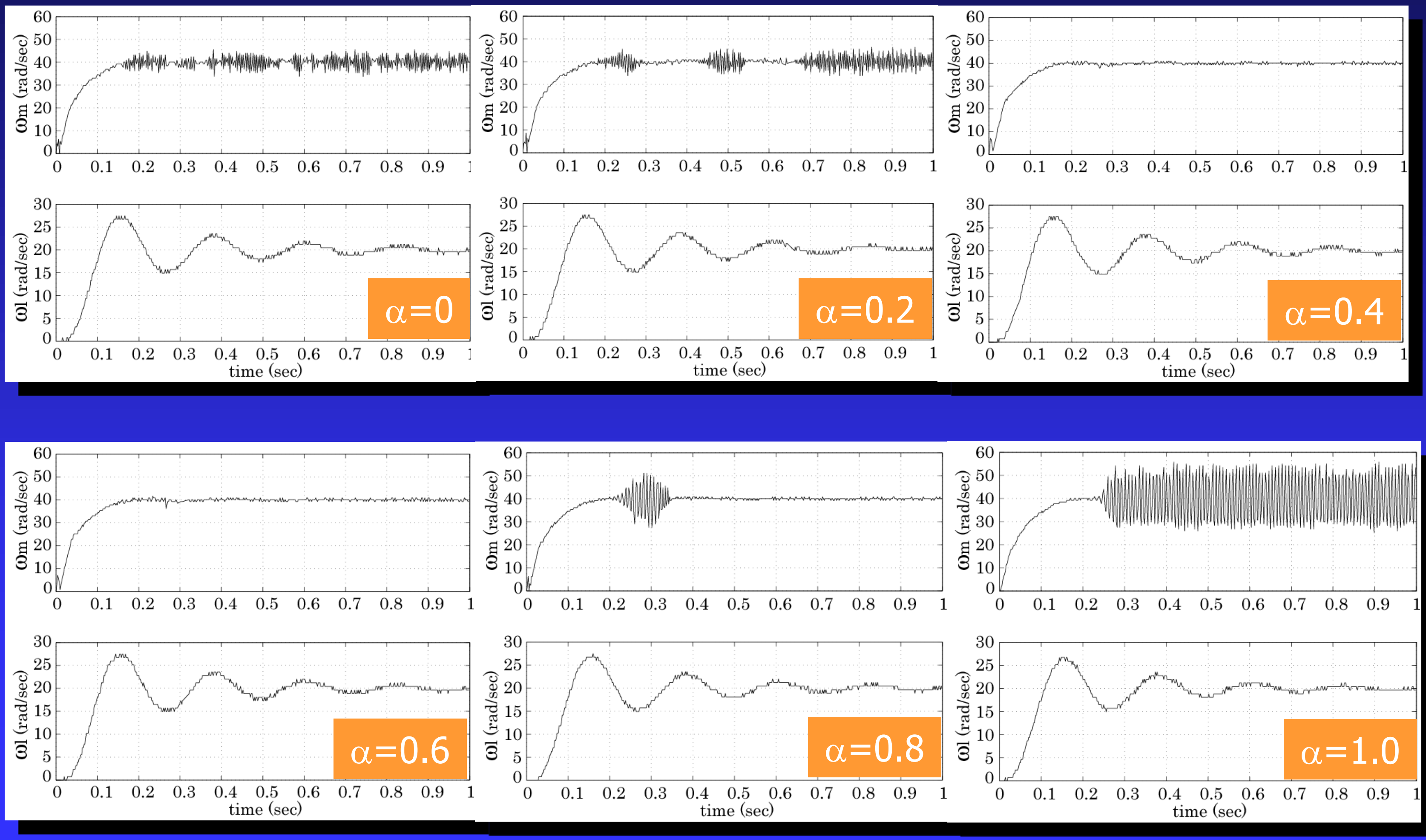
Controller of 1st-order: $0.2091\dfrac{(s+3092.4949)}{(s+646.7270)}$

Experiment: Load inertia variation (5 load flywheels)



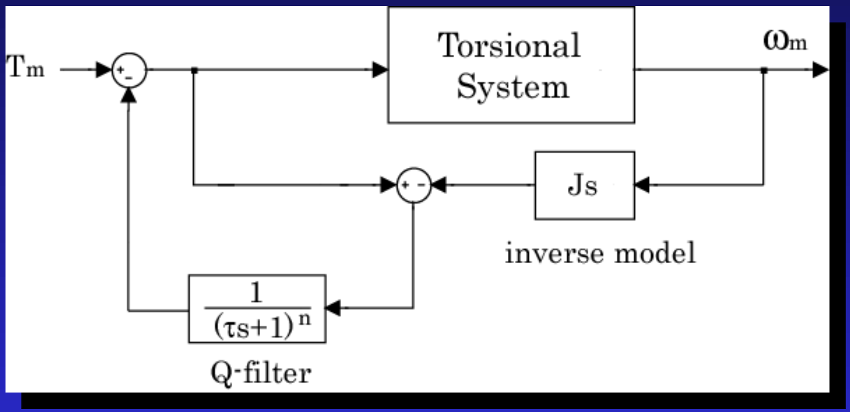


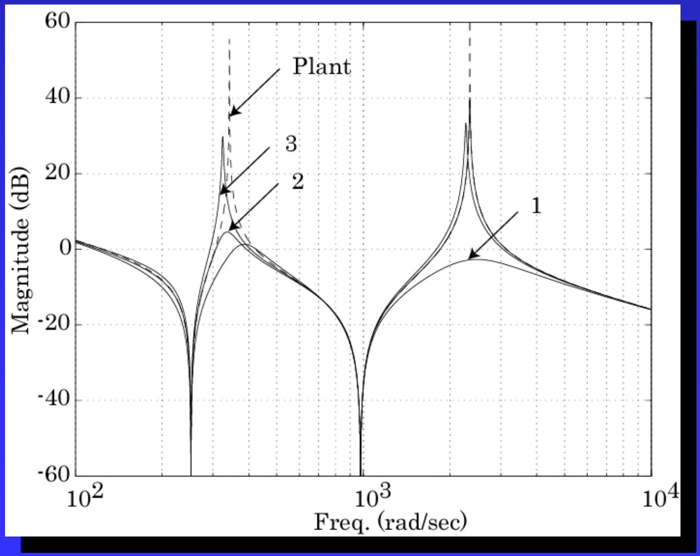
Experiment: Shaft elasticity variation (4mm shaft)



Fractional order disturbance observer

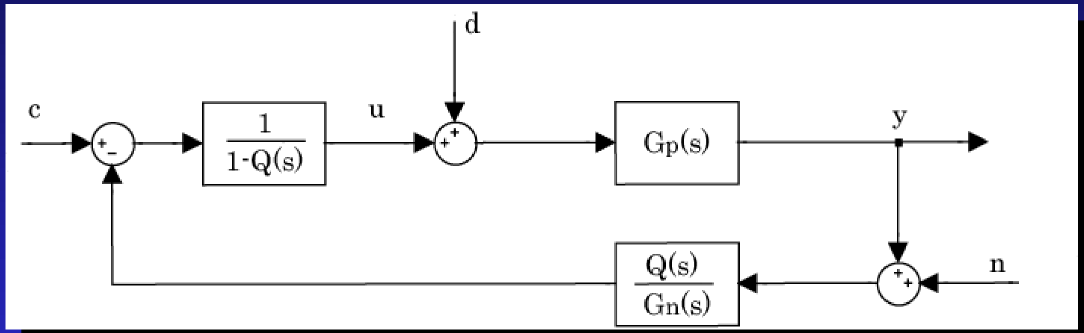
Conventional DOB





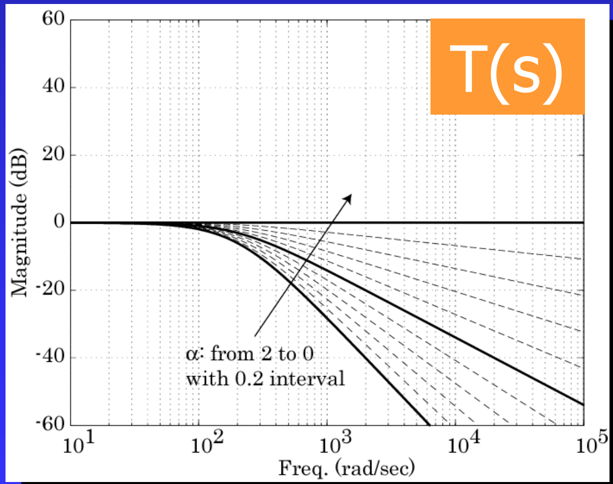
- Q-filter: a low-pass filter to restrict the effective bandwidth of DOB.
- DOB: a loop-shaping of adding more attenuation in the lower frequency range.
- Tradeoff: the reduced phase margin. Smaller n, better vibration suppression performance; however, poorer relative stability.

Robust stability

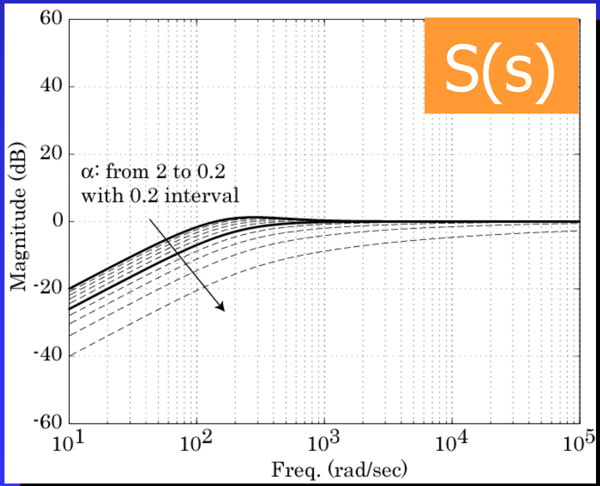


The open-loop transfer function

$$L(s)=\frac{Q(s)}{1-Q(s)}$$



Complementary sensitivity function



Sensitivity function

Q(s): complementary sensitivity function

1-Q(s): sensitivity function

An example

Multiplicative perturbation:

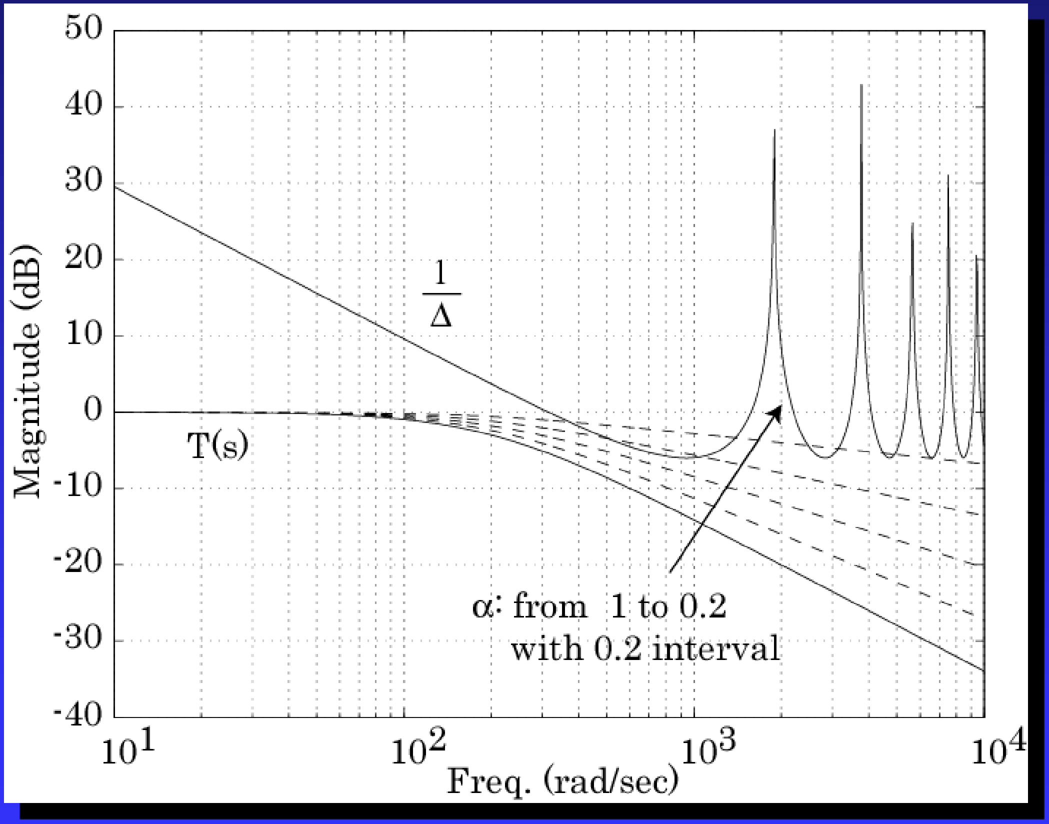
$$\Delta(s) = e^{-sT_d} - 1$$

Robust stability:

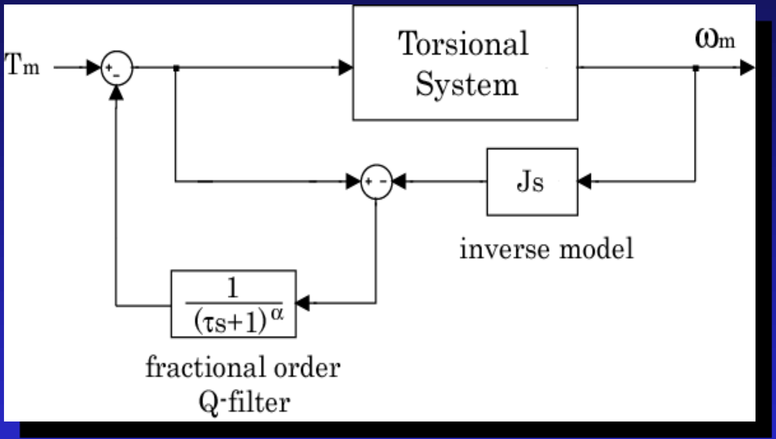
$$\left\| T(j\omega)\Delta(j\omega) \right\|_\infty \leq 1$$

Fractional Q-filter:

$$Q(s) = \frac{1}{(\tau s + 1)^{\alpha}}$$



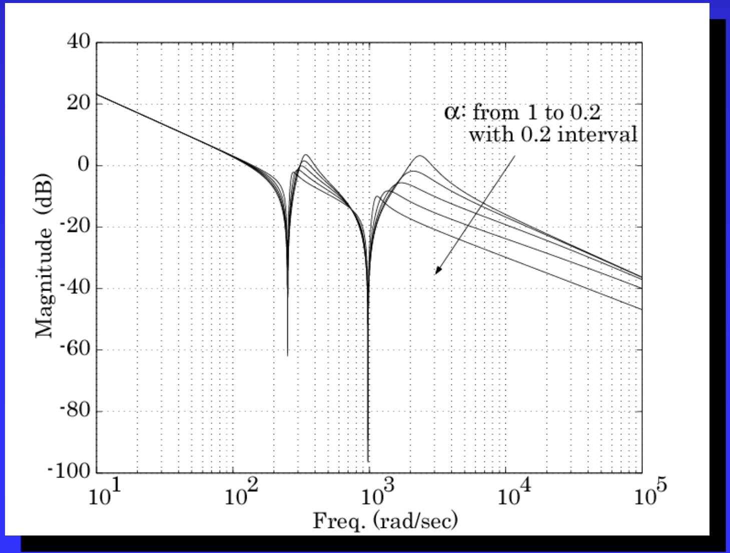
Direct tradeoff adjustment

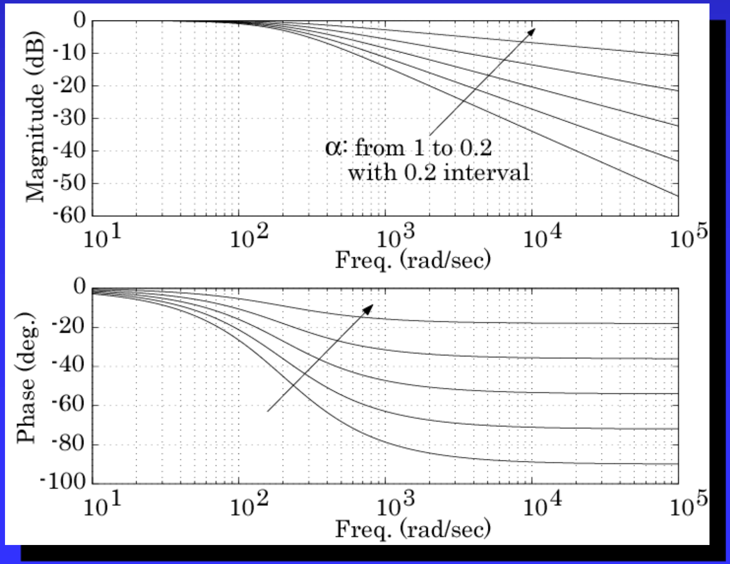


Fractional Q-filter

$$\frac{1}{(\tau s+1)^{\alpha}}$$

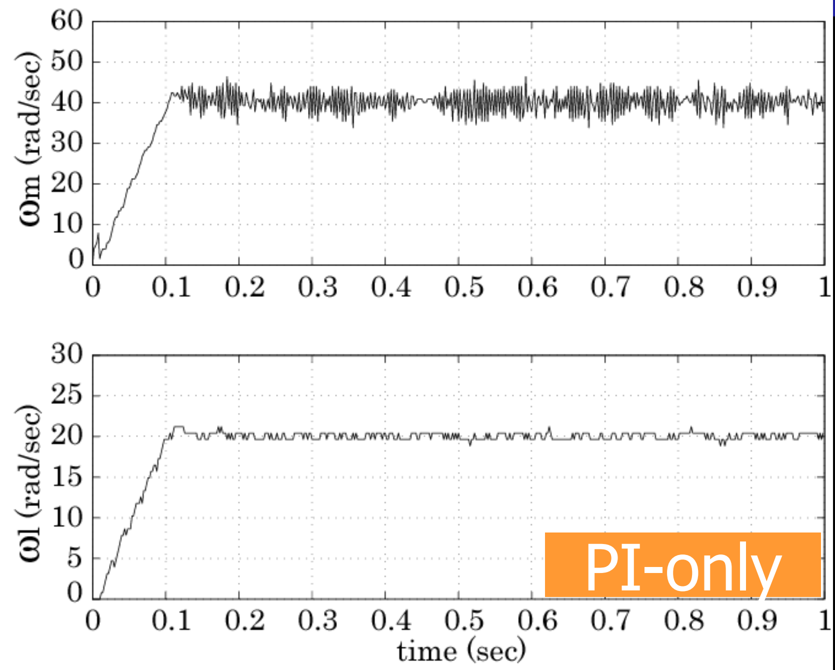
Any real number!

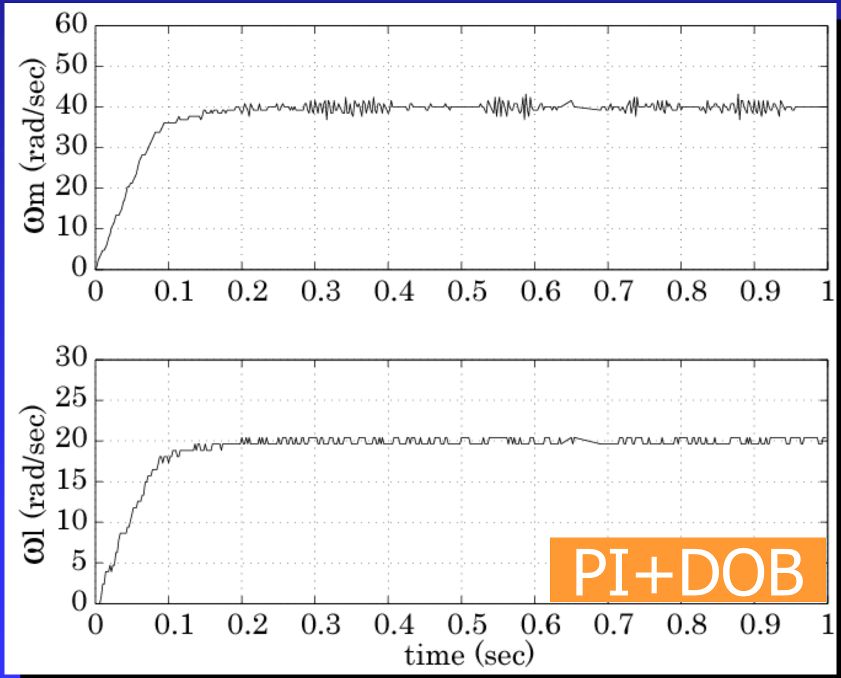




Experiment: PI+Conventional DOB

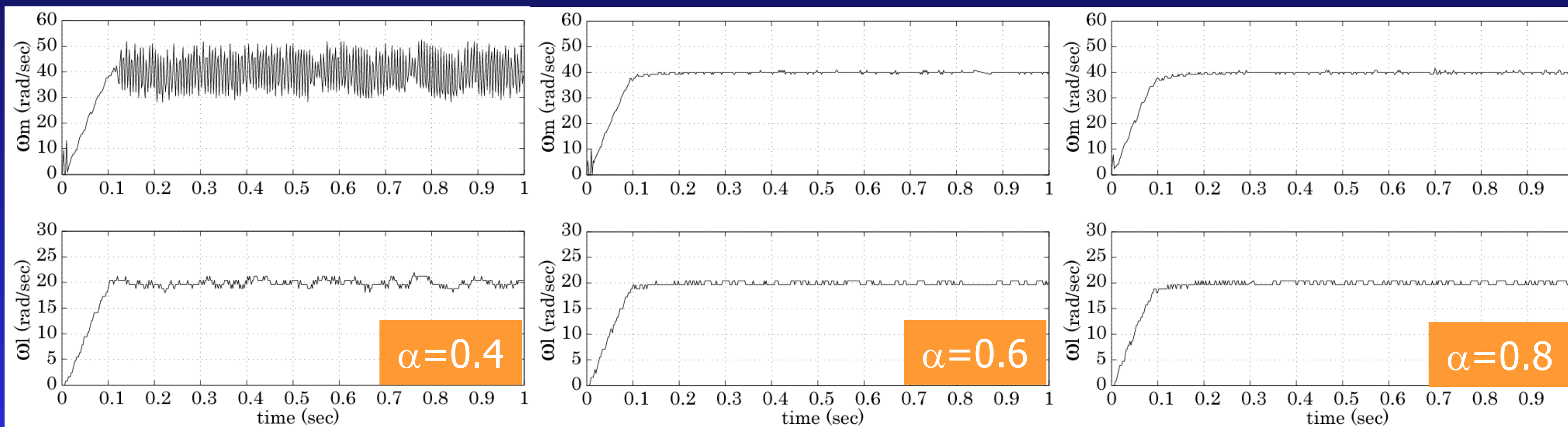
- Vibration suppression is improved but not enough yet.



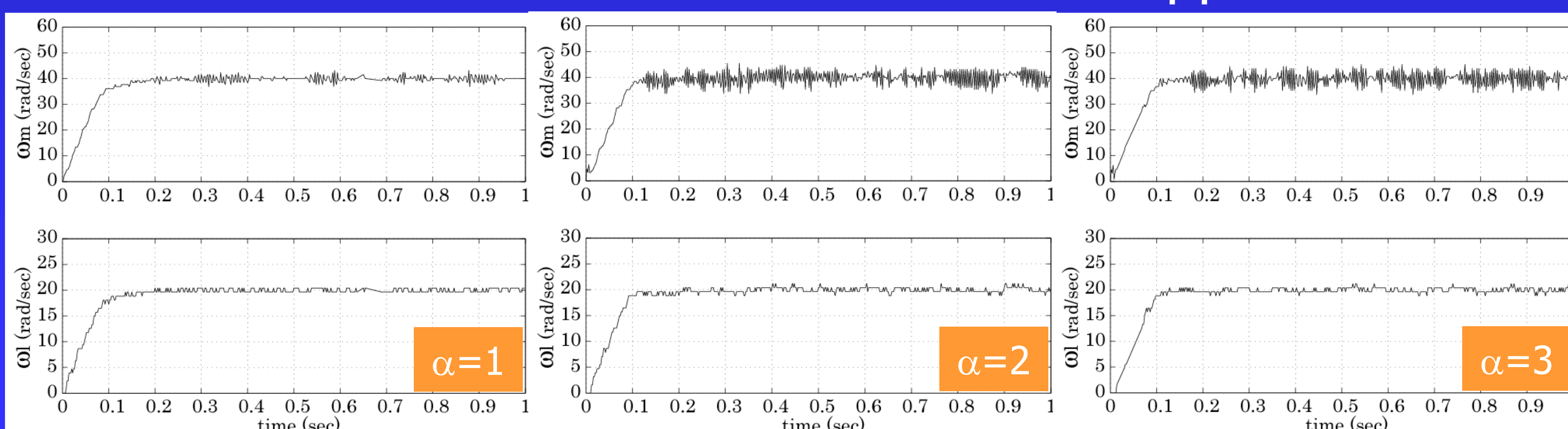


Experiment: PI+fractional order DOB

Poor robust stability

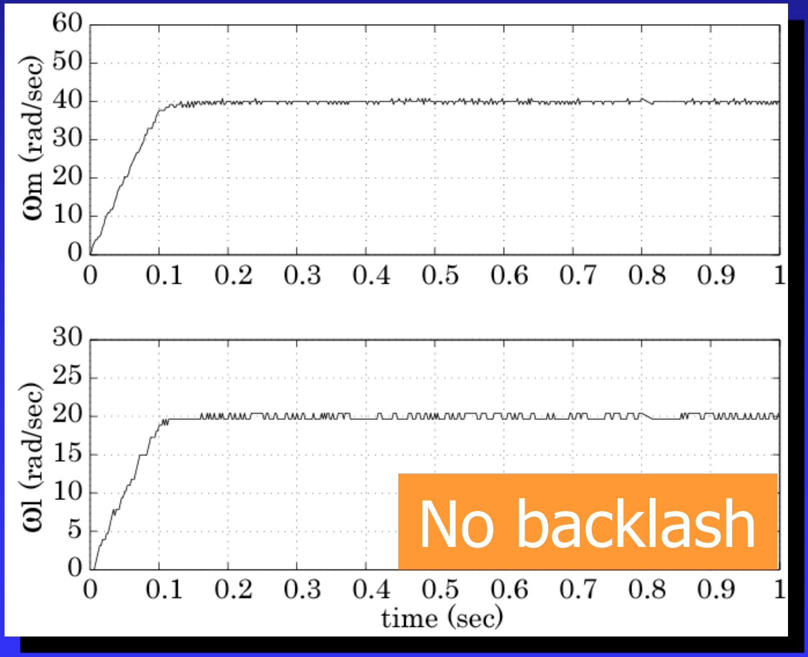


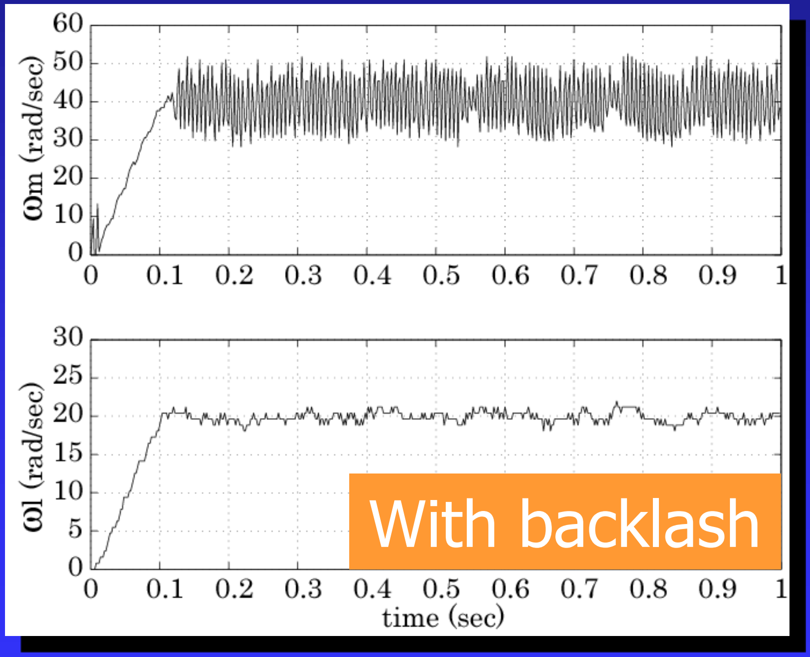
Poor vibration suppression



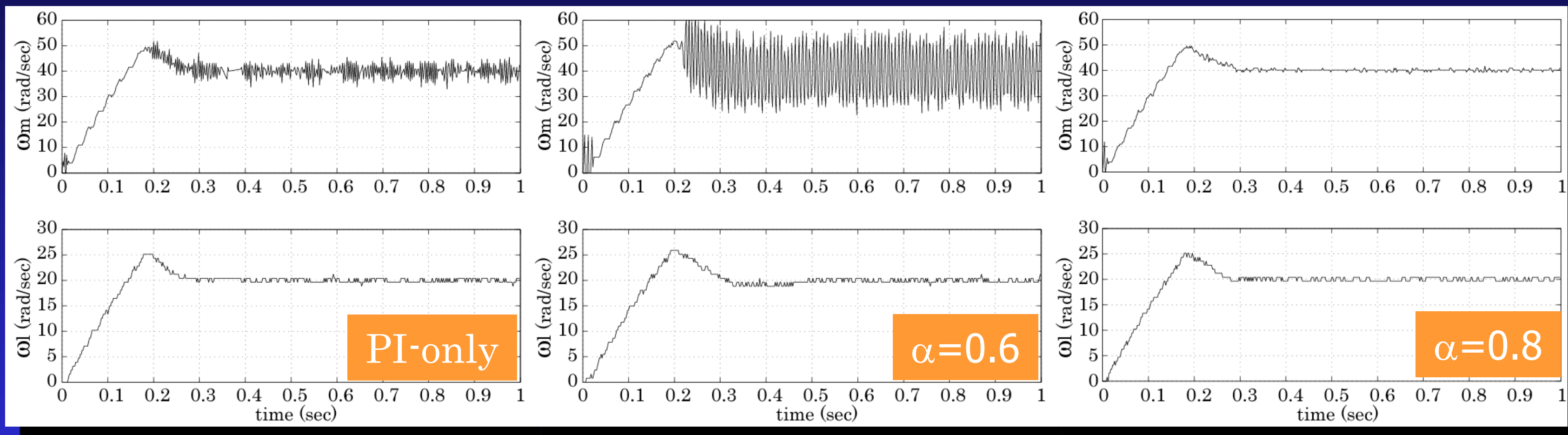
Experiment: 0.4 order Q-filter

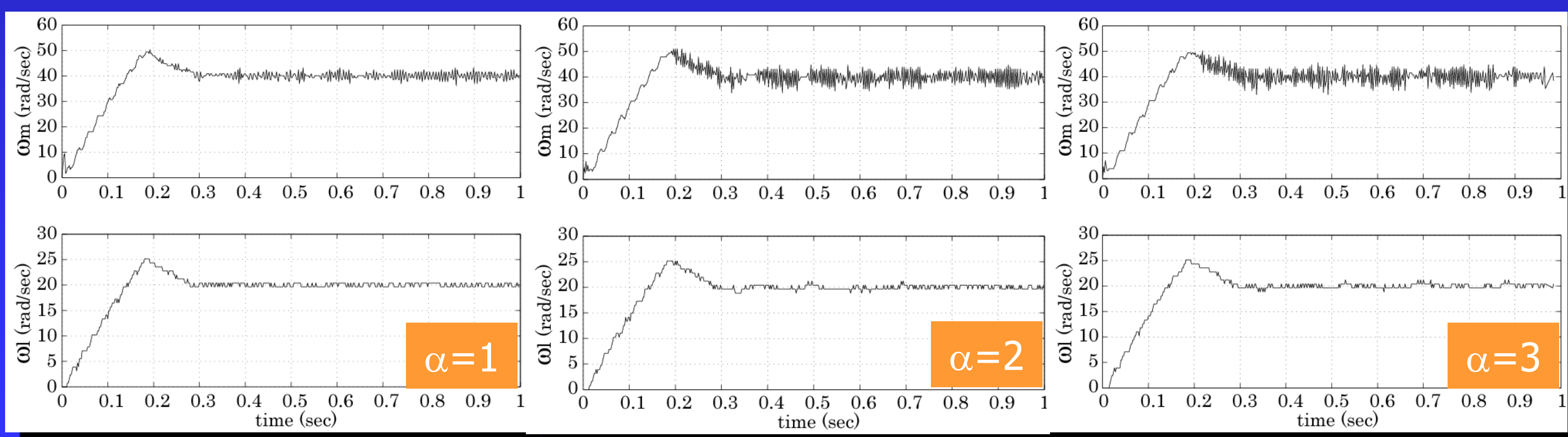
- Reason: bad robust stability



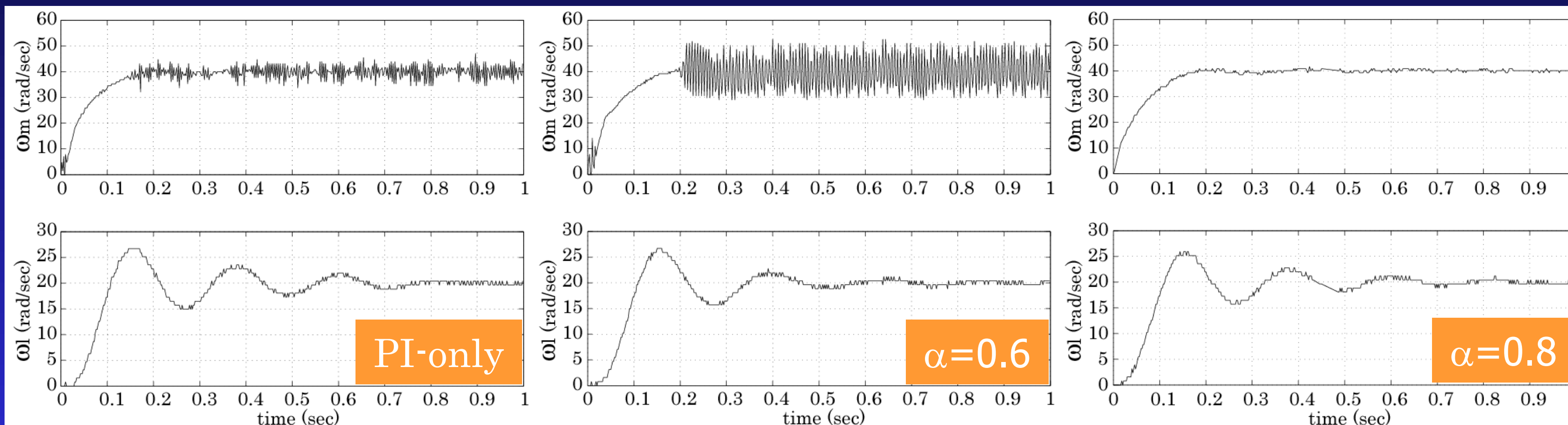


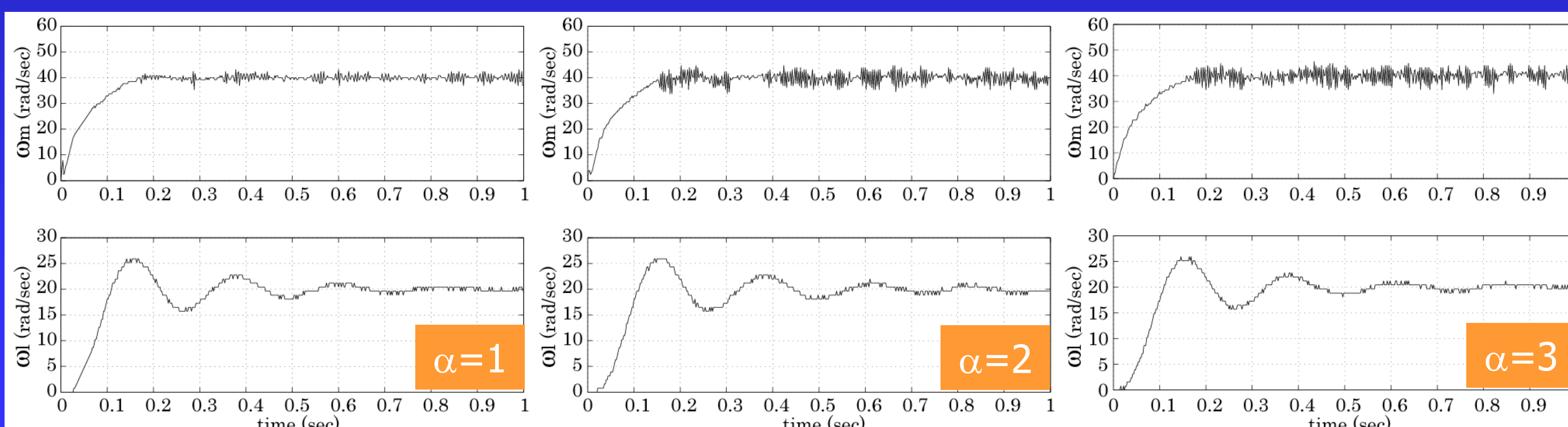
Experiment:Load inertia variation (5 load flywheels)





Experiment: Shaft elasticity variation (4mm shaft)





Section 3

- Brief Review
- My Contributions
- Conclusions & future works
- Publications

Position of FOC

- Theoretical position: FOC opened a new dimension for control theory. FOC is also a nice generalization of IOC theory.

- Practical advantages: "design by FOC and realize by IOC" are inevitable. The practical advantages for FOC is to provide more flexibility and insight in control design.

Unfamiliar but natural choice

- Modeling and identification: The dynamic features of “real” systems can be described more adequately by fractional order models.
- Control design: By introducing FOC, a better tradeoff between different prescribed control demands could be more easily obtained compared to conventional IOC approaches.

Clear-cut and effective design

- Powerful s^{α} operator: in FOC, the tuning knob can be reduced significantly compared to high-order transfer functions designed by conventional IOC approaches.

- Two-stage design approach: IOC design method gives a good sense of direction and novel FOC design method further improves control performance.

Reasonable realization

- Various approaches: Several realization methods were proposed for the realization of fractional order controllers.

- Reasonable approximation: The experimental results verify the reliability of fractional order controller's realization.

Future works …

- Applying FOC in MIMO system: using transfer function matrix should be an interesting research field.
- Fractional order z^{α} operator: Generalizing present digital control techniques based on FOC concept should be a quite challenging and meaningful research.
- Expansion of application field: FOC could be a general and effective approach with “in-between” characteristics. Especially, a human-friendly control for welfare control may be realized based on FOC.

Section 4

- Brief Review
- My Contributions
- Conclusions & future works
- Publications

Journals (4)

- Chengbin Ma, Yasumasa Fujii and Kenji Yamaji: China’s electric power sector’s options considering its environmental impacts, Vol. 5, Environmental Economics and Policy Studies, 2002, Springer-Verlag academic publisher (published)
- Chengbin Ma, Yoichi Hori, Backlash Vibration Suppression Control of Torsional System by Novel Fractional Order PID^k Controller, IEEJ Transactions on Industry Applications, , Vol.~124, No.~3, pp.~312-317, 2004 (published)
- Chengbin Ma, Yoichi Hori, Time-domain Evaluation of Fractional Order Controllers’ Direct Discretization Methods, IEEJ Transactions on Industry Applications (will be published in August, 2004)
- Chengbin Ma, Yoichi Hori, The Time-Scaled Trapezoidal Rule for Discrete Fractional Order Controllers, Nonlinear Dynamics, Kluwer Academic Publishers (accepted)

Conferences (8)

- 馬　澄斌，堀　洋一，「離散型非整数次制御器による軸ねじれ系の振動抑制制御効果の実験的評価」，平成15年産業計測制御研究会，pp.39-44, 2003年3月
- Chengbin Ma, Yoichi Hori, Design of Robust Fractional Order PI$^{\alpha}$D Speed Control for Two-inertia System, Japan Industry Applications Society Conference, Aug. 2003
- Chengbin Ma, Yoichi Hori, Geometric Interpretation of Discrete Fractional Order Controllers based on Sampling Time Sampling Property and Experimental Verification of Fractional $1/s^{\alpha}$ Systems' Robustness, 19th Biennial Conference on Mechanical Vibration and Noise, ASME Design Engineering Technical Conferences & Computers and Information In Engineering Conference, Sept. 2003, Chicago, Illinois, USA
- Chengbin Ma, Yoichi Hori, The Application of Fractional Order Control to Backlash Vibration Suppression, 2004 American Control Conference

Conferences (8)

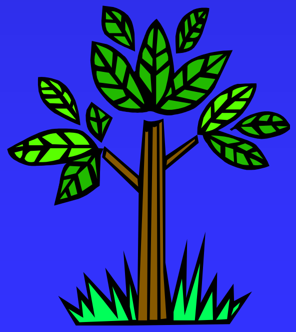
- Chengbin Ma, Yoichi Hori, Tradeoff Adjustment of Fractional Order Low-pass Filter for Vibration Suppression Control of Torsional System, First IFAC workshop on Fractional differentiation and its applications, Bordeaux, France, July 2004
- Chengbin Ma, Yoichi Hori: ``An Introduction of Fractional Order Control and Its Applications in Motion Control", International session, The 23nd Chinese Control Conference, August~10-13, 2004, Wuxi, China
- Chengbin Ma, Yoichi Hori, Design of Fractional Order PI$^{\alpha}$D Controller for Robust Two-inertia Speed Control to Torque Saturation and Load Inertia Variation, The 4th International Power Electronics and Motion Control Conference, Xi'An, PRC, August 2004
- Chengbin Ma, Yoichi Hori, Speed Control of Multi-inertia System by Fractional Order Control Approach, the 8th IEEE International Workshop on Advanced Motion Control, Kawasaki, Japan, March 25 - 28, 2004

Another impressive quotation for the end

"... We may express our concepts in Newtonian terms if we find this convenient but, if we do so, we must realize that we have made a translation into a language which is foreign to the organism which we are studying ..."

G. W. Scott Blair, Measurements of Mind and Matter, Dennis Dobson, London, 1950

Nature works with fractional time derivatives. With **fractional order calculus**, we may be able to extend a lot of new things …



Thanks! UT

2004

2003

2002

2001

2000

1999